Fatigue analysis of crack like geometrical discontinuities

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ABSTRACT

Practical rules exist for fatigue analysis, except in the case of crack like discontinuities. A practical rule which uses the design curves contained in design codes is described. This rule enables to predict the experimental results. It can also be used for analysis of small cracks.

1 INTRODUCTION-PURPOSE OF THIS PAPER

Cyclical loading (in particular thermal shocks) can cause the initiation of crack and their growth. Current practice in reactor technology is to prevent the initiation of fatigue cracks. The design codes contain rules for checking whether cracking has started during the life time of the component.

Fatigue cracks are especially likely to appear in areas of stress concentration (stress raisers). The rules in the design codes apply in particular to these areas, with one exception. This exception concerns geometrical design features in the form of a crack (see figure 1). As stress and deformation concentration is infinite at the tip of these crack like discontinuities, normal calculations would indicate that the initiation of cracking was immediate. This is not physically the case.

The purpose of this paper is to explain a practical method of assessing the number of allowable cycles, while avoiding the initiation of a crack. This method is covered by a rule in the French design code for fast breeder reactor components (RCC-MR) (1). It is developed in more detail in the report (2) and therefore we give only the brief description here .

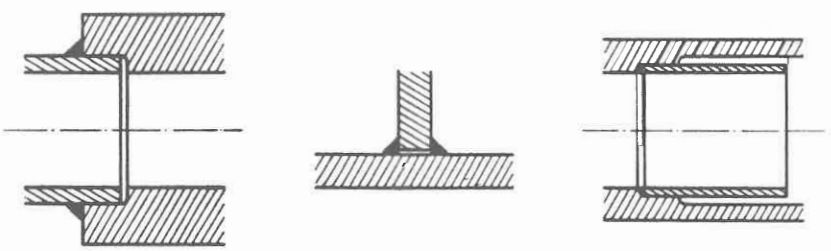

Fig.1-Examples of Crack like Discontinuities

2 BASES OF THE METHOD PROPOSED

2.1 Criterion for initiation of cracking

It is assumed that the number of cycles at which the crack may start depends on the strain range equivalent $\Delta\varepsilon$ .This assertion confirms the results of the work of Coffin (3) and Manson (4)

2.2 Large gradient of strain range-Characteristic length of the material

The above assertion applies when $\Delta\varepsilon$ is more or less uniform in the area considered. When this condition is not met and $\Delta\varepsilon$ varies considerably in the area considered, this is no longer the case. A practical rule has been suggested by Peterson (5). Peterson suggests that in the case of fatigue at a high number of cycles, we should base the analysis not on the value on the wall ,but on the value existing at a finite distance d from the wall. A particularly good description of the method is given by B.Langer (6):" A somewhat different concept has been described by Peterson" (5). The hypothesis is that fatigue failure does not occur when the peak stress reaches the endurance limit, but rather when the endurance limit is reached at some finite distance d,below the surface. The rationale for this hypothesis is that,due to the granular structure of the material, a finite volume of the material must be at a stress equal to the endurance limit in order to cause failure. The dimension d is more nearly a true material property than is q , and has been found to be smaller for high strength metals than for low strength metals. The available test data indicate that for pressure vessel steels conservative fatigue strength reduction factors will be obtained if a d = 0.002 in is used".

The only innovation here is to extend this method to low cycle fatigue.

2.3. Effect of cyclical plasticity

If the behaviour of the material were elastic, the value of $\Delta\varepsilon$ at a distance d would easily be determined by the stress intensification factor . If the variation in this factor is ΔK_I ,we would have $\Delta\varepsilon_o = \Delta K_I / E\sqrt{2\eta d}$.

This is not justified as the tip of the crack like discontinuity is the centre of cyclical plastic deformations. Luckily, in the case of small scale yielding, the solution is already well known (7).It is called HRR singularity and in this case (small scale yielding) produces the Neuber rule (8)

$$\Delta\varepsilon.\Delta\sigma = E\,\Delta\varepsilon_o^{\,2} = (\Delta K_1)^2 / 2\eta E d$$

in which $\Delta\sigma$ and $\Delta\varepsilon$ match on the cyclical curve .

2.4 Safety margin with respect to initiation

This three bases enable us to calculate $\Delta\varepsilon$ and assess the number of admissible cycles. Curves are provided in the design codes giving the number of cycles permissible with respect to the strain range $\Delta\varepsilon$.

These design curves include a margin with respect to the experimental best fit curves. This margin is not a true safety margin. It takes into account the scatter of the results as well as other effects, and in particular non-detected defects. For crack like discontinuities, this latter effect can be ignored and the margin can be reduced (if there is no corrosion phenomenon). This margin of two on $\Delta\varepsilon$ can ,for

example, be reduced to 1.4. This is the same thing as dividing $\Delta\varepsilon$ by 1.4 before using the design curves giving the number of permissible cycles.

3 DESCRIPTION OF THE PRACTICAL RULE PROPOSED

3.1.General definition

The method is completely described in the above paragraph. From a general point of view, it can be expressed as follows "calculate the strain range equivalent to a distance d from the tip of the crack like discontinuity. Divide it by 1.4 in order to use it in the design curve giving the number N of cycles permissible" .

3.2 Step by step description

It is ,however,useful to detail the consecutive stages in a more practical fashion :

a Calculate the variation in the stress concentration factor ΔK_I

b From it,deduce the elastic strain range $\Delta\varepsilon_o = \Delta K_1 / E\sqrt{2\pi d}$ in which E is the Young's modulus and d the characteristic distance from the material

c Make the plasticity correction to obtain the true strain range $\Delta\varepsilon$:

$\Delta\varepsilon\Delta\sigma = E(\Delta\varepsilon_o)^2$

($\Delta\varepsilon$ and $\Delta\sigma$ match on the material cyclical curve)

d Include $0.7\,\Delta\varepsilon$ in the design curve to determine the number of permissible cycles N .

3.3. Comments

-Where stresses are also modes II and III, allowance should be made in the calculation of $\Delta\varepsilon_o$.

-If the crack like discontinuity is somewhat blunt, this can be taken into account (radius ρ) .For example :

$$\Delta\varepsilon_o \sim (1 \frac{\rho}{4d}) \Delta K_I / E \sqrt{2\pi d}$$

-Thus,as indicated previously, d is of the order of 0.05mm for steels (except for high elastic limit steels). Some writers state that an approximation for steels is $d = (100/\sigma_Y)^2$, d being expressed in mm and proof stress in MPa .

4 VALIDATION OF THE PRACTICAL RULE

Validation consists in comparing the experimental results and this method. The experimental results (9) (19) give the number of cycles N on initiation for a known material, shape and loading. These data enable us to calculate $\Delta\varepsilon$ using the above method and the choice of d . We can then include $\Delta\varepsilon$ as a function of N in the best fit curve for the material. For a given material, it is always possible to determine d such that scatter does not exceed the normal scatter for tests on smooth samples .

Figure 2 gives the relationship between N and $\Delta\varepsilon_o$ for steel 316L with d=0.05mm. Clearly the relationship is highly satisfactory .

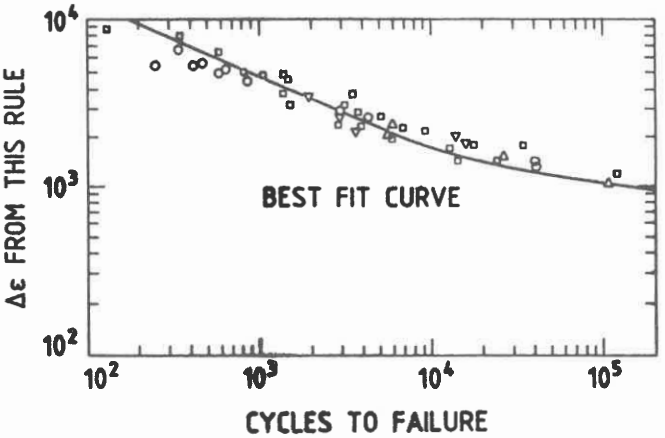

Figure 2- Validation of the method

5 USE FOR ANALYSIS OF SMALL CRACKS

The method can be applied to small cracks. The essential difference is that the strain range is not produced solely by the feature defined by ΔK. In fact, for a small crack of a short length a, ΔK_I only has a significant value if the variation in stress ΔS is high. As a result, we must also take into account the effect of the variation ΔS, i.e. at the distance d from the tip of the crack :

$$\Delta\varepsilon_o = \frac{\Delta S}{E} + \frac{\Delta K_I}{E\sqrt{2\pi d}} = \frac{\Delta S}{E}\left(1 + F\sqrt{\frac{a}{2d}}\right) = \frac{\Delta K_I}{E\sqrt{2\pi d}}\left(1 + \frac{1}{F}\sqrt{\frac{2d}{a}}\right)$$

(in which F is a coefficient usually around one). This is the value to which plastic correction must be applied. This can be written $\Delta\varepsilon_o = (1 + \sqrt{2d/a}/F)\,\Delta K_I / E\sqrt{2\pi d}$. (suggesting to correct Paris Law in multipliing ΔK_I by $(1 + \sqrt{2d/a})$.

If we limit ourselves to high cycle fatigue, this correction is of no use and we note that when a = 0 (no crack $E\,\Delta\varepsilon_o$) is double the endurance limit. When the crack is deep (a/d much greater than one) $E\,\Delta\varepsilon_o$ is equal to $\Delta K_{th}/\sqrt{2\pi d}$ (ΔK_{th} being ΔK threshold). We can eliminate $E\,\Delta\varepsilon_o$ and d , with gives :

$$\frac{\Delta S}{2L} + \frac{\Delta K}{\Delta K_{th}} = 1$$

as an initiation condition. We can also write :

$$\frac{\Delta S}{2L}\left(1 + \frac{2FL\sqrt{\pi a}}{\Delta K_{th}}\right) = 1$$

in order to express the variation in stress causing initiation as a function of the length of the crack (relation which depends on the parameters 2L and ΔK_{th} . The comparison of Figure 3 with the results of (12) is clearly suitable.

Note that this defines d by $\Delta K_{th} = 2L\sqrt{2\pi d}$

6 CONCLUSIONS

A practical method for fatigue analysis of crack like discontinuities has been established. It is limited to crack initiation .

It uses known elements as a basis : stress intensification factors, strain range at a characteristic distance d from the discontinuity tip ,plastic correction.

This method has been validated by comparison with the experimental results available.

It can also be applied to the analysis of small cracks where it produces results similar to those of the experiment.

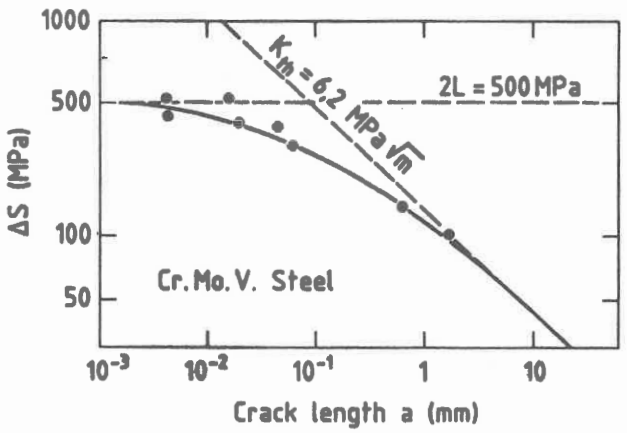

Figure 3-Small crack behaviour

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