PRIMARY VARIABLES INFLUENCING GENERATION OF EARTHQUAKE MOTIONS BY A DECONVOLUTION PROCESS

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In many engineering problems, the analysis of potential earthquake response of a soil deposit, a soil structure or a soil-foundation-structure system requires the knowledge of earthquake ground motions at some depth below the level at which the motions are recorded, specified, or estimated. A process by which such motions are commonly calculated is termed a deconvolution process.

This paper presents the results of a parametric study which was conducted to examine the accuracy, convergence, and stability of a frequency used deconvolution process and the significant parameters that may influence the output of this process. Parameters studied in included: soil profile characteristics, input motion characteristics, level of input motion, and frequency cut-off.

The results of the parametric study and examination of the theoretical formulation of the deconvolution process indicated the following:

1. The results of a deconvolution analysis are controlled by the transferr function between the input level and the level at which response is being calculated.
2. The transfer function is affected mainly by the following factors:
   a) Dynamic characteristics of the soil deposit under consideration;
   b) Level at which the input motion is specified;
   c) Characteristics of the input motion;
   d) The highest frequency (i.e., frequency cut-off) included in the computations.
3. The effects of the factors listed above are interdependent.

In particular, the results of the cases analyzed as part of this study and other cases emphasize the need for careful specification of the input motion and the depth at which the motion is imposed.

1. Introduction

Earthquake motions at a site result from a complex pattern of waves whose characteristics depend on many significant factors. Among these factors are the source mechanism, geologic conditions along the travel path between the source and the site, and the local site conditions. For engineering purposes, however, the motions in a soil deposit are often evaluated at the present time by analysis procedures which are based on the assumption that motions in the soil are due primarily to vertically propagating waves. Thus, for a deposit consisting of essentially horizontal soil layers overlying bedrock, horizontal ground motions can then be treated using theories applicable to the vertical propagation of shear waves through a semi-infinite medium. Mathematical models have been used to calculate the response of a soil deposit due to an input motion prescribed at the base of the deposit (i.e., soil-rock interface). In such cases, the results of an analysis for any specific deposit, with specific dynamic soil stress-strain characteristics, are unique and stable providing accuracy and stability considerations are correctly accounted for.

In the past few years, however, there has been a great deal of interest in computing motions at various points in a soil deposit due to a motion that is prescribed at a location other than the soil-rock interface. Most available recordings have been obtained at or near the ground surface. Therefore, in many instances it is desirable to evaluate the response throughout the soil depsoit using as control (input) motion the record obtained at or near the surface of the soil deposit during the earthquake. In addition, the present US NRC criteria [1] stipulate that a control motion be specified either at the ground surface or at some depth below the ground surface in the free field. Some methods of analyzing soil-structure interaction effects for nuclear plants require the determination of the spatial variations of ground motions within the soil. A deconvolution process for the deposit below the control point has been used for this purpose.

This paper examines the theoretical formulation of this deconvolution process and the parameters that influence the results through the analysis of a selected number of examples.

2. The Deconvolution Process

For a viscoelastic semi-infinite medium and considering only vertical propagation of shear waves, the equation of motion can be readily derived [e.g., 2, 3]. The displacement $u(x,t)$ at depth x and time t for a harmonic motion with frequency ω may be expressed as:

$$u(x,\ t) = E\ e^{i(kx + \omega t)} + F\ e^{-i(kx - \omega t)} \qquad (1)$$

where ω = circular frequency; $i = \sqrt{-1}$; k = complex wave number $= \sqrt{\frac{\rho\omega^2}{G^*}}$; G^* = complex shear modulus = $(G + i\omega\eta) = G(1+2i\lambda)$; λ = damping ratio; G = shear modulus; and η = viscosity coefficient.

The first term in eq. (1) represents the incident wave travelling upwards and the second term represents the reflected wave travelling downwards. E and F are constants that can be determined to satisfy the boundary conditions and continuity of stresses and displacements at all sublayer interfaces. Application of recursion formulas along with appropriate boundary and continuity conditions yield expressions for the m layer E and F (E_m and F_m) in term of those at the free surface (E_1 and F_1). In a similar way, the amplitudes at the top of another sublayer n can also be related to those at the free surface. Therefore; the relationships between displacement at the top of sublayer m and at the top of sublayer n can be readily obtained as follows:

$$A_{n,m}(\omega) = {Um}/{Un} \qquad (2)$$

The function $A_{n,m}$ (ω) is the transfer function between sublayer m and n. It can be readily calculated between any two sublayers in the semi-infinite medium once the sublayer thickness, h, and material properties (modulus, damping and mass density) are defined. If the motion is known in any of the sublayers of the medium, then the motion in any other sublayer can be calculated using eq. (2).

The transfer function is a complex variable indicating that there is a change in both amplitude and in phase-angle between any two points within the soil deposit. The change in amplitude $A_{n,m}$ (ω) is commonly referred to as the Amplification Function and the value of $|A|$ at a specific frequency is defined as the Amplification Ratio at that frequency.

The above discussion pertains to the case of steady state harmonic excitation of a point in a soil deposit. The theory can be readily extended to transient motion by decomposing this motion into a finite sum of harmonic motions, solving for each of these harmonic components in the frequency domain and then superimposing these solutions and transforming them back into the time domain to obtain the desired solution (e.g., 2, 3).

3. Cases Studied

A parametric study was conducted to examine the deconvolution process and factors influencing accuracy, convergence and stability of its results. The factors and their range of values examined in this study are summarized below.

Soil Deposits: Two soil deposits were considered. A relatively shallow soil profile consisting of 100 ft of sand overlying bedrock was used to represent a relatively stiff site condition. The second deposit consists of 300 ft of sand overlying bedrock; this deposit was selected to represent a relatively deep soil deposit. The maximum moduli assigned to each deposit are listed in Tables I and II, respectively. Variations of modulus and damping with strains were based on average published correlations for sands [4].

Input Motions: Three accelerograms were selected for use as input motions in the analyses. One accelerogram was an artificial time history whose spectrum provides a reasonable fit to the spectrum specified in US NRC Reg. Guide 1.60. The other accelerogram was the EW component recorded at the Caltech Athenaeum in Pasadena during the 1952 Kern County (California, USA) earthquake; the third accelerogram was the S69E component recorded at the Taft Lincoln School Tunnel during the 1952 Kern County earthquake.

The artificial accelerogram represents a time history having a relatively broad-banded spectrum. The Pasadena and Taft accelerograms are natural time histories and each has a relatively narrow-banded spectrum. The Pasadena record has a predominant period of approximately 0.65 sec and the Taft record a predominant period of approximately 0.35 sec.

Input Level: For the 100 ft profile, all analyses were conducted imposing the input motion at a depth of 10 ft below the ground surface. The input motion was imposed at the ground surface, at a depth of 20 ft, at a depth of 40 ft, or at the base rock interface (depth of 300 ft) for the 300 ft profile.

Frequency Cut-off: A frequency cut-off equal to either 10 cps or 25 cps was used.

A total of 48 cases were analyzed as part of this study.

4. Results of the Analyses

The analyses of all of the cases studied, were conducted using the computer code SHAKE [3]. The SHAKE code calculates the response of a horizontally layered, linear, viscoelastic system to an input earthquake motion by the method of wave propagation. Each soil layer is assumed to consist of a homogeneous isotropic material having thickness, h, mass density, ρ , shear modulus, G, and damping ratio, λ . The input earthquake motion in the time domain is transformed into the frequency domain and transfer functions are analytically determined for each soil layer in the viscoelastic continuum. The transfer functions and input earthquake motion are combined in the frequency domain and then transformed to the time domain. The strain-dependent nature of the shear modulus and damping in soils is accounted for by the use of equivalent linear soil properties [4].

The results of the analyses included computed peak acceleration and shear stresses at the top and middle of each sublayers of the soil deposit and the following:

a) Acceleration time histories and associated response spectra at the level of input motion, at the ground surface, at bedrock and at several selected depths below the surface of the soil profile. Figure 1 shows typical acceleration time histories and response spectra of the input motion and the computed motions at the surface and 40 and 300 ft (bedrock) below the ground surface when the artifical record is input at a depth of 20 ft and normalized to a peak acceleration of 0.10g.

b) Amplification ratios between the level of input motion and the surface $|A_{c,s}|$, input motion and bedrock $|A_{c,r}|$, and bedrock and the surface $|A_{r,s}|$. Figure 2 presents typical distribution of these amplification ratios with frequency for the artificial record normalized to 0.1 and 0.3 g and input at a depth of 10 and 20 ft of the 100 and 300 ft soil deposits, respectively.

The amplification ratios $|A|$ shown in Fig. 2 provide a convenient indication of how the amplitude of motions are varying from point to point. For example, the unknown amplitude of motion at the surface, s, is related to the known amplitude at the control depth, c, as $|U_s| = |U_c|\ |A_{c,s}|$. Similarly, the unknown amplitude at the base rock interface, r, is related to the known amplitude at the control depth as $|U_r| = |U_c|\ |A_{c,r}|$.

Note that the amplification ratio $|A_{r,s}|$ is the amplitude of the transfer function that is used to calculate the motion at the ground surface when the control motion is at the base rock interface (i.e., the usual upward propagation site response calculation). Also, note that this ratio is always a well-behaved function of frequency.

Examination of the data presented in Fig. 2 for the 100 ft deposit indicate:

1. The amplitude of the transfer function relating the control point and the ground surface (i.e., $|A_{c,s}|$) is always a well behaved function of frequency and has maximum peak that occurs at a frequency approximately equal to the fundamental shear frequency of the part of the deposit above the control point (in this case, the upper 10 ft of the deposit).

2. The amplitude of the transfer function relating the control point and the base rock interface (i.e., $|A_{c,r}|$) is well behaved for all frequencies when the input motion has a peak of 0.1 g. When the input motion has a peak of 0.3 g, $|A_{c,r}|$ is well behaved for all frequencies, but begins increasing rapidly for frequencies greater than approximately 18 cps.

Examination of the data presented in Fig. 2 for the 300 ft deposit indicate:

1. The amplitude of $|A_{c,s}|$ is also well behaved for all frequencies and its maximum peak occurs at a frequency corresponding to that of the fundatmental frequency of the part of the deposit above the control point.

2. The amplitude of $|A_{c,r}|$ is well behaved for all frequencies when the input has a peak of 0.1 g. It becomes very large for frequencies longer than approximately 3 to 5 cps and excessively large for frequencies longer than 8 cps when the input has a peak of 0.3g. The latter suggests that this deposit cannot transmit these higher frequencies when the input has such a large peak.

5. Conclusions

The intent of this study was to examine trends and reasonableness of the results of a deconvolution analysis as various key parameters are changed. The study was not tailored to provide design guidelines and the results presented in this paper should be viewed only in relative terms. Nevertheless, the theoretical considerations of the deconvolution process and the results of the cases analyzed provided the means to draw a series of conclusions. Among the most pertinent of these conclusions are the following:

1. The results of a deconvolution analysis are controlled by the transfer function between the input level (or control point) and the level at which response is being calculated.

2. The transfer function is affected mainly by the following factors:

   a. Dynamic characteristics of the soil deposit under consideration; these characteristics are controlled by: (i) the total depth of the soil profile; and (ii) the dynamic material properties of the soils comprising the deposit; of particular importance are the strain-compatible modulus and damping values used in the last interation (note that these values are affected by other factors such as those listed in conclusions 2b, 2c and 2d below).

b. Level at which input motion is specified.

c. Characteristics of the input motion (in particular, frequency content and maximum peak acceleration).

d. The highest frequency included in the computations (this is commonly referred to as the frequency cut-off).

3. Peak acceleration and spectral values (especially for frequencies higher than the frequency cut-off) are significantly affected by the frequency cut-off used in the deconvolution analysis. However, peak shear stresses are not much affected, especially in the upper half of the deposit, by this frequency cut-off.

4. The effects of frequency cut-off, dynamic material properties and characteristics of the input motion are inter-dependent. Of particular concern is the attempt to deconvolute a time history through a deposit that, because of its dynamic material properties, is incapable of transmitting frequencies higher than a certain range. In such a case, high amplitudes of motion are required at depths below the control point to recreate the control motion. These high amplitudes of motion induce high shear strains, which in turn require that the strain-compatible moduli be decreased. As these moduli decrease, the ability of these parts of the deposit to transmit higher frequencies is reduced further again leading to the need for an increase in the amplitudes of motion to reproduce the control motion, etc. On occasions, such analyses could result in unrealistically high motions computed with depth or could result in an unstable solution. The results of such analyses should be discounted.

5. Because of the effects discussed in conclusion (4), it is necessary that attention be paid to the selection of the input motion and the frequency cut-off be consistent with the dynamic characteristics of the soil deposit under consideration.

6. Except when the input motion is specified at the ground surface or at the base rock interface, spectra of motions calculated at various depths by the deconvolution analysis are dominated by the period corresponding the fundamental shear period of the deposit above the control point.

7. When the input motion is specified at the ground surface, spectra of motions calculated at depths below the surface have valleys (which are significant when the input has a broad-band spectrum) that occur at periods corresponding to the

fundamental shear period of the part of the deposit above this depth.

In particular, the results of the cases analyzed as part of this study and other cases emphasize the need for careful specification of the input motion and the depth at which the motion is imposed. This specification must be guided by the frequency transmission characteristics of the soil deposit under consideration.

ACKNOWLEDGEMENT

The support given by the Professional Development Committee of Woodward-Clyde Consultants and by Ebasco Services to this study is gratefully acknowledged.

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Table I - MATERIAL PROPERTIES FOR 100 FT DEPOSIT

Sublayer No.	Ave Depth Below Ground Surface - ft	Max Shear Modulus - ksf	Properties Used in Last Iteration[a]		Properties Used in Last Iteration[b]	
			Shear Modulus - ksf	Damping - %	Shear Modulus - ksf	Damping - %
1	2.5	985	755	5.1	543	9.1
2	7.5	1706	1208	6.2	769	11.7
3	12.5	2202	1583	6.0	997	11.6
4	17.5	2605	1957	5.4	1245	11.0
5	24	2922	2169	5.6	1333	11.5
6	32	3176	2149	6.8	1336	12.5
7	40	3412	2244	7.1	1231	14.0
8	48	3633	2360	7.3	1136	15.3
9	56	3841	2361	7.9	1126	15.8
10	65	4062	2369	8.5	1202	15.7
11	75	4294	2513	8.5	1311	15.5
12	85	4515	2755	8.0	1443	15.1
13	95	4725	2840	8.2	1507	15.1

(a) Control Motion at 10 ft depth; peak acceleration = 0.1 g

(b) Control Motion at 10 ft depth; peak acceleration = 0.3 g

Table II - MATERIAL PROPERTIES FOR 300 FT DEPOSIT

Sublayer No.	Ave Depth Below Ground Surface - ft	Max Shear Modulus - ksf	Properties Used in Last Iteration[a]		Properties Used in Last Iteration[b]	
			Shear Modulus - ksf	Damping - %	Shear Modulus - ksf	Damping - %
1	2.5	985	740	5.4	540	9.2
2	7.5	1706	1098	7.4	666	13.3
3	15	2412	1431	8.3	747	15.3
4	25	2955	1722	8.5	878	15.7
5	35	3267	2003	8.0	1029	15.2
6	45	3552	2334	7.1	1231	14.4
7	55	3815	2197	8.7	1078	16.2
8	70	4180	2176	9.9	1000	17.7
9	90	4621	2563	9.1	1163	17.3
10	110	5024	2753	9.2	1018	19.0
11	130	5397	2920	9.3	936	20.0
12	150	5746	3287	8.7	579	22.9
13	170	6074	3319	9.2	364	24.8
14	190	6386	3204	10.4	383*	24.8*
15	212.5	12000	7228	7.7	691*	24.8*
16	237.5	12628	6845	8.9	727*	24.8*
17	262.5	13227	6590	9.9	762*	24.8*
18	287.5	13800	4680	14.2	795*	24.8*

(a) Control Motion at 20 ft depth; peak acceleration = 0.1 g

(b) Control Motion at 20 ft depth; peak acceleration = 0.3 g

* Strain in sublayer significantly greater than 10% reaching several thousand percent in sublayer 18.

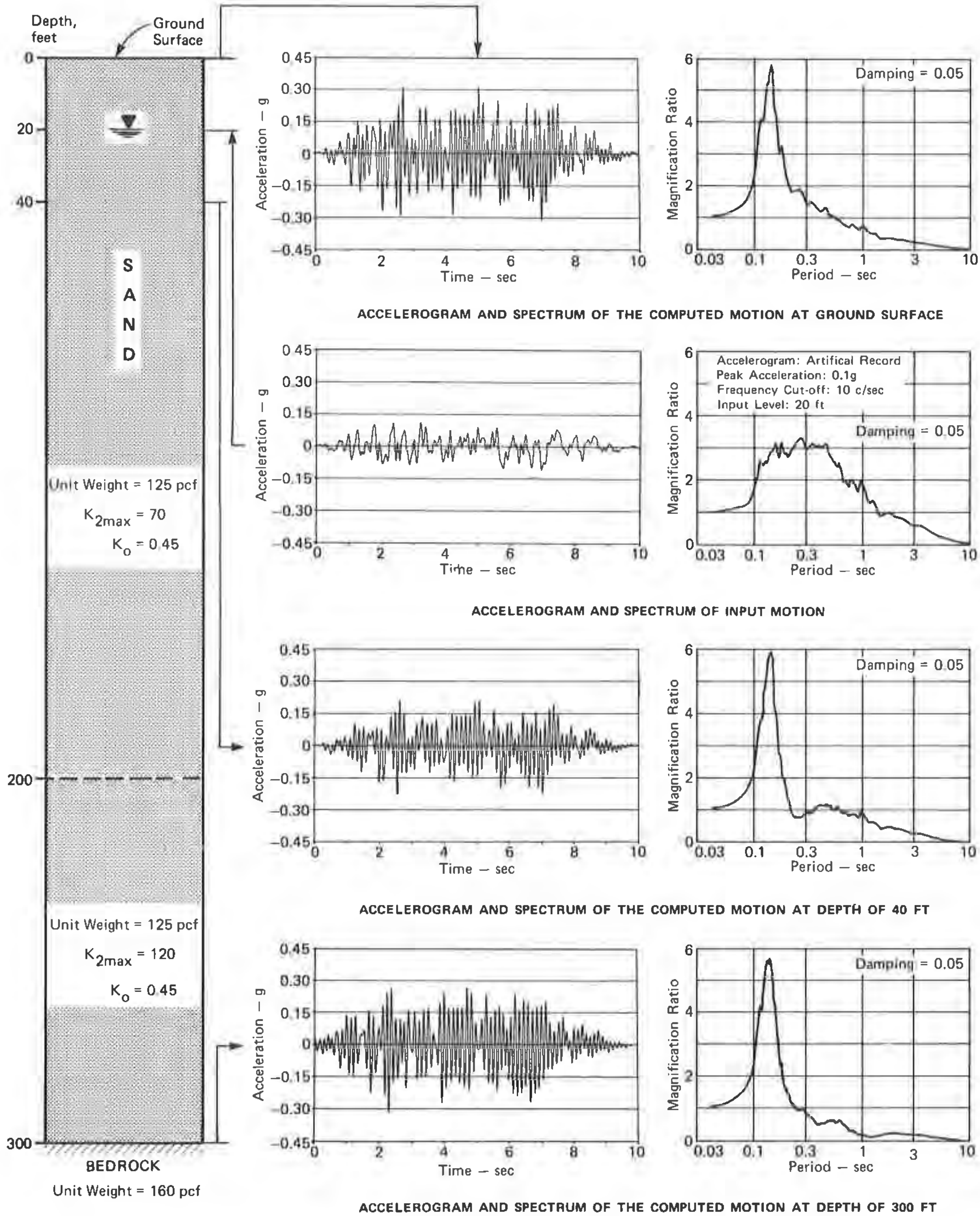

Fig. 1 Typical Computed Acceleration Time Histories and Response Spectra at Various Depths in the Soil Deposits.

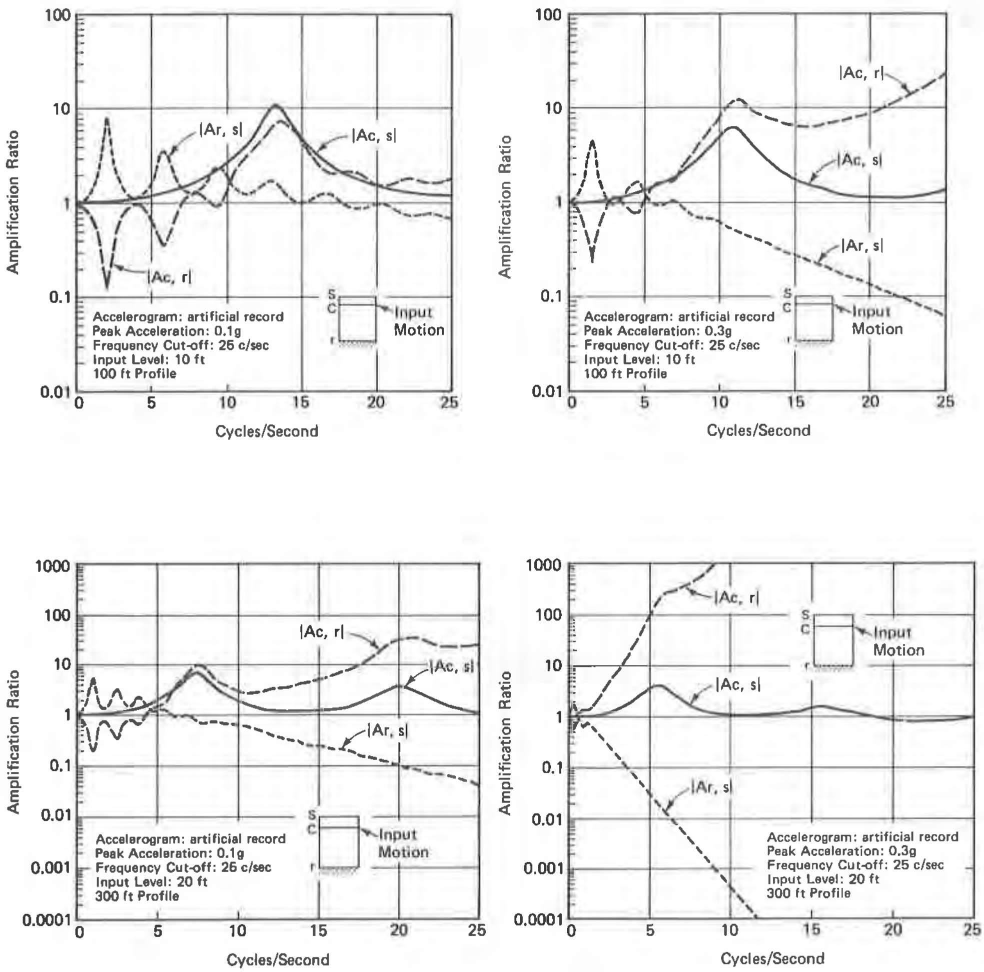

Fig. 2 Typical Computed Transfer Functions Between the Level of Input Motion and the Ground Surface, Input Motion and Bedrock, and Bedrock and Ground Surface.