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APPLICATION OF THE ELASTIC-PLASTIC CORRECTION FACTOR K_E IN TUBE PLATES (TP)

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ABSTRACT

When we carry out a fatigue analysis in a tube plate and we exceed the $3S_m$ rule in the ligament between 2 holes, the question which then arises is: how to consider the elastic-plastic correction factor K_e? As a reminder, the $3S_m$ analysis aims at limiting primary plus secondary stress intensity ranges (paragraph B3234.2 of RCC-M code).

The objective of our study is to propose an answer to the question by carrying out an elastic and an elastic-plastic analysis of the tube-sheet Elementary Representative Portion (ERP), made of 9 holes (see Figure 1).

The calculations are conducted by varying the stresses applied to the ERP and by analyzing, for different stress states and from the fatigue curve, the number of allowable cycles obtained with:

A - An elastic calculation and a fatigue curve based on an elastic alternate stress (S_{alt}, N),
B - An elastic-plastic calculation and a fatigue curve based on an alternate elastic-plastic strain (ε_{alt}, N)

From these calculations we can determine if a K_e must be considered (Billon and Jourden 2024) and how this K_e must be considered in the equivalent homogeneous medium. Actually stress analysis in Tube Plate perforated medium is generally based on modelling this zone by a homogeneous material to which equivalent properties are assigned ($\{\sigma\} = [D^*].\{\varepsilon\}$, see Figure 1, Billon 1986, Billon and Jourden 2022).

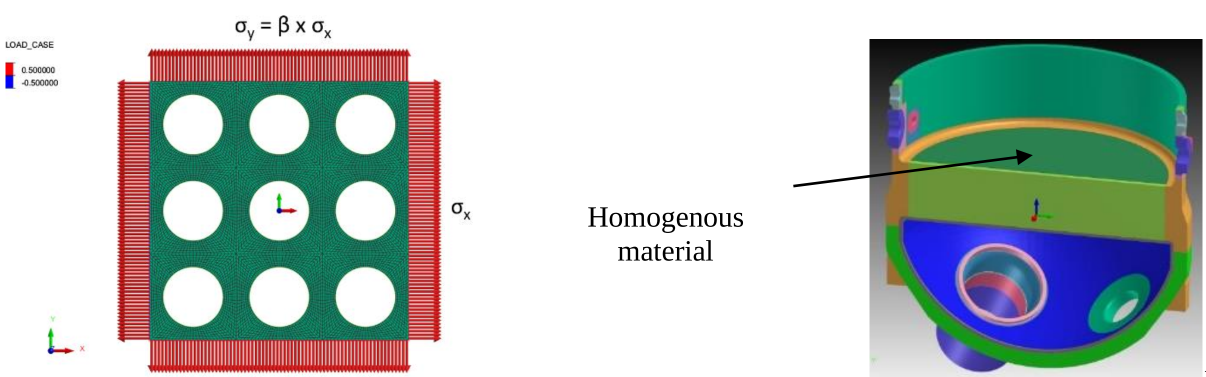

Figure 1. Models

The study is conducted according to the following main steps:

Step 1: elastic analysis of the ERP in order to directly calculate the circumferential stresses at hole edge $\sigma_{\phi\phi} = a_\phi.\sigma_x + b_\phi.\sigma_x = (a_\phi + \beta.b_\phi).\sigma_x$ by incrementing σ_x such that the mean stress amplitude in the small ligament of the ERP varies between 3 S_m and 6 S_m. Parameter β is a biaxiality coefficient that is variable from -1 to 1. Given the amplitude $\Delta\sigma_{\phi\phi}$ we calculate an elastic Usage Factor (UF_e).

Step 2: elastic-plastic analysis of the ERP from the stress-strain curve (σ-ε) of the material and as previously by incrementing σ_x within the limits obtained to reach 3 S_m and 6 S_m, in order here to calculate the elastic-plastic circumferential strain $\varepsilon_{\phi\phi}$ at the hole edge.
Given the amplitude $\Delta\varepsilon_{\phi\phi}$ we calculate an elastic-plastic Usage Factor (UF_{ep}).

Step 3: by comparing the usage factors UF_e and UF_{ep} we can determine whether an elastic-plastic correction factor K_e must be applied, and which correction value must be considered for the fatigue analysis.

The fatigue analysis in a Tube Plate is based on the stress amplitude $\Delta\sigma_{\phi\phi}$ at the hole edge through coefficients a_ϕ, b_ϕ and c_ϕ applied to the stress calculated in the equivalent homogeneous medium, essentially at the surfaces, so that the UF is obtained at each point around the hole edge.

Regarding the use of the elastic-plastic correction factor K_e for fatigue analysis there are 2 possibilities that we intend to explore with a study case:

- A global one which consists in the application at each point around the hole edge of the maximum value $K_{e,max}$ estimated in the small ligament,
- A detailed one (or at local position) which consists in using the $K_e(\phi)$ at each angle ϕ around the hole edge.

INTRODUCTION

In a previous paper, Billon and Jourden (2024) showed that when the value $3S_m$ is exceeded in the small ligament of the ERP, an elastic-plastic correction factor K_e* must be considered for the TP's fatigue analysis but surely different from the value calculated according to the Code. The approach is summarized in the next part of this paper.

However, the accurate method of application of K_e* needs to be detailed because the analysis shows that when the angle ϕ varies away from the small ligament at ϕ = 0, and particularly between ϕ = 0 and 45°C, the plasticity at the hole edge varies.
Defining the modalities of application of the K_e* is the subject of this paper and is developed hereafter.

SUMMARY OF PREVIOUS STUDY

Approach

The approach adopted to evaluate the considering of the elastic-plastic correction factor is based on the FEM elastic and elastic-plastic analysis of the ERP of the tubesheet according to the steps described previously (steps 1 to 3 described in the Abstract).

We start from a model representing 9 holes of a tubesheet with square pitch, subjected to a state of biaxial stress (σ_X, σ_Y) so that $\sigma_Y = \beta.\sigma_X$, see Figure 2. β is a coefficient of bi-axiality that varies between -1 and 1. σ_X is the controlling stress and σ_y is the controlled stress.

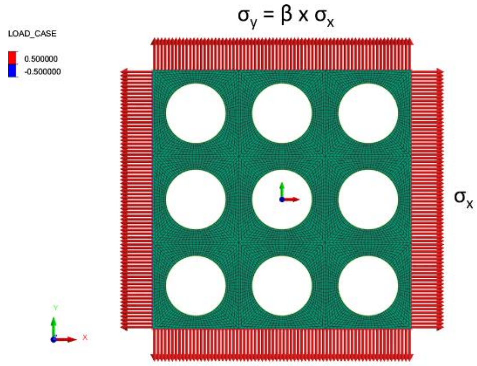

Figure 2. Stresses applied to the model

Mesh, loads and boundary conditions

Using the numerical tool SYSTUS™, a 2D translation model representing 9 holes of a square pitch drilling tubesheet (ERP) was used, see Figure 3.
The drilling pitch is 27.43mm and the diameter of the hole is 19.28 mm. Tubes are not considered.
The finite elements are 8-nodes quadrangles and sixteen elements are arranged in the small ligaments (thickness 8.15mm).

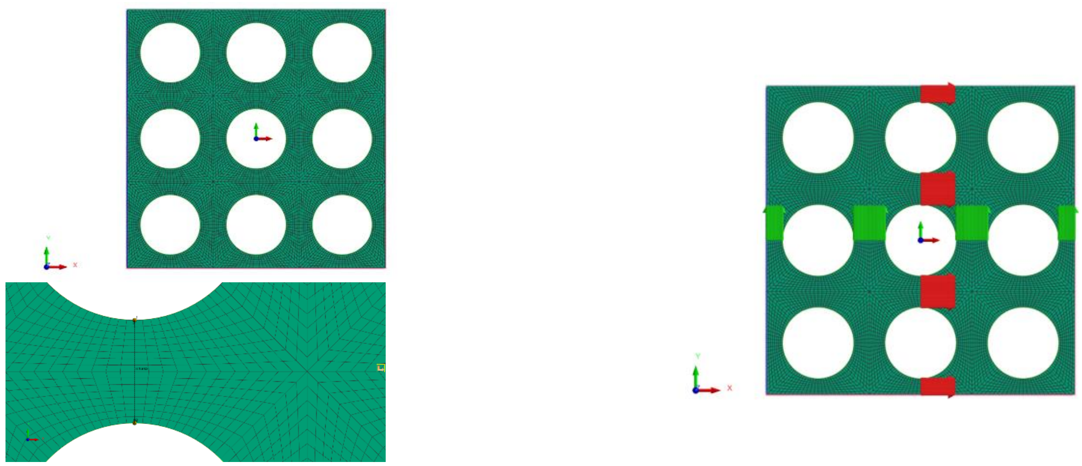

Figure 3. Mesh and boundary conditions

The model is subjected to a state of biaxial stress (σ_X, σ_Y) so that $\sigma_Y = \beta.\sigma_X$, see Figure 2. β is a coefficient of bi-axiality varying between -1 and 1. For a given value of β, the value of σ_X is incremented so that the amplitude of stress variation in the small ligament reaches 6Sm.

The boundary conditions are as follows:

- For nodes belonging to the horizontal symmetry axis of the mesh: blocking of vertical displacement.
- For nodes belonging to the vertical symmetry axis of the mesh: blocking of horizontal displacement.

These boundary conditions allow a homothetic symmetrized expansion of the central hole. In addition, it is required that the 4 faces of the model remain flat.

Elastic and elastic-plastic calculations

The application of the $3S_m$ rule involves an amplitude of stress variation. Here the applied load being of monotonic type, the stresses identified at each increment of the calculation correspond to a half-amplitude of stress variation.
Thus, the range for which the amplitude of stress variation in the small ligament varies from $3S_m$ to $6S_m$ corresponds to the range where the stresses found in this same ligament vary from $1.5S_m$ to $3S_m$.

Elastic-plastic calculation involves the reduced cyclic plastic law ($\Delta_{\varepsilon stab}/2$, $\Delta_{\sigma stab}/2$) of RCC-MRx for a low alloy steel type 18MND5. The law is shown in Figure 4.

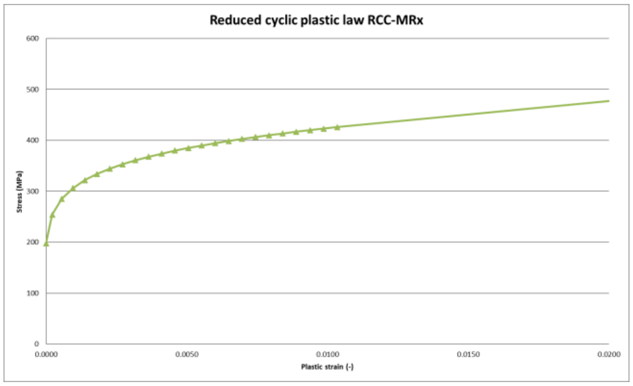

Figure 4. Reduced cyclic plastic law

The N_{adm} are then calculated according to the RCC-M Appendix ZC using the design ($S_{alt} – N$) fatigue curve of the material.

Results for β = 1 and first conclusions

The results, given hereafter, are associated to a value of $\beta = 1$ which means $\sigma_Y = \sigma_X$ at each time step of the calculation.

From the results of the elastic calculation, 2 curves giving the number of allowable cycles N_{adm-e} as a function of the applied load are built as follows:

- Without elastic-plastic concentration factor K_e
- With elastic-plastic concentration factor K_e calculated according to paragraph B 3234.6 of RCC-M code

The curve giving the number of allowable cycles N_{adm-ep} according to the applied load is also built for elastic- plastic calculation.
The resulting curves are then compared to determine whether, or not an elastic-plastic correction is justified, see Figure 5.

Numbers of allowable cycles N_{adm} are plotted as a function of the ratio $\zeta = S_n / 3S_m$, ζ varying from 1 to 2. It corresponds to the validity domain of formula giving K_e as a function of ζ, see paragraph B 3234.6 of RCC-M code.

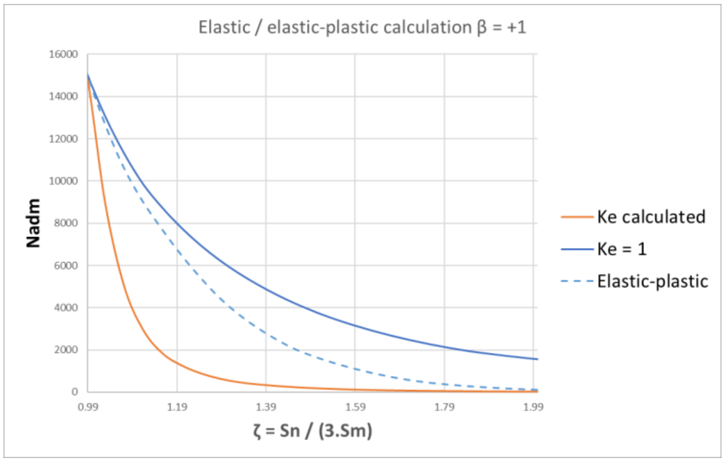

Figure 5. N_{adm}=f(ζ) curves comparison

At this stage we note that an elastic-plastic correction factor K_e* must be considered but depending on the biaxiality of the load of the ERP, its value differs (more or less) from the value obtained with the parameters codified in the RCC-M Code.
However, the accurate method of application of K_e* needs to be detailed because the analysis shows that when the angle ϕ varies away from the small ligament at ϕ = 0, and particularly between ϕ = 0 and 45°C, the plasticity at the hole edge varies.

PRELIMINARY OBSERVATIONS FOR THE CALCULATION OF K_E*

The aim of this part is to develop a method for considering an elastic-plastic concentration factor K_e* at hole edge. Analyses presented in what follows are more qualitative than quantitative.

Before presenting a method for K_e* calculation, important observations are made and illustrated hereafter:

- The development of plasticity at hole edge depends on biaxiality coefficient β
- The level of plasticity at hole edge depends on ϕ angle around hole

Development of plasticity at hole edge

Nine calculations were performed on the model described before with nine different β values: β varying from –1 to 1 with a step of 0.25.

For each β value, the total plastic strains contours are plotted for one given σ_X value, thus strains contours are plotted for the same σ_X value for each of the nine calculations.

The contours are plotted close to the ERP central hole.

Plastic strains contours are given below for five β values: 1 / 0.5 / 0 / -0.5 / -1, see Figure 6.

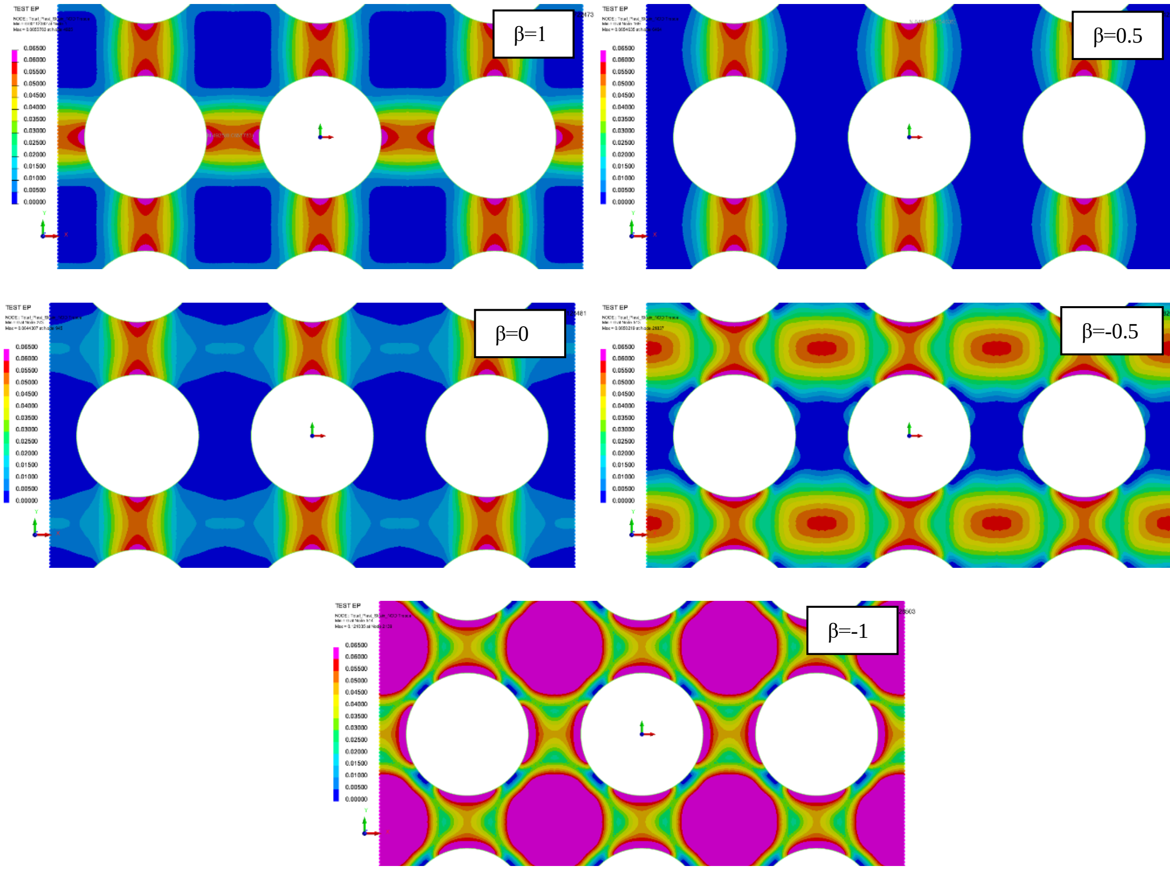

Figure 6. Total plastic strains contours for different β values

From these figures, observation is done that the development of plasticity at hole edge depends on biaxiality coefficient β. The formulation of K_e* should depend on β, so:

$$K_e^* = f(\beta) \tag{1}$$

Level of plasticity at hole edge

In addition, total plastic strains profiles are plotted for the same β values and same σ_X value, around central hole edge, see Figure 7.

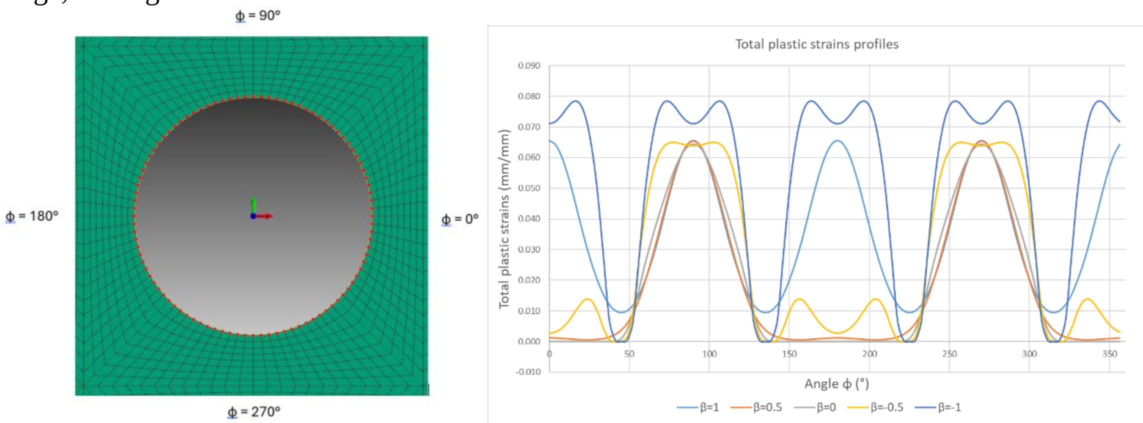

Figure 7. Total plastic strains profiles for different β values

From these profiles, observation is done that the level of plasticity at hole edge widely depends on ϕ angle. Moreover, observation is also done that level of plasticity in small ligaments located at $\phi = 0°$ and $\phi = 180°$ depends on biaxiality coefficient β while this is not the case (or less) for small ligaments located at $\phi = 90°$ and $\phi = 270°$.
The plasticity in ligaments located at $\phi = 0°$ and $\phi = 180°$ is mainly driven by $\sigma_Y = \beta.\sigma_X$, loading dependent on β, while plasticity in ligaments located at $\phi = 90°$ and $\phi = 270°$ is mainly driven by σ_X, loading independent of β.

The formulation of K_e* should also depend on angle ϕ, so:

$$K_e^* = f(\beta, \phi) \tag{2}$$

APPROACH FOR THE DETERMINATION OF K_E*

In this part we treat the case $\beta = +1$ ($\sigma_Y = \sigma_X$) but the same principles apply for other β values.

In a first step, from the elastic and elastic-plastic calculations, we can calculate the K_e* value at each time step of the calculation according to paragraph ZC 3440 of RCC-M.
At each time step of the calculation, $S_{n,ref}$ value being known, we can plot a curve giving K_e* as a function of $\zeta = S_{n,ref} / 3S_m$.
This curve is compared to the one obtained by calculating $K_{e\text{-}code}$ according to paragraph B 3234.6 of the RCC-M, see Figure 8 .
This K_e* value is calculated at hole edge at the small ligament position; $S_{n,ref}$ is the S_n value calculated in the small ligament perpendicular to σ_X load.

The formula for K_e* calculation is the following: $K_e^* = \Delta\varepsilon_t$ (elastic-plastic) / $\Delta\varepsilon_t$ (elastic).

In addition, we plot the curve giving $\gamma = K_e^* / K_{e\text{-}code} = f(\zeta)$. This parameter γ can be used to express K_e* as a function of K_e codified.

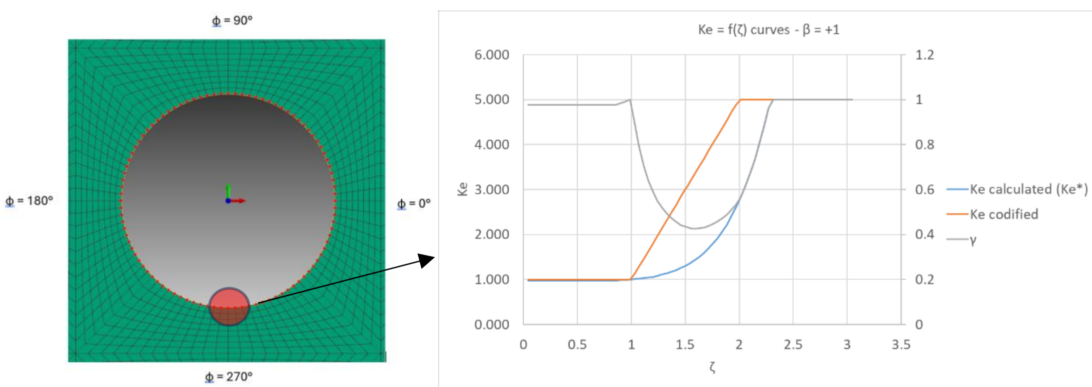

Figure 8. $K_e^* = f(\zeta)$ curves – $\beta = +1$

As expected in the previous paper (Billon and Jourden, 2024), K_e calculation according to paragraph B 3234.6 of RCC-M ($K_{e\text{-}code}$) seems to give a bounding value of the calculated one (K_e*).
From these curves we can write:

$$K_e^* = \gamma \times K_{e\text{-}code} = \gamma \times f(\zeta) = \gamma \times f(S_{n,ref} / 3S_m) \tag{3}$$

With function "f" as defined in the RCC-M.

In a second step, to treat the angles ϕ around hole, our proposal is to calculate $K_e*(\phi)$ at each angle around hole, see Figure 9. We start from the small ligament (270°) and go to the large one (315°).

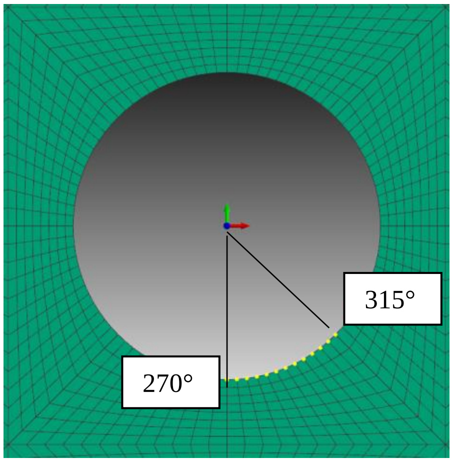

Figure 9. Locations for $K_e*(\phi)$ calculation

Curves giving $K_e*(\phi)$ are plotted as a function of $\zeta = S_{n,ref} / 3S_m$, see Figure 10.
We also plot on Figure 10 the curves $\Lambda(\phi) = K_e*(\phi) / K_e*(270°)$.

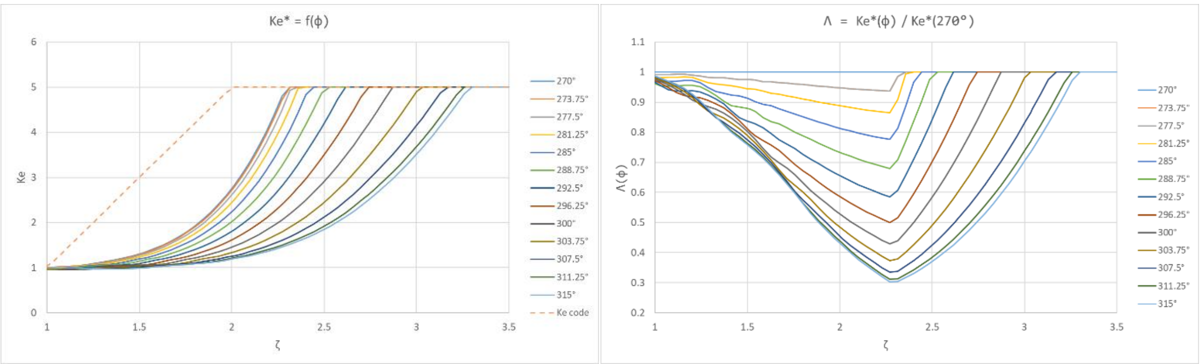

Figure 10. $K_e*(\phi)$ and $\Lambda(\phi)$ curves

From these curves we can write:

$$K_e* = \Lambda(\phi) \times K_e*(270°) = \Lambda(\phi) \times \gamma \times K_{e\text{-}code} = \Lambda(\phi) \times \gamma \times f(\zeta) = \Lambda(\phi) \times \gamma \times f(S_{n,ref} / 3S_m) \qquad (4)$$

This formula is obtained by analysing the case $\beta = +1$ but the method can be extended to other β values. Finally, K_e* should depend on β, ϕ and $S_{n,ref}$.

APPROACH FOR THE APPLICATION OF K_E*

In this last part, we suggest an approach for the implementation of K_e* for fatigue analysis. As reminded in the abstract, stress analysis in Tube Plate perforated medium is usually based on a modelling by a homogeneous material with equivalent properties.

Several linearization paths (cross-sections) are arranged in the homogenous material to calculate stresses through the TP thickness, particularly on primary and secondary surfaces.
From these stresses, see Figure 11, we can calculate, through specific multiplying functions:

- Mean stress in the small ligament (for progressive deformation analysis),
- Tangential stress around hole edge (for fatigue analysis).

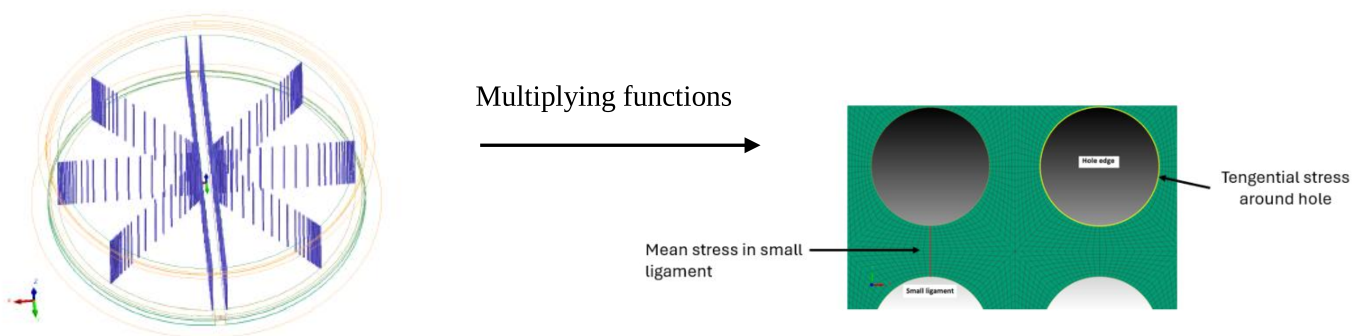

Figure 11. Use of multiplying functions

For the implementation of the approach, the phasing would be as follows:

Step	Action
1	For each time step of each thermal transient, stress calculation along cross-sections arranged in the TP homogenous material (nominal stresses)
2	From these nominal stresses and multiplying functions, calculation of: - normal stress in small ligament - tangential stress around hole edge: $\sigma_{\phi\phi}$
3	From $\sigma_{\phi\phi}$ stresses calculated in 2, run of a first fatigue analysis and extraction of thermal transient combinations contributing to Usage Factor: UF
4	For each combination identified in 3, extraction of normal stress intensity range in small ligament S_n and comparison to $3S_m$
5	If $S_n \leq 3S_m$, criterion related to progressive deformation is reached and no K_e needs to be considered -> $K_e = 1$ is affected to associated transient combinations (S_{alt}). If $S_n > 3S_m$, criterion related to progressive deformation is not reached and K_e needs to be considered for associated transient combinations -> go to 6
6	For each transient combination leading to $S_n > 3S_m$, extraction of associated in-plane nominal stress state (σ_X, σ_Y)
7	From in-plane nominal stress state (σ_X, σ_Y), determination of biaxiality coefficient β
8	Biaxiality coefficient β, angle ϕ around hole and $S_{n,ref}$ being known -> K_e* calculation for all ϕ angles
9	K_e* affectation to the analysed transient combination (S_{alt})
10	Run of a second iteration of fatigue analysis taking K_e* into account

CONCLUSION

In a previous paper (Billon and Jourden, 2024), we showed that when the value $3S_m$ is exceeded in the small ligament, an elastic-plastic correction factor K_e* must be considered for the TP's fatigue analysis but different from the value calculated according to the RCC-M code.

However, given the development and level of plasticity at hole edge (see Figure 6 and Figure 7) the accurate method of application of K_e* needed to be detailed.

The suggested approach is summarized hereafter, for one given β value (biaxiality coefficient):

1- Direct calculation of K_e*(270°) at hole edge right to the small ligament and expression of this calculated K_e* value as a function of $K_{e\text{-code}}$, itself function of $S_{n,ref}$ / $3S_m$,
2- To treat ϕ angles around hole: direct calculation of K_e*(ϕ) values and expression of these values as a function of K_e*(270°).

It is then possible to determine K_e* values at each ϕ angle only as a function of $S_{n,ref}$ and apply theses values to fatigue analysis. Biaxiality coefficient and behaviour of the small ligament perpendicular to σ_X load (controlling stress) are the only data which are needed to determine K_e* values.

Note: A further work would be to try to express the values of Ke*(ϕ) around the hole directly as a function of the values of Sn(ϕ). Our idea would be to calculate S_n on ϕ-oriented support segments, as shown in Figure 12, and to relate S_n(ϕ) to $S_{n,ref}$ in the small ligament.

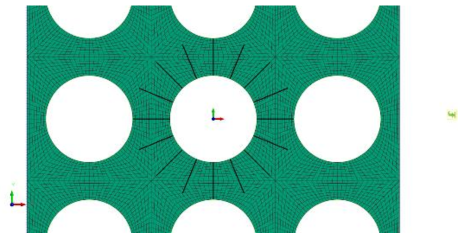

Figure 12: view of the support segments for the calculation of Sn(ϕ)

NOMENCLATURE

2D	Two-dimensional
$3S_m$ rule	Rule of RCC-M paragraph B3234.2 that aims at limiting the amplitudes of variation of the sum of the primary and secondary stresses at 3Sm against progressive deformation failure mode.
a_ϕ, b_ϕ	Multiplying functions associated to σ_x and σ_y
β	Biaxiality coefficient such as $\sigma_y = \beta$ x σ_x, β varying from -1 to 1
γ	Ratio K_e* / $K_{e\text{-code}}$
$\Delta\varepsilon_{stab}$, $\Delta\sigma_{stab}$	Strain and stress amplitudes related to reduced cyclic curve
ε	Strain
ε_{alt}	Alternate elastic-plastic strain
$\varepsilon_{\phi\phi}$	Circumferential strain at the hole edge
ζ	Ratio S_n / $3S_m$
Λ	Ratio K_e*(ϕ) / K_e*(270°)
σ	Stress
$\sigma_{\phi\phi}$	Circumferential stress at the hole edge equal to $a_\phi.\sigma_x + b_\phi.\ \sigma_y$

σ_x, σ_y	Stress along X and Y directions
ϕ	Angle around hole
ERP	Elementary Representative Portion of the tubesheet, here made of 9 holes, analyzed by the FEM
FEM	Finite Elements Method
K_e	Elastic-plastic correction factor for fatigue analysis. Ratio between elastic-plastic strain and elastic strain.
$K_{e\text{-code}}$	Elastic-plastic correction factor for fatigue analysis, defined in RCC-M
K_e^*	Elastic-plastic correction factor applied for tubesheet fatigue analysis
$K_e^*(270°)$	K_e^* in small ligament
N_{adm}	Number of allowable cycles
$N_{adm\text{-}e}$	Number of allowable cycles calculated from elastic calculation
$N_{adm\text{-}ep}$	Number of allowable cycles calculated from elastic-plastic calculation
Progressive deformation	Progressive deformation, or progressive plastic deformation or ratchetting is a failure mode with regard to accumulation of plastic strain under alternate cyclic stress
S_{alt}	Alternating stress
S_m	Equivalent allowable stress
S_n	Primary + secondary stress intensity range
TP	Tube Plate
Tubesheet	Perforated part of the TP crossed by the exchanger tubes. The other non-perforated parts of the TP ensure permanent connections by welded joints with the shell (shell and tubes heat exchanger) and the inlet channel head
UF	Usage Factor

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