Departure Process for Periodic Real-Time Traffic of an ATM-SMX in the NCIH

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Abstract

In this paper, we analyze the departure process for periodic real-time loss-insensitive traffic of an ATM Service Multiplexer (ATM-SMX) in the North Carolina Information Highway (NCIH). The ATM-SMX is modeled as a discrete-time finite-buffer single-server queueing system, where two types of traffic represent the inputs of the multiplexer: i) real-time loss-insensitive traffic; and ii) nonreal-time loss-sensitive traffic. The real-time loss-insensitive traffic is assumed to be a periodic batch arrival with a general distribution for the batch size, while the nonreal-time loss-sensitive traffic consists of N identical Bernoulli processes. We adopt a scheduling policy which suits the characteristics of these two types of traffic. All the slow-speed inputs of the ATM-SMX are multiplexed onto a single high-speed output link, and the service rate is constant. An exact analysis is developed to obtain the steady-state queue length distribution and the interdeparture time distribution of the periodic real-time loss-insensitive traffic. The result in this paper enables the per-session performance evaluation of the NCIH.

1 Introduction

In August of 1994, the first statewide information highway was initiated by the North Carolina State Government. It is a high-speed wide area network (WAN) which utilizes Asynchronous Transfer Mode (ATM) technology and Synchronous Optical Network (SONET) transmission equipment. This network, the North Carolina Information Highway (NCIH), connects universities, community colleges, schools, hospitals, sheriffs' offices, and other government agencies together into one ATM WAN [16]. The services provided by the NCIH are designed to support numerous emerging applications, and applications having different quality of service (QOS) requirements, such as end-to-end packet delay, delay jitter, and cell loss probability. Hence, the performance evaluation of each individual application on the NCIH becomes an issue. An important part of the performance modeling of a queueing network is to obtain the departure process of each individual node, then characterize the departure process so it can be used as an arrival process of a downstream node. In this paper, we obtain the exact interdeparture time distributions of an ATM Service Multiplexer (ATM-SMX) by using discrete-time queueing analysis.

Previous related work was focused primarily on a queueing system with homogeneous speed links between inputs and outputs. Murata et al. [11] adopted the technique developed in [5] to analyze a discrete-time $GI + M^{[X]}/D/1/K$ queue. An exact analysis of the waiting time and cell loss probability is obtained for both new call and existing calls. In Ohba et al. [12], a discrete-time $GI + M^{[X]} + N \times IPP/D/1/K$ queue is presented. An exact analysis is developed to derive the waiting time and interdeparture time distributions for the GI-stream cells at the first queue. An approximate end-to-end delay is also obtained for the designated traffic stream. Chang et al. [6] studied a multi-severs queueing system with two types of input traffic: i) periodic and correlated real-time traffic; and ii) batch Poisson nonreal-time traffic. The system has two separate finite buffers for each type of traffic. They adopted the complete rejection strategy, i.e., when the message length of an arrival is larger than the available buffer space, the whole message (all packets in the message) is rejected. The blocking probabilities and mean delays are obtained by using a Markov analytical method. Wang et al. [21] derived the interdeparture time distribution of a discrete-time single-server queueing system which consists two priority queues. Their analysis is mainly devoted to the derivation of the interdeparture time distribution for nonreal-time traffic.

Several researchers have studied queueing models with heterogeneous speed links. In the early years, Eckberg [7] studied the delay distribution of a continuous-time infinite buffer single server queue with $N+1$ equivalent periodic arrivals. He introduced the idea of a recursive form solution. In [13], we studied the departure process of an ATM multiplexer by using the moment generating function and autocorrelation coefficients. Recently, Balay and Nilsson analyzed an ATM multiplexer with discrete-time non-renewal arrival processes. They obtain the steady-state performance results of the session of interest, and the exact departure process of the session is also presented [2, 3]. However, no batch arrivals were considered as the session of interest in their researches, and no scheduling policies were implemented.

In this paper, we consider a queueing system which is denoted as a $N \times BP(L) + D^{[X]}(TL)/D/1/K$ system. The interest session is modeled as a periodic batch arrival, and the cross-traffic is represented by N sessions of identical Bernoulli process. We assume heterogeneous speed input and output links. We also take a scheduling policy and partial rejection strategy into consideration in our queueing model.

This paper is organized as follows. In section 2, we describe our analytical model. In section 3, the steady-state queue length distribution and the departure process are obtained. In section 4, we present some numerical results of the analysis and simulation. Section 5 concludes our paper.

2 Model Description

Recently, discrete-time queueing models have been studied extensively because of the increasing attention on ATM networks where each data unit is a fixed size cell (53 bytes). Lots of traffic models have been proposed for the traffic in ATM networks [18]. Interrupted Bernoulli Process (IBP), Markov Modulated Bernoulli Process (MMBP), and Discrete-time Batch Markovian Arrival Process (D-BMAP) are among the most famous ones (see Park [14] for IBP and MMBP, and Blondia et al. [4] for D-BMAP.) The latest research by Leland et al. [10] suggests that data traffic is self-similar or fractal, and the paper shows that the burstiness (degree of self-similarity) of LAN traffic typically intensifies as the number of active traffic sources increases. As does Partridge [15], we believe the self-similar analysis is yet another reminder that we still do not really understand how data communications traffic behaves. It has been a long struggle for researchers to develop accurate traffic models, and there is a serious need for further research in this area.

Due to lack of real measurements on the NCIH, the distribution of the arrival process of each application is not known exactly. Without loss of the generality, it is still reasonable to assume renewal processes as the arrival processes of the ATM-SMX in our model, since one of the main function of the ATM-SMX is to provide ATM cell assembly and disassembly for the provision of services [1], so the traffic before the ATM-SMX is not ATM traffic.

Initially, the NCIH supports Switched Multimegabit Data Service (SMDS) and Circuit Emulation Service (CES) [8]. A very important characteristic of SMDS is its compatibility with ATM [17]. Since both SMDS and ATM use 53-octet cells for transport, a discrete-time cell scale model would best characterize the nature of input and output traffic of the ATM-SMX. Hence, we model each SMDS session as a Bernoulli Process (BP) with parameter λ. Furthermore, due to the fact that the input link speed of CES is faster than that of SMDS

[8] and the periodic nature of video frames, we assume the arrival process of a CES session (the session of interest) as a batch arrival and the interarrival time between each batch is a constant. In what follows, we will refer to SMDS as M-stream (cell arrival streams with a discrete-time Markovian interarrival time distribution) and CES as P-stream (periodic arrival.)

The queueing model analyzed in this paper is denoted as $N \times BP(L) + D^{[X]}(TL)/D/1/K$ queue, where there are N sessions of SMDS (M-stream) and one session of CES (P-stream.) We define the time to transmit an SMDS cell on an SMDS input link as a big slot and the time to transmit an ATM cell on the output link as a small slot. We also assume the ratio of the output link speed to the SMDS input line speed is L, i.e., 1 big slot $= L$ small slot; hence $N \times BP(L)$ denotes there are at most N M-stream arrivals for every L small slots (i.e., 1 big slot) and $D^{[X]}(TL)$ represents a periodic arrival (P-stream) with batch size X for every $T \times L$ small slots, where T and L are constants. We also assume that all the input lines are synchronized and cell arrivals are considered to occur at slot boundaries. The protocol processing time is assumed to be negligible in our model.

Moreover, we assume M-stream cells are loss-sensitive and P-stream cells are delay-sensitive. Under this assumption, the scheduling policy of the queue acts as follows:

- The queue is a first-come-first-serve (FCFS) queue for cells arriving at different time slots.
- If M-stream cells and P-stream cells arrive at the same time, i.e., for every $T \times L$ small slots:
    1. The ATM-SMX will accept M-stream cells first;
    2. if there is buffer space available after all M-stream cells being accepted, then the ATM-SMX will accept P-stream cells;

3. for those accepted M-stream and P-stream cells which arrived at the same time slot, the P-stream batch will be served before those M-stream cells.

- A cell is discarded when it arrives to a full buffer.

3 Analysis of the $N \times BP(L) + D^{[X]}(TL)/D/1/K$ Queue

3.1 Derivation of Queue Length Distribution

We consider the P-stream arrival instants as the embedded (or observation) points in our system. We observe the queue length distribution at $(n+1)$st embedded point and relate to that of the previous embedded point (i.e., the nth embedded point). Hence, we need to obtain the queue length distribution in each small slot following the nth P-stream arrival, but before the $(n+1)$st arrival from the P-stream.

Let M be a random variable representing the number of cells arriving from M-stream during each big slot and $m(k)$ be the probability density function (p.d.f.) of M, i.e., $m(k) = Prob\{M = k\}$. Since cell arrivals from the P-stream occur in batches, let X be a random variable representing the size of a batch. The batch size follows a general distribution with the p.d.f. $x(k)$, and we assume that there is at least one cell in each batch, i.e., $X \geq 1$.

Let $C_{i,j}^{(n)}$ be a random variable denoting the queue length at the end of the jth small slot which belongs to the ith big slot following the nth arrival of a batch from the P-stream. $C_{i,j}^{(n)}$ is obtained before any arrivals in that small slot, but after the departure of a cell, if there is any, in that small slot. $C_{0,0}^{(n)}$ is defined as the queue length immediately before the nth batch arrival from the P-stream and associated M-stream arrivals. This subsection's purpose is to derive the distribution of $C_{0,0}^{(n)}$. Furthermore, we introduce another random variable, $\overline{C}_i^{(n)}$, which represents the queue length observed at the beginning of the ith big slot following

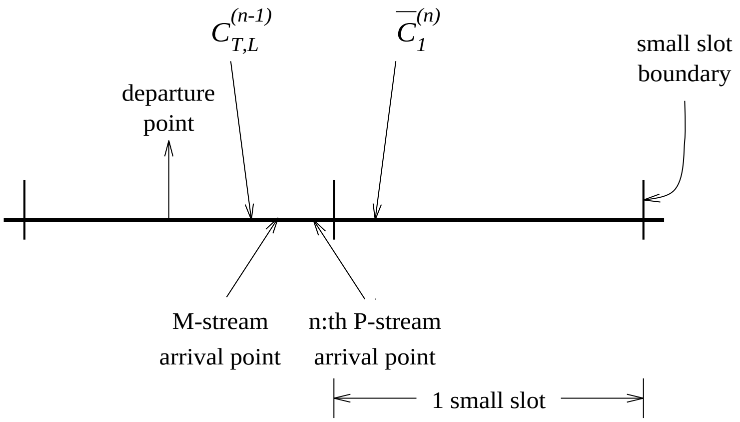

Figure 1: Relative order of events at the embedded point.

the nth arrival of a batch from the P-stream. The relative order of events at the embedded point is shown in Figure 1. Figure 2 illustrated the sample path of these random variables.

As shown in Figure 2, $\overline{C}_1^{(n)}$ represents the number of cells in the queue at the beginning of the first big slot following the nth batch of P-stream arrivals. We know that $\overline{C}_1^{(n)}$ is the sum of: i) the cells which have been present in the buffer before the arrival of the P-stream and associated M-stream cells; ii) newly arrived M-stream cells; and iii) newly arrived P-stream cells, or $\overline{C}_1^{(n)}$ is the queue capacity K if the sum of i)—iii) exceeds K. Hence, we have

$$\overline{C}_1^{(n)} = \min\left(C_{0,0}^{(n)} + M + X,\ K\right). \tag{1}$$

Next, we consider the relationships between $\overline{C}_i^{(n)}$ and $C_{i,j}^{(n)}$, we have

$$C_{i,1}^{(n)} = \max\left(\overline{C}_i^{(n)} - 1,\ 0\right), \quad 1 \le i \le T, \tag{2}$$

$$C_{i,j}^{(n)} = \max\left(C_{i,j-1}^{(n)} - 1,\ 0\right), \quad 1 \le i \le T, \quad 2 \le j \le L, \tag{3}$$

and

$$\overline{C}_i^{(n)} = \min\left(C_{i-1,L}^{(n)} + M,\ K\right), \quad 2 \le i \le T. \tag{4}$$

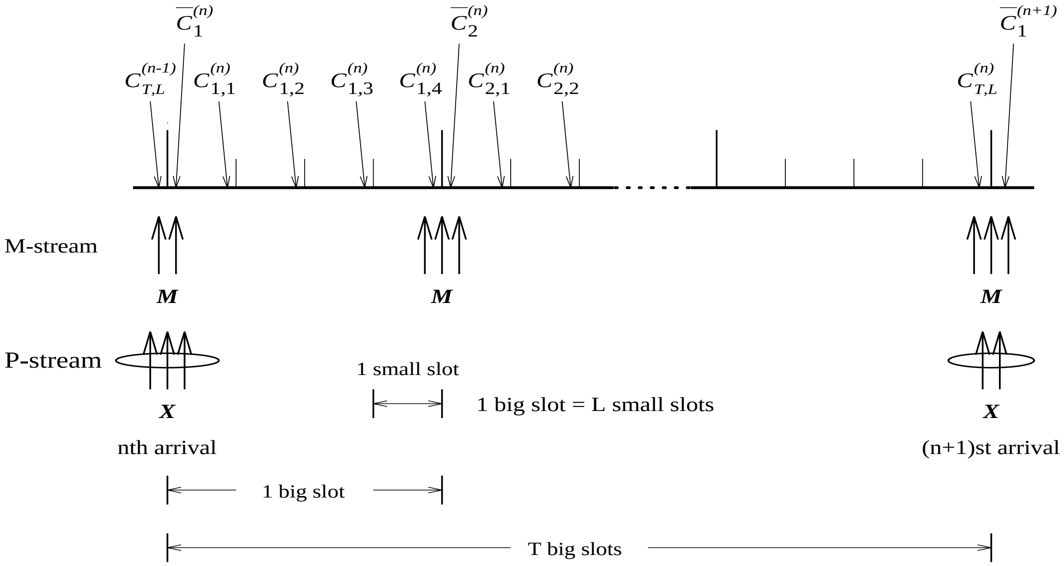

Figure 2: Sample path for random variables ($L = 4$).

From the definition of $C_{i,j}^{(n)}$, we have

$$C_{0,0}^{(n+1)} = C_{T,L}^{(n)}. \tag{5}$$

Before we look at the probability densities for the above random variables, we introduce some operators defined by Tran-Gia in [20]:

$$\boldsymbol{\pi}^K(y(k)) = \begin{cases} y(k) & \text{if } k < K \\ \sum_{i=K}^{\infty} y(i) & \text{if } k = K \\ 0 & \text{if } k > K, \end{cases}$$

$$\boldsymbol{\pi}_\circ(y(k)) = \begin{cases} 0 & \text{if } k < 0 \\ \sum_{i=-\infty}^{0} y(i) & \text{if } k = 0 \\ y(k) & \text{if } k > 0. \end{cases}$$

A convolution operator for two discrete distributions is denoted by a symbol $*$ as $y_1(a) * y_2(a) = \sum_{b=0}^{a} y_1(a-b)y_2(b)$.

Now, let $c_{i,j}^{(n)}(k)$ and $\overline{c}_i^{(n)}(k)$ be the probability density functions for $C_{i,j}^{(n)}$ and $\overline{C}_i^{(n)}$, respectively. By using convolutions for discrete distributions and the above operators on equations (1)–(5), we have

$$\overline{c}_1^{(n)}(k) = \boldsymbol{\pi}^K \left(c_{0,0}^{(n)}(k) * m(k) * x(k) \right), \quad 1 \leq k \leq K, \tag{6}$$

$$c_{i,1}^{(n)}(k) = \boldsymbol{\pi}_\circ \left(\overline{c}_i^{(n)}(k+1) \right), \quad 0 \leq k \leq K-1, \quad 1 \leq i \leq T, \tag{7}$$

$$c_{i,j}^{(n)}(k) = \boldsymbol{\pi}_\circ \left(c_{i,j-1}^{(n)}(k+1) \right), \quad 0 \leq k \leq K-j, \quad 1 \leq i \leq T, \quad 2 \leq j \leq L, \tag{8}$$

$$\overline{c}_i^{(n)}(k) = \boldsymbol{\pi}^K \left(c_{i-1,L}^{(n)}(k) * m(k) \right), \quad 0 \leq k \leq K, \quad 2 \leq i \leq T, \tag{9}$$

$$c_{0,0}^{(n+1)}(k) = c_{T,L}^{(n)}(k), \quad 0 \leq k \leq K. \tag{10}$$

Note that equations (6)–(10) show the analysis of the transient behavior of the queue. The steady-state probability that there are k cells in the queue before a P-stream and associated M-stream arrivals is given by

$$c_{0,0}(k) = \lim_{n \to \infty} c_{0,0}^{(n)}(k), \quad 0 \leq k \leq K.$$

Hence, it requires an iterative calculation and we need to start with arbitrary initial values $c_{0,0}^{(1)}(k)$, $0 \leq k \leq K$, with the condition $\sum_{k=0}^{K} c_{0,0}^{(1)}(k) = 1$. We repeat the iterative steps until the difference of the queue length distribution at two consecutive embedded points is less than a pre-defined value, i.e.,

$$\left| c_{0,0}^{(n+1)}(k) - c_{0,0}^{(n)}(k) \right| < \epsilon, \quad \forall k.$$

Note that at each small slot, convolution operations are required and it may take lots of computations to get steady-state results. Thus, as indicated in [20], efficient algorithms like FFT could be employed to save computing costs.

3.2 Derivation of Interdeparture Time Distribution of P-Stream Cells

In this subsection, we derive the P-stream cells' interdeparture time distribution by using the queue length distribution, $c_{0,0}(k)$, which we obtained in the previous subsection. In order to do so, we observe a P-stream arriving at the non-saturated steady-state point, i.e., at least one cell from the P-stream's will be accepted by the queue. First, we condition the previous defined random variables $C_{i,j}^{(n)}$ and $\overline{C}_i^{(n)}$ by:

1. q, the number of cells found in the buffer when the M-stream and P-stream cells arrive;
2. l, $(0 \leq l \leq N)$, the number of cells from the M-stream's arriving together with the P-stream cells;
3. x_a, $(1 \leq x_a \leq x)$, the number of cells from the P-stream's which will be accepted by the queue, where x is the number of cells from the P-stream's.

As mentioned earlier, we assume that the queue is not saturated before the P-stream arrives, i.e., $q + l < K$, and this is our given condition throughout this subsection. The random variables $C_{i,j,\{q+l<K\}}$ and $\overline{C}_{i,\{q+l<K\}}$ will be used in replacement of $\lim_{n\to\infty} C_{i,j}^{(n)}$ and $\lim_{n\to\infty} \overline{C}_i^{(n)}$, respectively. Since q and l are given, x_a can be described as

$$x_a = \min\left(K - (q+l)\,,\ x\right), \quad 0 \leq q < K, \quad 0 \leq l < \min\left(N,\ K-q\right), \quad 1 \leq x \leq X_{max}.$$

By a similar discussion in deriving equations (1)–(5), we have

$$\overline{C}_{1,\{q+l<K\}} = q + l + x_a, \tag{11}$$

$$C_{i,1,\{q+l<K\}} = \max\left(\overline{C}_{1,\{q+l<K\}} - 1,\ 0\right), \quad 1 \leq i \leq T, \tag{12}$$

$$C_{i,j,\{q+l<K\}} = \max\left(C_{i,j-1,\{q+l<K\}} - 1,\ 0\right), \quad 1 \leq i \leq T, \quad 2 \leq j \leq L, \tag{13}$$

$$\overline{C}_{i,\{q+l<K\}} = \min\left(C_{i-1,L,\{q+l<K\}} + M,\ K\right), \quad 2 \le i \le T. \tag{14}$$

The corresponding probability densities $c_{i,j,\{q+l<K\}}(k)$ and $\overline{c}_{i,\{q+l<K\}}(k)$ are obtained as

$$\overline{c}_{1,\{q+l<K\}}(k) = \begin{cases} 1 & \text{if } k = q+l+x_a \\ & \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

$$c_{i,1,\{q+l<K\}}(k) = \boldsymbol{\pi}_0\left(\overline{c}_{i,\{q+l<K\}}(k+1)\right), \quad 0 \le k \le K-1, \quad 1 \le i \le T, \tag{16}$$

$$c_{i,j,\{q+l<K\}}(k) = \boldsymbol{\pi}_0\left(c_{i,j-1,\{q+l<K\}}(k+1)\right), \quad 0 \le k \le K-j, \quad 1 \le i \le T, \quad 2 \le j \le L, \tag{17}$$

$$\overline{c}_{i,\{q+l<K\}}(k) = \boldsymbol{\pi}^K\left(c_{i-1,L,\{q+l<K\}}(k) * m(k)\right), \quad 0 \le k \le K, \quad 2 \le i \le T. \tag{18}$$

Let $SID_{\{q,l,x_a\}}$ be a random variable which represents the interdeparture time of the observed P-stream cells in terms of small slots, given the condition that there are q cells in the queue when the M-stream and P-stream cells arrive, and l cells from the M-stream's arriving together with the P-stream cells, where $q + l < K$, and there are x_a cells from the P-stream's which are accepted. Furthermore, we define another random variable X_A to be the number of P-stream cells being accepted by the queue. Since $C_{0,0}$, M, and X_A are all discrete random variables, it is convenient to define the joint p.d.f. of $C_{0,0}$, M, and X_A by

$$f(q,l,x_a) = Prob\{C_{0,0} = q, M = l, X_A = x_a\},$$

and

$$f(q,l,x_a) = \begin{cases} \displaystyle\sum_{i=x_a}^{X_{max}} c_{0,0}(q)m(l)x(i) & \text{if } q+l+x_a = K \\ & \\ c_{0,0}(q)m(l)x(x_a) & \text{if } q+l+x_a < K \\ & \\ 0 & \text{otherwise.} \end{cases}$$

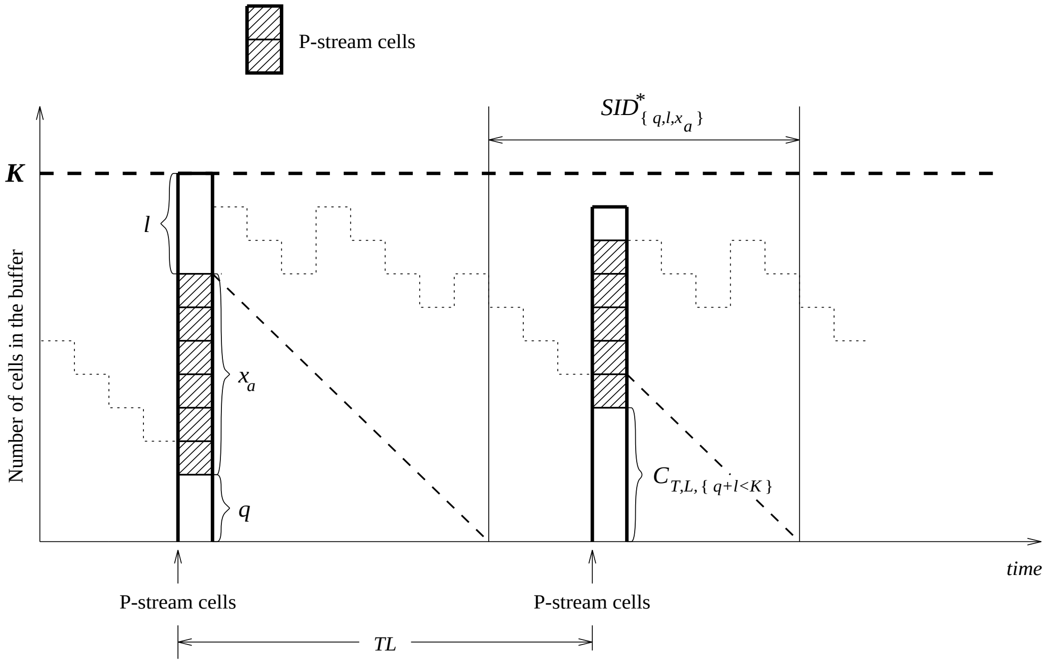

Figure 3: Sample for interdeparture time between two P-streams.

Due to the fact that there is a given condition, $q + l < K$, we have the conditional p.d.f.

$$\begin{aligned} f_{\{q+l<K\}}(q, l, x_a) &= Prob\{C_{0,0} = q, M = l, X_A = x_a | q + l < K\} \\ &= \frac{f(q, l, x_a)}{\sum_{k=0}^{K-1} [c_{0,0}(k) * m(k)]}. \end{aligned}$$

Based on the scheduling policy we described in section 2, a batch of P-stream cells are served consecutively as we can see from Figure 3. Then, by using the renewal type arguments [9], we can obtain the interdeparture time between two P-stream cells as

$$SID_{\{q,l,x_a\}} = \begin{cases} 1 & \text{w.p.} \quad \frac{1}{m_1}(x_a - 1) f_{\{q+l<K\}}(q, l, x_a) \\ SID^*_{\{q,l,x_a\}} & \text{w.p.} \quad \frac{1}{m_1} f_{\{q+l<K\}}(q, l, x_a), \end{cases} \tag{19}$$

where

$$m_1 = \sum_{q=0}^{K} \sum_{l=0}^{N} \sum_{x_a=1}^{X_{max}} x_a f_{\{q+l<K\}}(q, l, x_a),$$

and $SID^*_{\{q,l,x_a\}}$ denotes the interdeparture time between the last cell of the P-stream and the first cell of the next accepted P-stream. To derive $SID^*_{\{q,l,x_a\}}$, we further introduce the following two random variables. The random variable $C'_{i,j,\{K\}}$ denotes the queue length at the end of the jth small slot which belongs to the ith big slot following a P-stream arrives to a saturated queue, i.e., all cells from the P-stream's are discarded. The other random variable $\overline{C}'_{i,\{K\}}$ represents the queue length at the beginning of the ith big slot following a P-stream arrives to a saturated queue. We have

$$\overline{C}'_{1,\{K\}} = K,$$

and the derivation of $C'_{i,j,\{K\}}$ is similar to equations (12)–(14). The corresponding probability densities $c'_{i,j,\{K\}}(k)$ and $\overline{c}'_{i,\{K\}}(k)$ can be obtained as

$$\overline{c}'_{1,\{K\}}(k) = \begin{cases} 1 & \text{if } k = K \\ 0 & \text{otherwise,} \end{cases}$$

$$c'_{i,1,\{K\}}(k) = \boldsymbol{\pi}_0 \left(\overline{c}'_{i,\{K\}}(k+1) \right), \quad 0 \le k \le K-1, \quad 1 \le i \le T,$$

$$c'_{i,j,\{K\}}(k) = \boldsymbol{\pi}_0 \left(c'_{i,j-1,\{K\}}(k+1) \right), \quad 0 \le k \le K-j, \quad 1 \le i \le T, \quad 2 \le j \le L,$$

$$\overline{c}'_{i,\{K\}}(k) = \boldsymbol{\pi}^K \left(c'_{i-1,L,\{K\}}(k) * m(k) \right), \quad 0 \le k \le K, \quad 2 \le i \le T.$$

Now, we can express $SID^*_{\{q,l,x_a\}}$ as (see Figure 3)

$$SID^*_{\{q,l,x_a\}} = \begin{cases} TL + C_{T,L,\{q+l<K\}} + 1 - (q + x_a) & \text{w.p. } p_{s_1} \\ (1+R)\,TL + C'_{T,L,\{K\}} + 1 - (q + x_a) & \text{w.p. } 1 - p_{s_1}, \end{cases}$$

where

$$p_{s_1} = \sum_{k=0}^{K-1} \left(c_{T,L,\{q+l<K\}}(k) * m(k) \right),$$

and the p.d.f. of R is given by

$$Prob\{R = r\} = (1 - p_{s_2})^{r-1} p_{s_2}, \quad r \geq 1,$$

where

$$p_{s_2} = \sum_{k=0}^{K-1} \left(c'_{T,L,\{K\}}(k) * m(k) \right).$$

Then, the corresponding p.d.f. of $SID^*_{\{q,l,x_a\}}$ can be obtained as

$$\begin{aligned} sid^*_{\{q,l,x_a\}}(k) &= c_{T,L,\{q+l<K\}}(k - TL - 1 + q + x_a)\, p_{s_1} + \\ & \left\{ \sum_{r=1}^{\infty} c'_{T,L,\{K\}}\left[k - (1+r)TL - 1 + q + x_a\right] (1 - p_{s_2})^{r-1} p_{s_2} \right\} (1 - p_{s_1}). \end{aligned}$$

Furthermore, we have the indicator function [19]

$$\mathbf{1}(k = i) = \begin{cases} 1 & \text{if } k = i \\ 0 & \text{otherwise.} \end{cases}$$

Then, we can obtain the p.d.f. $sid_{\{q,l,x_a\}}(k)$ for the random variable $SID_{\{q,l,x_a\}}$, equation (19), as

$$sid_{\{q,l,x_a\}}(k) = \frac{1}{m_1} f_{\{q+l<K\}}(q, l, x_a) \left[(x_a - 1)\, \mathbf{1}(k = 1) + sid^*_{\{q,l,x_a\}}(k) \right].$$

Finally, by unconditioning the above equation, the probability density $sid(k)$ is given as

$$sid(k) = \sum_{q=0}^{K} \sum_{l=0}^{N} \sum_{x_a=1}^{X_{max}} \frac{1}{m_1} f_{\{q+l<K\}}(q, l, x_a) \left[(x_a - 1)\, \mathbf{1}(k = 1) + sid^*_{\{q,l,x_a\}}(k) \right].$$

4 Numerical Results

In this section, we present some numerical results of the queue length distributions and P-stream's interdeparture time distributions by using the approach presented above. Although our analysis allows a general distribution for the P-stream's batch size, we assume a truncated geometric distribution for the batch size X which follows

$$x(k) = (1-g)^{X_{max}-k}g, \quad \text{if} \;\; 2 \leq k \leq X_{max},$$

and

$$x(1) = 1 - \sum_{k=0}^{X_{max}-2} (1-g)^k g,$$

where g is the parameter of the truncated geometric distribution. We set $K = 50$, $N = 10$, $L = 4$, $T = 15$, $X_{max} = 30$, $g = 0.5$, and $\epsilon = 10^{-12}$ throughout this section.

Figure 4 and 5 show the queue length distribution seen by P-stream. Figure 4 illustrates the case that there is no cell loss ($\lambda = 0.2$) and Figure 5 shows an unstable system ($\lambda = 0.5$) , i.e., the queue is saturated most of the time. Notice that there is almost no difference between the analytical and simulation results except for some points where the probability densities are less than 10^{-5}.

Figure 6 shows the interdeparture time p.d.f. of P-stream when $\lambda = 0.2$. Since $N\lambda$ is relatively small in this case, we have $SID < TL = 60$ all the time. In Figure 7, 8, and 9, we plot the interdeparture time p.d.f. of P-stream when $\lambda = 0.5$. The reason for this is because when the total traffic load becomes larger, the tail of the departure process becomes longer. In order to get a better view of the comparison between the analytical and simulation results, we separate the first three portions of the p.d.f. into these three figures. Again, we notice that the analytical results are consistent with the simulation results.

5 Conclusion

In this paper, we have analyzed the P-stream's departure process of a discrete-time $N \times BP(L) + D^{[X]}(TL)/D/1/K$ queueing system. We considered the session of interest (P-

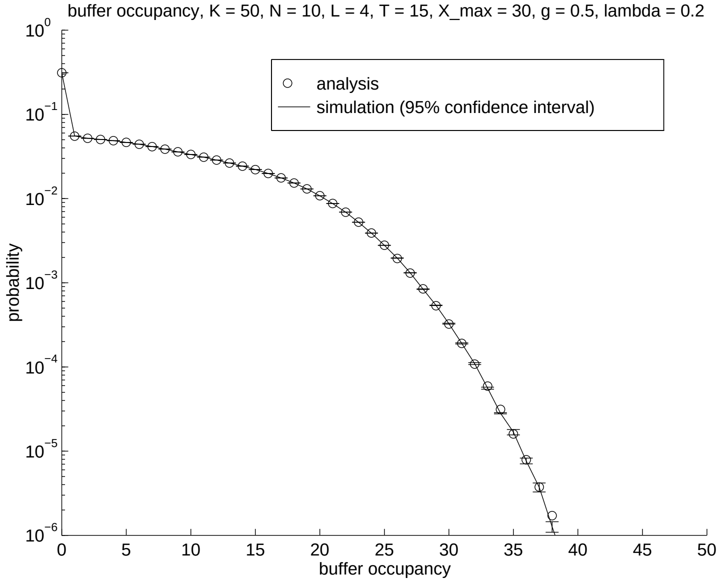

Figure 4: Queue length distributions.

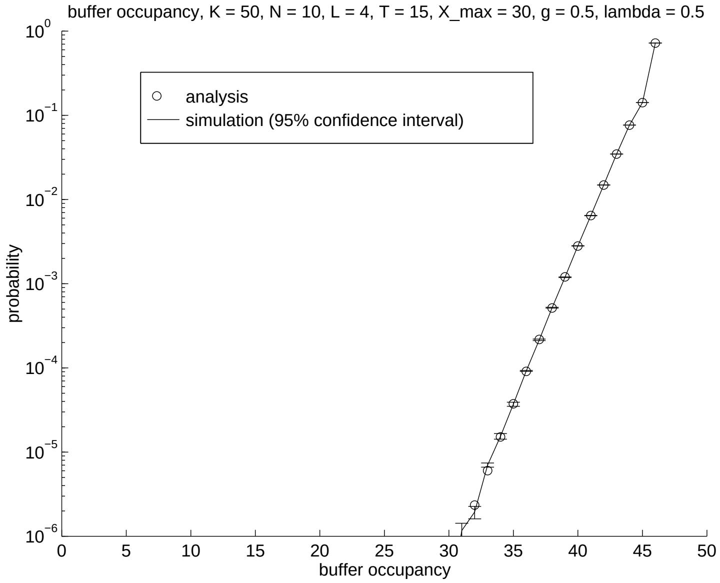

Figure 5: Queue length distributions.

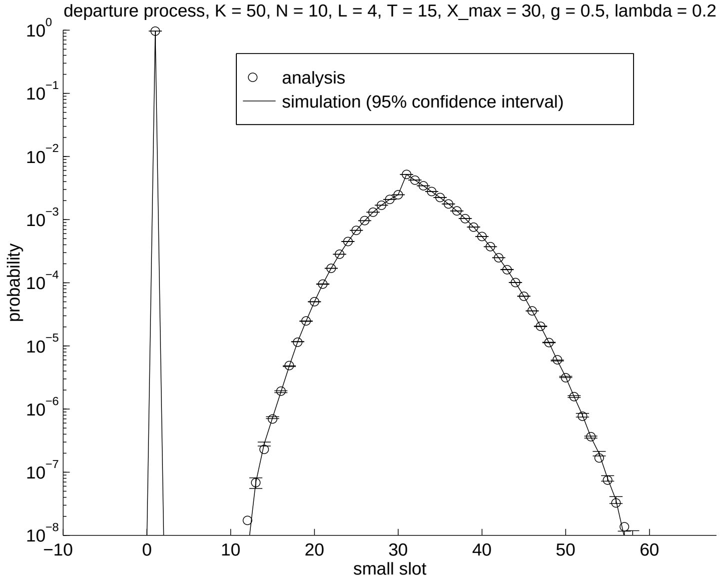

Figure 6: Interdeparture time distributions for P-stream.

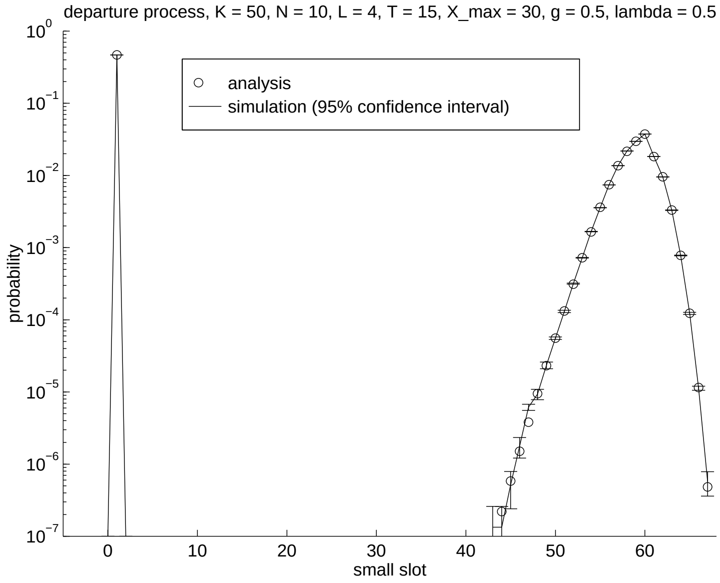

Figure 7: Interdeparture time distributions for P-stream.

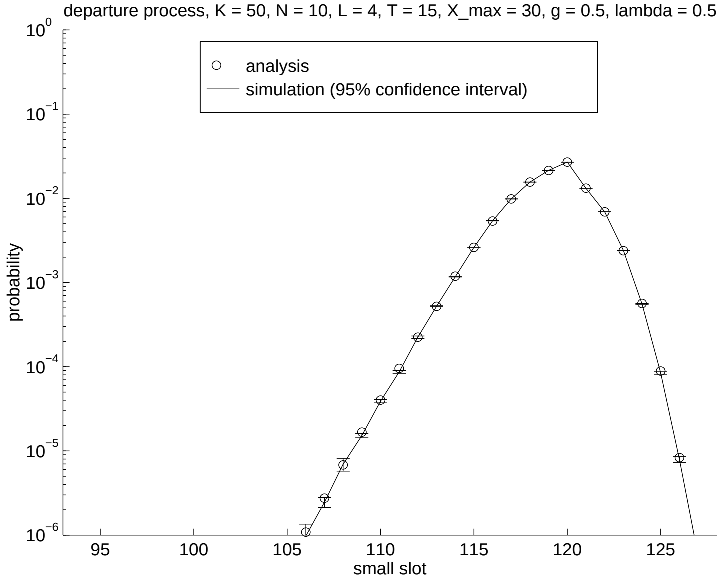

Figure 8: Interdeparture time distributions for P-stream.

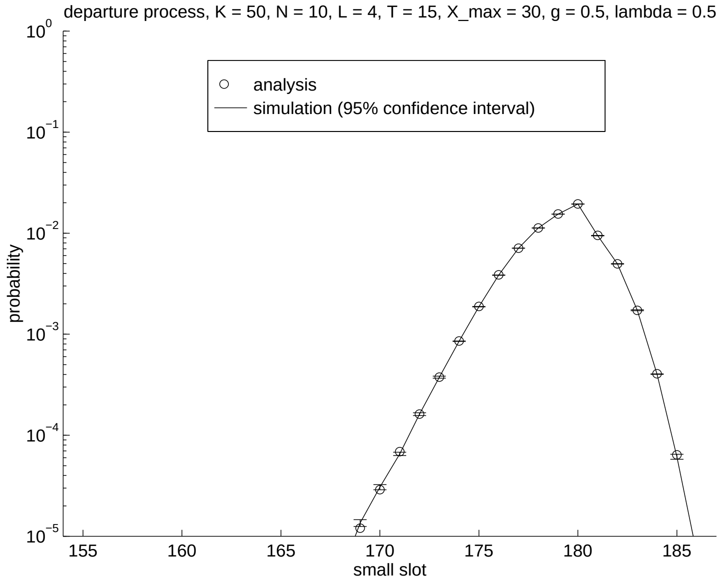

Figure 9: Interdeparture time distributions for P-stream.

stream) as a periodic batch arrival while the cross-traffic (M-stream) was modeled as N identical Bernoulli processes. We also adopted a scheduling policy which suits the characteristics of these two types of traffic. Our analysis has been devoted to the queue length distribution seen by P-stream and interdeparture time distributions of accepted P-stream cells. Through numerical examples, we showed that the analysis is very accurate. We also showed how the queue length and departure process of P-stream are affected by the total traffic load. Since the output traffic from the ATM-SMX will become a part of the input traffic to a downstream node (e.g., an ATM switch,) our work here enables us to study the per-session end-to-end performance of the NCIH in the future.

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