

## A New Edge Loaded Fracture Mechanics Pipe Specimen

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### ABSTRACT

The demand for pressurizing pipes to determine their  $K_{IC}$  values was pointed out during the SMIRT-5 post conference on fracture. However, special hydraulic setups are required for such testing and special precautions have to be taken to account for the explosions (large energy released) from steel pipes.

In this work a specimen which is a short length of the pipe loaded at the edges is presented. The problem of a short cylinder with radial and axial loads at the inner diameter is first investigated. Different outer to inner diameter ratios are studied to determine the stress distribution without any precracks to obtain the range of applicability of such specimens. Closed form solution is presented for thin walled specimens while for thick ones finite element analysis is used. The tangential component of stress  $\sigma_{\theta}$  at the mid plane simulates the effect of internal pressure while the axial stress component  $\sigma_z$  represents the end constraints.

With the presence of a crack on the compressive side of  $\sigma_z$  the problem becomes a three-dimensional one. Superposition together with two-dimensional special crack tip finite element analysis are used to determine the load-stress intensity relation. In the experimental setup the loading is made by two cones on a compression machine. The load versus crack opening displacements are recorded to test if a "valid" test is obtained.

This specimen has the advantage of being made on a regular compression machine with no special hydraulic or explosion precautions and with loading that simulates the service loading of the pipe.

Valid test results have been obtained with thick cylinders ( $t/D > 0.4$ ). For thin ones large scale deformation at the ends appears and stable rather than unstable crack propagation is more pronounced.

## INTRODUCTION

It has been demonstrated elsewhere [1,2,3] that it is important to determine the fracture toughness of pipes from specimens cut off from the required pipes. The difference in manufacturing techniques between the pipe and specimens made from the same pipe material may affect the critical stress intensity factor. Moreover, the geometry and kind of loading could affect the plastic zone size and hence the  $K_Q$  value obtained.

Clifton et al [3] determined  $K_{IC}$  for rocks & PMMA using internally pressurised thick walled pipes with an internal radial crack. Jones solved the above problem by presenting [4,5] two specimens cut from cylinders thereby keeping the same properties as the produced pipe. The first is a 3-point curved bend specimen cut from a segment of the pipe with a crack on the concave side. The other is a compact tension specimen machined off the thickness. The later specimen is limited to the thickness and curvature of the pipe. In the above mentioned work the authors used either finite element results [6], modified mapping collocation [7] or theoretical analysis [8] to determine the  $K_I$ - load relation.

In reference (1), the authors compare several specimen geometries cut off the pipe. The best specimen would be a precracked pipe internally pressurised since such a specimen simulates closely the service conditions. However, the following simple computations show that special precautions are necessary to perform such a test.

Consider a thin walled precracked cylinder made of a steel with  $K_{IC} = 110 \text{ MN}/\text{M}^{3/2}$  having the following dimensions  $D_o = 50 \text{ mm}$ ,  $D_i = 45 \text{ mm}$ , length  $L = 150 \text{ mm}$  and crack depth = 1.25 mm. Then using the results from reference [1],  $K_I/K_o = 2.4$ .

Hence the maximum bursting pressure which is obtained from the relation ;

$$q_{\max} = \frac{2t K_I}{2.4 D \sqrt{\pi c}} \quad (1)$$

would be 740 atm. This pressure is very large and special precautions in the design of the pressuring system are required.

Moreover, the energy released from the fracture of the pipe material would be 10.3 NM. The energy released from the pressuring fluid may be estimated using a Bulk's modulus  $K$  of the fluid of  $K = 2.06 \text{ GN}/\text{M}^2$ . The amount of energy in the fluid would then be 376.7 NM.

That is the energy released from the pressurising fluid is 36 times that released from the pipe material and is close to an explosion.

## Scope of Work

In this work, a new edge loaded radially cracked specimen is proposed (Fig.1). This specimen is cut from the pipe and axially loaded through two cones. This specimen may be used on a compression machine. It is required to determine the geometry of the specimen and the cone angles " $\alpha$ " such that the tangential stress  $\sigma_\theta$  is similar in form to that obtained

from internal pressure. Moreover, it is required to keep the distribution of stress as uniform as possible over the gage length to be as close as possible to plane strain conditions.

A two-dimensional finite element fracture mechanics [9,10] computer program may thus be used to determine the relation between the load and stress intensity factor. That is, it is required to determine the length  $L$  & the ratio of  $F_r/F_a$  to give the stress distribution for the uncracked pipe close to that of an internally pressurised cylinder over a significant part of the cylinder length. The cone angle " $\alpha$ " may thus be obtained by solving the relations ;

$$F_r = \frac{P}{\pi D_i} \frac{(\cos \alpha - \mu \sin \alpha)}{\sin \alpha + \mu \cos \alpha} \quad (2)$$

$$F_a = \frac{P}{\pi D_i}$$

A large spectrum of  $D_o/D_i$  ratios has been investigated ranging from very thin to  $D_o/D_i = 5$ .

#### Thin Walled Cylinders Edge Loaded

The problem of a thin walled uncracked cylinder radially loaded at the edges is a classical thin shell theory problem<sup>[11,12]</sup>. It may be shown that the circumferential bending moment due to a radial edge load may be given by ;

$$M = \frac{p}{\lambda} e^{-\lambda x} \sin \lambda x$$

where

$$\lambda = 4 \sqrt{\frac{3(1-\nu^2)}{R^2 h^2}} \quad (3)$$

The maximum bending & membrane tangential stresses at the inner & outer fibers may be given by ;

$$\sigma_{in \max} = \frac{E F_r}{2R D \lambda^3} e^{-\lambda x} \cos \lambda x + \frac{6 \nu F_r}{h^2 \lambda} e^{-\lambda x} \sin \lambda x \quad (4)$$

$$\sigma_{out \max} = \frac{E F_r}{2R D \lambda^3} e^{-\lambda x} \cos \lambda x - \frac{6 \nu F_r}{h^2 \lambda} e^{-\lambda x} \sin \lambda x$$

where

$$D = \frac{E t^3}{12(1-\nu^2)}$$

Figure (2) gives the distribution of the tangential stresses at the inner & outer fibers for a specific long thin walled cylinder of  $D_i = 144$  mm and  $D_o = 156$  mm.

It may be observed from figure (2) that due to the term  $e^{-\lambda x} \sin \lambda x$  that the tangential stress quickly decays & becomes compressive. Precracking such a specimen beyond ( $x/D_i = 0.2$ ) from the edge will not give successful results since the crack will be present in a compressive field. Then for such a case a very short specimen must be used if it required to have a positive tangential stress. Moreover, plane conditions for such a short

specimen cannot be achieved. An edge longitudinally cracked specimen may be more successful for this case.

Thick Walled Cylinders Edge Loaded

No closed form solution of a thick uncracked cylinder radially or axially edge loaded is available in the literature to the best knowledge of the authors. Hence an axisymmetric general purpose<sup>[9,10]</sup> finite element computer program was used to determine the stress distribution in a cylinder due to an edge radial load and axial load separately. Several ratios of  $D_o/D_i$  were investigated.

For a given ratio of  $D_o/D_i$  a certain length was chosen and a finite element solution for the edge radial load  $F_r$  & the axial load  $F_a$  is obtained separately. Several ratios of  $F_a$  to  $F_r$  are superimposed & the resulting stress distribution obtained. The original chosen length may then be changed and the same procedure repeated until the optimum length and ratio of  $F_a/F_r$  are obtained. The suitable crack geometry may be determined such that the crack tip lies in a field close to that of an internally pressurized pipe under plane strain conditions.

The relation between the stress intensity factor  $K_I$  & the applied load may hence be obtained by the two dimensional finite element program using procedure similar to that discussed in reference [1].

That is an equivalent pressure  $q$  is obtained using the relation for an internally pressurized cylinder ;

$$\sigma_{\theta} = \frac{q R_i^2}{R_o^2 - R_i^2} \left[ 1 + \frac{R_o^2}{R^2} \right] \tag{5}$$

such that the stress distribution from equation (5) is as close as possible to that obtained from the superimposed finite element solutions of the axial & radial edge loads. Once  $q$  is determined, the same procedure as in reference (1) may be used to determine the relation between the stress intensity factor  $K_I$  & the applied compression load  $P$ .

Results & Discussion

Results for one case are presented here. For a given ratio of  $D_o/D_i = 2$ , several finite element solutions for different  $L/D_i$  ratios were investigated. Figure 3 & 4 give the stress distributions obtained due to a radial  $F_r$  and an axial load  $F_a$  for the case of  $L/D_i = 2$ . Note that in figure (3) the stress distribution  $\sigma_{\theta}$  has a negative part close to the mid-plane due to the horizontal force component. The  $\sigma_z$  distribution in figure 3 is close to that of pure bending. In figure 4, however, the tangential stress due to the vertical load is positive and increases towards the mid-plane.

Several ratios of  $F_r/F_a$  were superimposed to try to obtain the optimum conditions. Figure "5" shows the ratios  $F_r/F_a$  of 1 : 2 superimposed. This ratio seemed to be the best

with the given specimen dimensions to give the closest distribution to that of an internal pressure in the zone close to the mid-plane. The stress distribution for an equivalent pressure of  $8 \text{ N/mm}^2$  is also shown on figure 5 for comparison near the mid-plane.

The suitable crack dimensions may thus be chosen such that the crack tip lies in a region as close as possible to that of the internal pressure. The final geometry and crack dimensions and cone angle are given in table 1. The cone angle  $\alpha = 57.7^\circ$  was determined using equations "2" and a coefficient of friction  $\mu = 0.1$  (boundary lubrication).

Table (1)

$D_o/D_i$	$L/D_i$	$l/D_i$	$c/D_i$
2.0	1.6	0.4	0.2

From reference [1] for  $c/t = 0.4$  and  $D_o/D_i = 2$ ,

then  $\frac{K_I}{K_o} = 1.6$  where  $K_o = \frac{2 q R_i^2}{R_o^2 - R_i^2}$  (6)

For this case using an equivalent pressure of  $8 \text{ N/mm}^2$ , then

$$K_o = 150 P \sqrt{c} \quad \text{where } K_o \text{ is in } \text{MN/M}^{3/2}, \quad P \text{ in MN, } c \text{ in M.} \quad (7)$$

Hence from (6) & (7)  $K_I = 240 P \sqrt{c}$  (8)

Equation (8) gives the relation between the stress intensity factor  $K_I$  & the load  $P$ .

It is important to note here that a complete finite element solution is required for each trial. An optimization technique is necessary instead of the trial and error technique used here.

#### CONCLUSION

A new edge loaded fracture mechanics pipe specimen is proposed in this study. The stress distribution obtained from the edge loading is close to that obtained from the internal pressure. Hence such a specimen has the advantage of testing the pipe in the "as is" condition on a simple compression machine without the need of pressurizing a precracked pipe, in which special testing facilities are required.

The optimum geometry for each  $D_o/D_i$  ratio has been determined using finite elements. The results for  $D_o/D_i = 2$  are presented in this paper. An optimization technique is required to minimize the effort in obtaining the suitable dimensions for each pipe diameter ratio instead of the trial and error technique used here.

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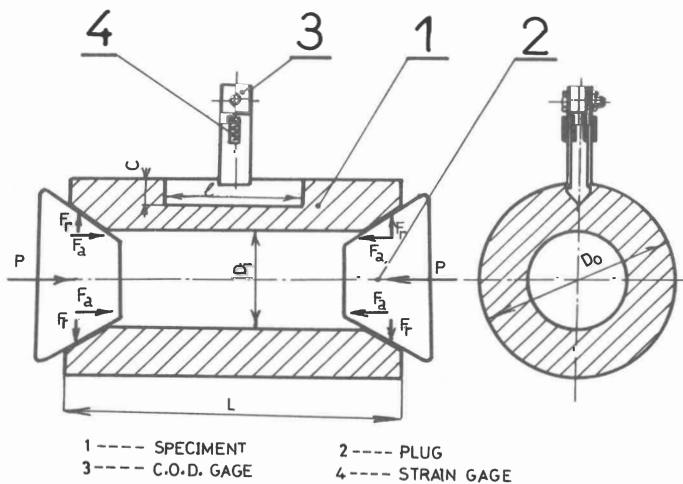


Figure (1) Proposed edge loaded fracture toughness pipe specimen

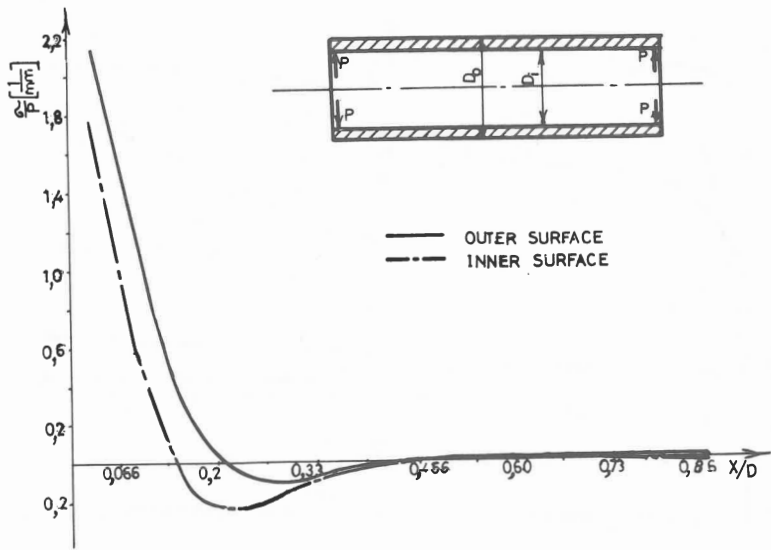


Figure (2) Outer and inner tangential stresses in a thin walled edge loaded specimen

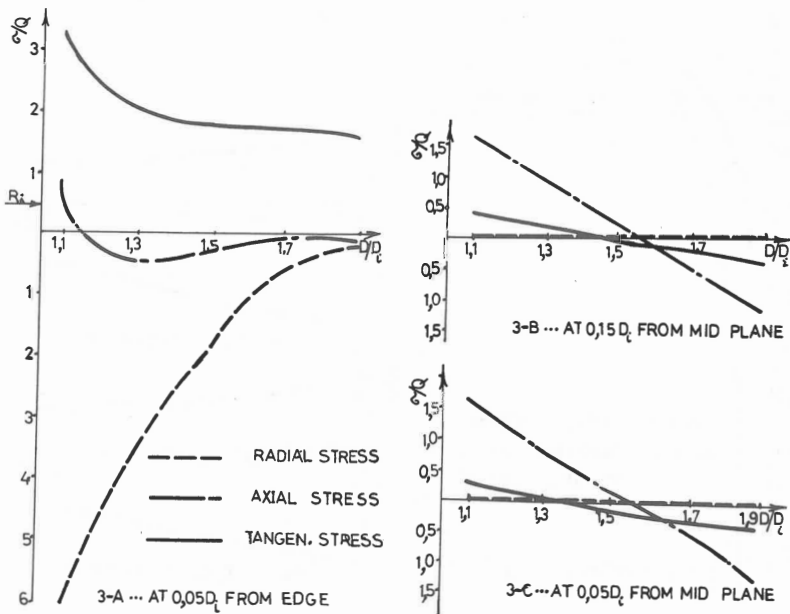


Figure (3) Stress distribution due to a radial edge load  
 $(F_r = 1000N/mm)$  ,  $D_o/D_i = 2$  ,  $L/D_i = 1.6$

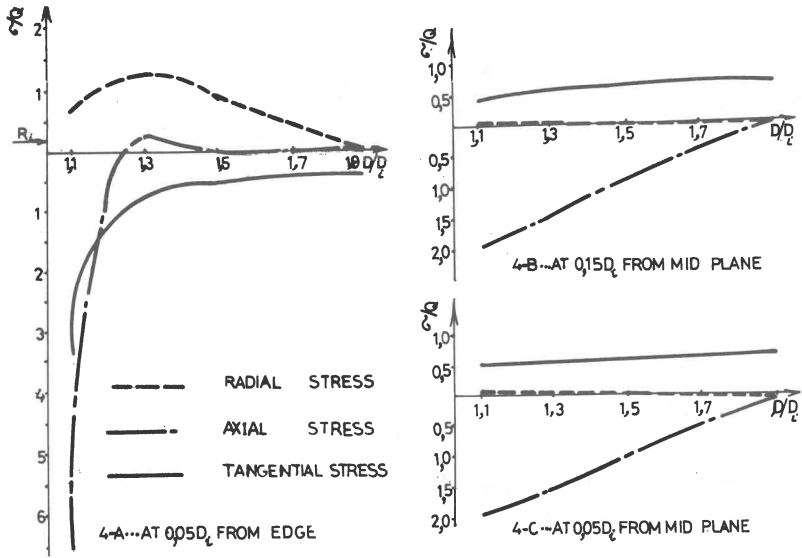


Figure (4) Stress distribution due to an axial edge load  
 $(F_a = 1000\text{N/mm})$ ,  $D_o/D_i = 2$   $L/D_i = 1.6$

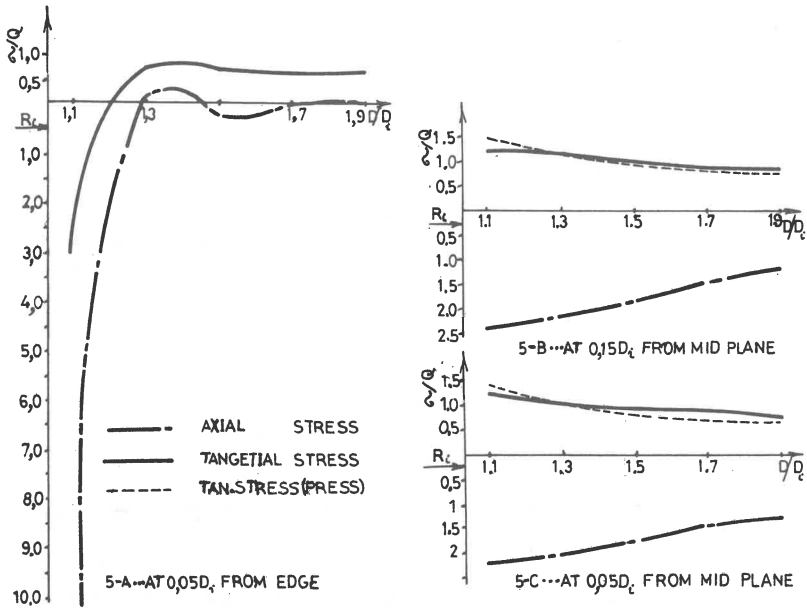


Figure (5) Resultant stress distribution for  $(F_r + 2 F_a)$ ,  $D_o/D_i = 2$   
 $L/D_i = 1.6$ , and equivalent internal pressure  
 $q = 8\text{-N/mm}^2$ .