Application of Post–Buckling Theory to HVAC Duct Design

Sayed H. Stoman
Bechtel Power Corporation, Pottstown, PA USA

ABSTRACT

Conventionally, HVAC duct spans range from 8'-0" to 10'-0". However, a utilization of duct panel post-buckling strength allows the use of significantly larger spans for Seismic Class I Rectangular Ducts. Duct behavior is better described when sheet panel post-buckling behavior is taken into consideration. The thin panel due to its large h/t or w/t ratio is unable to remain fully effective during the entire loading history. As the loading is applied, it may undergo local instability due to either excessive compressive stress, shear stress, or a combination thereof. However, such local instability does not constitute overall failure as duct panel post-buckling behavior is stable. Duct corner chords in conjunction with the transverse reinforcing stiffeners and the web panels develop a pratt truss-like behavior, capable of maintaining post-buckling stability. Once bifurcation of the web due to shear has occurred, the tension-field action in the web develops a band of tensile forces. Equilibrium is maintained by the transfer of stress to the transverse stiffeners and the adjacent panel chords. As a result of the application of the post-buckling method, HVAC hangers required by a conventional analysis can be reduced by up to 50 percent.

1. INTRODUCTION

HVAC duct systems are essential to the power industry. Given construction schedule constraints and the modern day demand for efficiency and cost-effectiveness, it is imperative that new, refined methodologies be developed to realistically analyze and optimally design Seismic Class I HVAC Ducts.

The conventional design approach based solely on linear-elastic laws and non-utilization of duct post-buckling strength is costly and unattractive. The actual capacity of ducts is much more than that predicted by a linear-elastic analysis. Its true behavior is better described by a nonlinear analysis that takes into consideration duct panel post-buckling strength or membrane action. Experimental studies have verified the adequacy of ducts using thinner sheet metal with height-to-thickness, h/t, or width-to-thickness, w/t, ratios of up to 1500 (Desai and Stoman, 1989). At these slenderness ratios, the duct panel is unable to remain fully effective during the entire loading history. With increasing load it may undergo local instability due to excessive compressive stress, shear stress or a combination thereof. However, such local instability does not constitute overall failure as duct panel post-buckling behavior is stable and its ultimate strength is very high.

2. POST-BUCKLING BEHAVIOR

The behavior of a rectangular duct with transverse stiffeners, under "beam shear," is very similar to plate girders or tension-field beams. Buckling of the web panels may occur at loads well below the ultimate. After buckling, the duct panels behave in a manner similar to that of a Pratt truss, with "corner chords" acting as truss chords, the transverse stiffeners as verticals and the webs as tension diagonals. So long as yielding has not precipitated, the duct will return to its original shape after the removal of the load.

Buckling of the duct web occurs perpendicular to the direction of the principal compressive stresses due to shear. However, as in plates stiffened by flanges and stiffeners, web panels of transversely stiffened rectangular ducts can carry shear loads considerably greater than their buckling load. As shown in Figure 1, elastic or inelastic buckling of plates under pure shear gives critical shear stress as illustrated by line ABCD. Plates appropriately stiffened by flanges and stiffeners may have their shear strength raised to near the shear yield in classical beam theory - indicated by line ABE. The cross-hatched area in the figure represents panel post-buckling strength (Basler, 1963 and Salmon and Johnson, 1980).

Similarly, since duct webs are stiffened, they are capable of developing the post-buckling strength. Once bifurcation of the web due to shear has occurred, the tension-field action in the web develops a band of tensile forces. Equilibrium is maintained by the transfer of stress to the transverse stiffeners and the adjacent panel chords - see Figure 2.

3. ANALYTICAL APPROACH

The ultimate shear strength of a duct panel may be expressed as

$$(V_u)_{\text{ultimate}} = (V_{cr})_{\text{buckling}} + (V_{tf})_{\text{tension-field action}} \tag{1}$$

The buckling strength may be expressed as a function of panel critical shear stress, τ_{cr}, as

$$V_{cr} = \tau_{cr} A_w = \tau_{cr} ht = \tau_y ht\ (\tau_{cr} / \tau_y) \tag{2}$$

where τ_y represents the panel shear yield stress. Defining a coefficient $C_v = \tau_{cr}/\tau_y$ and, using the Huber-Von Mises-Hencky "energy of distortion" theory, since shear yield stress is related to duct sheet tension-compression yield stress F_y by the relation $\tau_y = F_y / \sqrt{3}$,

$$V_{cr} = F_y ht\ C_v / \sqrt{3} \tag{3}$$

Using the plate girder analogy (Salmon and Johnson, 1980), the post-buckling strength of the duct panel, see Figures 3 and 4, may be expressed as

$$V_{tf} = F_y ht \left[\frac{(1 - C_v)}{2 \sqrt{1 + (a/h)^2}} \right] \tag{4}$$

where "a" represents duct transverse stiffener spacing. Adding Eqs. 3 and 4 results in

$$V_u = F_y ht \left[\frac{C_v}{\sqrt{3}} + \frac{(1 - C_v)}{2 \sqrt{1 + (a/h)^2}} \right] \tag{5}$$

The elastic buckling stress of duct panel for the case of pure shear may also be written in analogy to plate girders as

$$\tau_{cr} = \frac{\pi^2 E k}{12(1-\mu^2)(h/t)^2} \tag{6}$$

where
$$k = 4.0 + 5.34/(a/h)^2 \qquad \text{for } a/h \leq 1.0 \tag{7}$$
$$k = 4.0/(a/h)^2 + 5.34 \qquad \text{for } a/h \geq 1.0 \tag{8}$$

substituting $E = 29{,}000$ ksi and $\mu = 0.3$ in Eq. 6 and $\tau_y = F_y / \sqrt{3}$ result in (see Figure 5)

$$C_v = \frac{45{,}000\, k}{F_y (h/t)^2} \leq 0.8 \qquad \text{(elastic buckling)} \tag{9}$$

Residual stresses and imperfections cause inelastic buckling of the duct panel as critical stresses approach panel yield stress. Using Basler's (1963) transition curve between elastic buckling and yielding

$$\tau_{cr} = \sqrt{(\tau_{\text{prop. limit}})(\tau_{\text{cr ideal elastic}})} \tag{10}$$

With the proportional limit taken as 0.8 τ_y , substituting for τ_{cr} from Eq. 6 yields

$$C_v = \frac{190}{(h/t)} \sqrt{\frac{k}{F_y}} \geq 0.8 \qquad \text{(Inelastic Buckling)} \tag{11}$$

In the above formulation, the duct panel is conservatively assumed to have its edges simply supported. In order that the tension field may effectively develop, duct transverse stiffener spacing should be such that $(a/h) \leq 3.0$. If this limit is exceeded, no tension-field action may occur.

4. RESULTS AND DISCUSSIONS

As an illustration, assuming $F_y = 33$ ksi and $a = 48$ inches, duct shear capacity is evaluated for 16 and 18 gauge sheet metal panels and tabulated in Table I. It is evident that for the slenderness ratios (h/t) considered, V_{tf} exceeds V_{cr} by 22 to 71 times. Hence, the panels have a tremendous capacity in resisting shear, and failure of a duct panel due to shear is very unlikely. Further analysis (Stoman, 1985) reveals that about 5 to 11% of panel post-buckling capacity is actually utilized for duct spans up to 20'-0". Equation 6 is derived on the assumptions that the plate is initially perfectly flat and its transverse displacements are small compared to plate thickness. However, these assumptions are very restrictive in view of the high post-buckling strength of duct panels.

At the ultimate load, the slope of the diagonal tension-field, ϕ, is postulated to be such as to make V_{tf} a maximum. This angle corresponds to one-half the angle which the panel diagonal makes with the horizontal (McGuire, 1968), see Figure 3. For duct works, the tension-field develops over the band width "S" such that

$$S = h \cos\phi - a \sin\phi \tag{12}$$

where the angle of tension-field, ϕ, is such that

$$\sin 2\phi = \frac{1}{\sqrt{1 + (a/h)^2}} \tag{13}$$

With duct capacity in shear defined above, other design considerations can be based on the AISI Code (1986). For analysis of stresses due to pressure, the methodology of Desai and Stoman (1989) or one similar to it can be adopted.

Regardless of the geometry of a duct run, the analysis can be performed in two stages: First, stresses in the stiffeners and corner chords resulting from pressure loading are determined. Then, they are superimposed on stresses resulting from duct system "frame-action" solution, which includes stresses due to tension-field action. With respect to a set of well-defined allowable stresses, interaction values are evaluated for the duct corner chords as well as the stiffeners. A value exceeding unity indicates over-stress.

5. CONCLUSIONS

Criteria are formulated for utilizing duct panel diagonal tension in resisting panel shear. In order to ensure that diagonal tension can develop in the panel, duct corner chords must have adequate strength in maintaining stability. Likewise, the welding of traverse stiffeners to duct sheet shall be such that the stiffeners are capable of resisting pressure loading in addition to forces resulting from the components of the diagonal tension from the tributary panels of the duct.

As a result of the application of the post-buckling method, it is anticipated that the number of HVAC hangers required by a conventional analysis can be reduced by up to 50 percent, depending on system frequency requirements. This will result in a substantial savings of overall cost of the hangers. In addition, there are other indirect, intangible benefits such as: (a) Space availability improvement of other commodities, (b) accessibility improvement, and (c) lesser inspection and documentation.

REFERENCES

Basler, K. (1963). "Strength of Plate Girders in Shear." Transactions ASCE, Vol. 128, Part II, pp. 683-719.

Desai, S. C., Stoman, S. H., and Marano, D. (1989). "Structural Design of Seismic Category I HVAC Ducts." Proceedings of the Tenth Conference on Structural Mechanics in Reactor Technology (SMIRT), California, USA.

McGuire, W. (1968). "Steel Structures," Prentice-Hall, Inc., Englewood Cliffs, N.J.

Salmon, C. G. and Johnson, J. E. (1980). Steel Structures, Design and Behavior. Second Edition, Harper & Row, Publishers, New York.

American Iron and Steel Institute. (1986). Specification for Design of Cold-Formed Steel Structural Members. AISI, Washington, D.C.

Stoman, S. H. (1985). "Application of Post-Buckling Theory to HVAC Duct Design." Bechtel Power Corporation Report, Pottstown, Pennsylvania.

TABLE I - ULTIMATE SHEAR CAPACITY OF DUCT SHEET PANEL

GAUGE	h(in)	h/t	a/h	k	C_v	V_{cr}(kips)	V_{tf}(kips)	V_u (kips)
18	50	1,046	0.96	9.79	0.012	0.546	28.107	28.653
	60	1,255	0.80	12.34	0.011	0.601	36.546	37.147
	70	1,464	0.69	15.22	0.010	0.637	44.987	45.625
16	40	669	1.20	8.12	0.025	1.139	24.635	25.774
	50	836	0.96	9.79	0.019	1.082	34.913	36.0
	60	1,003	0.80	12.34	0.017	1.162	45.443	46.605
	70	1,171	0.69	15.22	0.015	1.196	56.0	57.193
	80	1,338	0.60	18.83	0.014	1.276	66.739	68.015

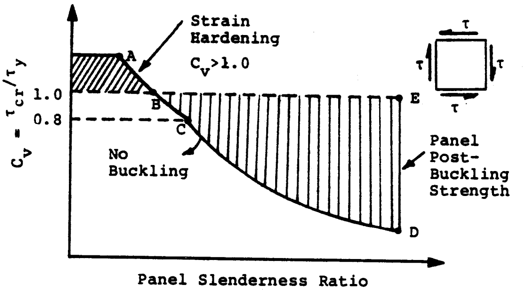
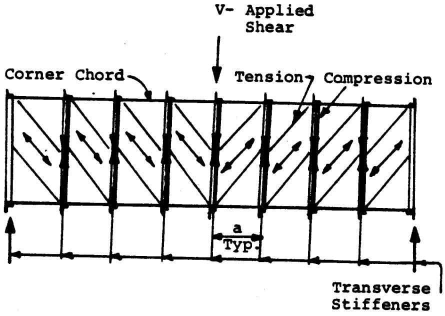


FIGURE 1 DUCT PANEL ULTIMATE SHEAR CAPACITY



DUCT PANELS UNDER TENSION-FIELD ACTION

FIGURE 2A

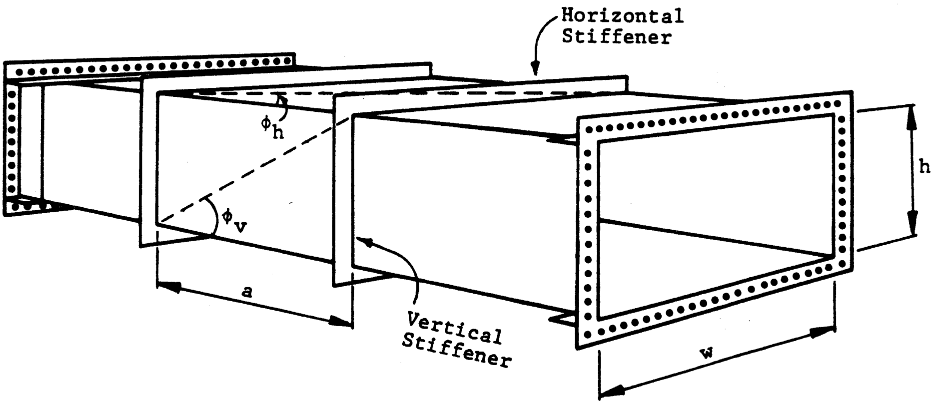


FIGURE 2B HVAC DUCT ANGLES OF DIAGONAL TENSION-FIELD

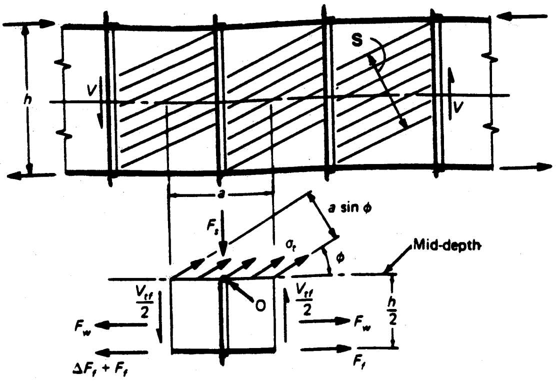
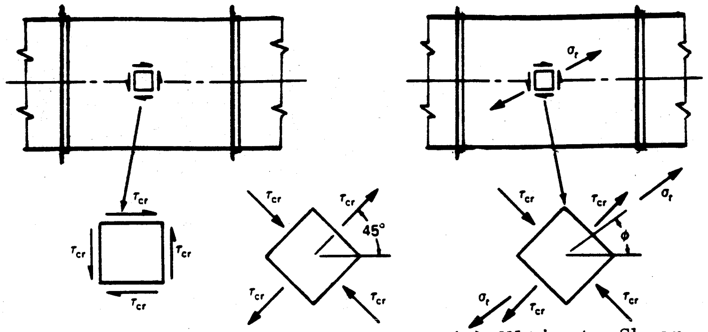
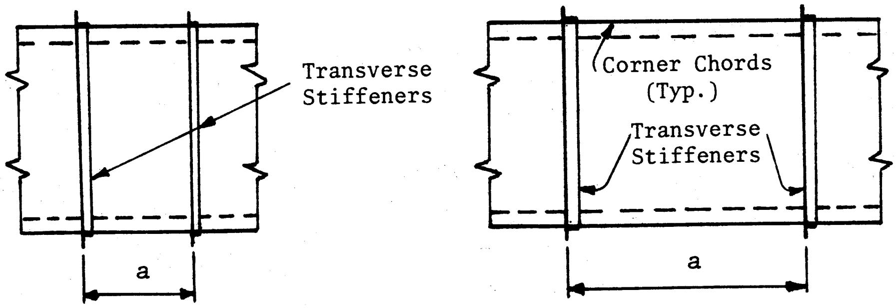


FIGURE 3 **FORCES IN DUCT TRANSVERSE STIFFENERS DUE TO TENSION-FIELD ACTION**



(a) Pure Shear (b) Principal Stress (c) Ultimate Shear

FIGURE 4 **DUCT PANEL STATE OF STRESS**



(a) a/h< 1.0

(b) a/h> 1.0

FIGURE 5 **DUCT PANEL TRANSVERSE STIFFENER SPACING**