Use of modal damping in direct time-history analysis

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1 INTRODUCTION

The analysis of complex structures subjected to arbitrary dynamic loading is usually performed either by mode superposition or direct integration (Bathe and Wilson 1976) method. The mode superposition method is generally preferred unless a large number of modes are required to adequately describe the system response to the loading under consideration. If higher modes are needed, the direct integration method may be cost effective. However, most dynamic problems encountered in complex structural design require only a few modes to compute dynamic responses with reasonable accuracy. The mode superposition method is therefore quite attractive to engineers.

On many occasions the extraction of complex system modes may be nearly impossible due to numerical difficulties encountered in analysis. A typical example is a structural system embedded in a soft soil deposit. The direct integration method can eliminate such difficulties. A major disadvantage, however, is the difficulty of applying damping values that are consistent with modal damping values.

The direct integration method forces the user to employ unique damping values at each modal frequency. On the other hand, the mode superposition method allows arbitrary damping at various modal frequencies. Since damping plays an important role in dynamic analysis, restricted use of modal damping in the direct integration method can be a major obstacle to engineers when using this method. The author's intent is to eliminate such difficulty.

In this approach, response characteristics of a complete structural system are obtained by using the direct integration method with any arbitrary set of mass and stiffness proportional (α-β) dampings. These response and force time-histories are then transformed into frequency domain to obtain transfer function as the ratio of response to load. Once the transfer function of a response component is known, the product of mode shape and modal participation factors, which is a scalar quantity, and eigenvalues associated with the response component can be extracted from the complex transfer function. It should be noted that these modal characteristics are independent of any structural damping used in the direct integration method. These modal characteristics can subsequently be used to reconstruct the transfer function of the system using any desired damping.

It should also be noted that it is unnecessary to compute mode shapes and modal participation factors separately to reconstruct transfer functions. The time-history response of the system can then be easily computed using the updated transfer function.

Two example problems have been analyzed to illustrate the convenience and effectiveness of the proposed method, and to demonstrate its practical value.

2 MATHEMATICAL FORMULATION

The transfer function of a multi-degree-of-freedom system (Singh et al. 1979) subjected to time dependent forcing function can be expressed as

$$T_k(\omega_\ell) = \sum_{j=1}^{N} \frac{x_{jk}}{\omega_j^2 - \omega_\ell^2 + 2i\omega_\ell\omega_j d_j} \tag{1}$$

in which

$$x_{jk} = \phi_{jk}\psi_j \tag{2}$$

$$\psi_j = \phi_j^T r \tag{3}$$

$$d_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2} \tag{4}$$

where ω_j, ϕ_j, and ψ_j, respectively, are the frequency, mode shapes, and participation factors for the jth mode; N is the number of modes; the vector r represents the relative magnitude of loads acting at various structural nodes; d_j is the mass and stiffness proportional damping at jth modal frequency; α, β are damping constants; and

$$i = \sqrt{-1} \ .$$

The variable x_{jk} in Equation 1 is a scalar quantity obtained by multiplying mode shape with participation factor. Therefore, a transfer function can be defined by only three scalar quantities (e.g., x_{jk}, ω_j, and d_j), as shown in Equation 1. It should be noted that mode shapes and participation factors of the system need not be computed separately in estimating a transfer function. If the complex transfer function of a given response component is known, then the variables x_{jk} and ω_j can be obtained. The frequency at which the transfer function magnitude is maximum represents a modal frequency (ω_j) of the system. Once the modal frequencies ω_j are determined, the scalar variable x_{jk} an be obtained by solving a set of complex simultaneous algebraic equations formed by expanding Equation 1.

The response of a system component k in the frequency domain can be obtained by multiplying the transfer function of the response component with the Fourier transform of the loading function, for example

$$R_k(\omega) = T_k(\omega) \times F(\omega) \tag{5}$$

where

$R_k(\omega)$ = frequency domain response of kth component,
$T_k(\omega)$ = transfer function of kth component, and
$F(\omega)$ = Fourier transform of the loading function.

The new transfer function $\bar{T}(\omega)$ for any desired damping $\bar{d}_j$ can be obtained from the following equation:

$$(6) \qquad \bar{T}_k(\omega_\ell) = \sum_{j=1}^{N} \frac{c_j x_{jk}}{\omega_j^2 - \omega_\ell^2 + 2i\omega_\ell \omega_j \bar{d}_j}$$

The modal damping value $\bar{d}_j$ can be arbitrarily chosen in Equation 6. If it is also desirable to update the response by a modal correction factor c_j for the jth mode, then the corrected transfer function can be expressed as

$$(7) \qquad \bar{T}_k(\omega_\ell) = \sum_{j=1}^{N} \frac{c_j x_{jk}}{\omega_j^2 - \omega_\ell^2 + 2i\omega_\ell \omega_j \bar{d}_j} .$$

In a multi-degree-of-freedom system of order N, the transfer function of any given response component may have only n(<N) number of peaks in the transfer function. In that case, the size of matrix in Equation 1 will only be n x n. This indicates that the response component under consideration has modal contribution from only n number of modes. In practical design problems, n is far less than N. Hence, little additional computation is needed to employ modal damping in the direct integration method. The method of using modal damping in the direct integration time-history analysis is therefore not cost prohibitive.

3 EXAMPLE PROBLEMS

To demonstrate the validity and effectiveness of the proposed method, two example problems were analyzed. The results obtained from these analyses are discussed in the following sections.

3.1 Uniaxial Beam

To demonstrate the validity of the method, a simple uniaxial cantilever beam with three-degrees-of-freedom (d.o.f.), as shown in Figure 1, is analyzed.

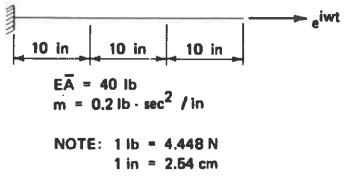

Figure 1. Uniaxial Beam Model

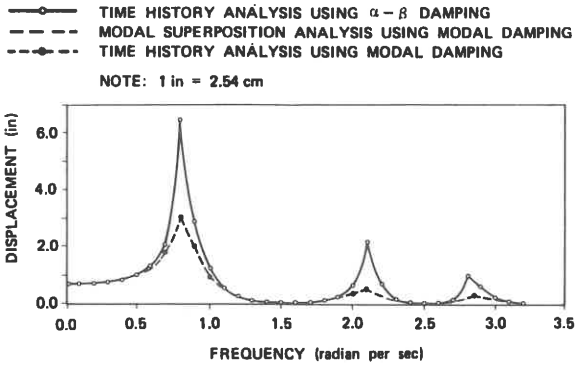

Figure 2. Tip Displacement Transfer Function of a Uniaxial Beam

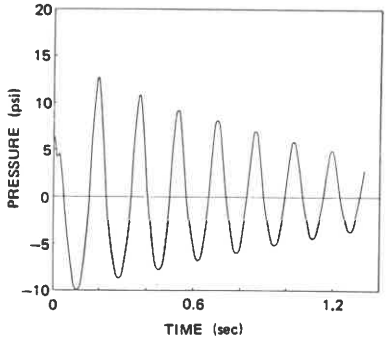

Figure 3. Typical Torus Shell Wall Pressure Time History (1 psi = 0.6894 N/cm^2)

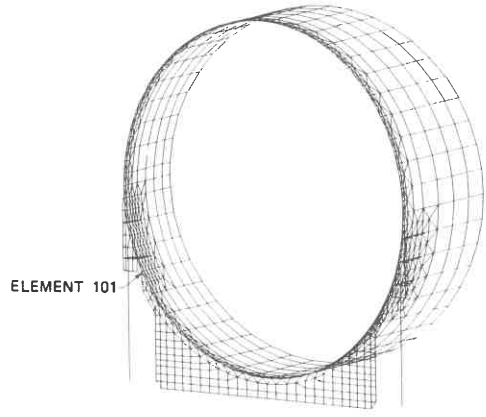

Figure 4. 1/32 Sector Suppression Chamber Finite Element Model

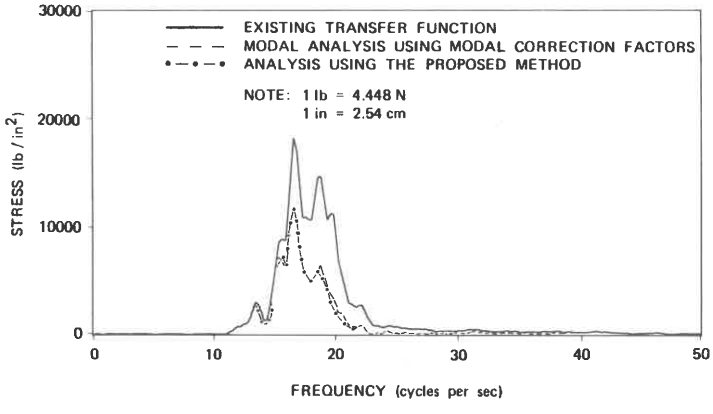

Figure 5. Transfer Function Comparison

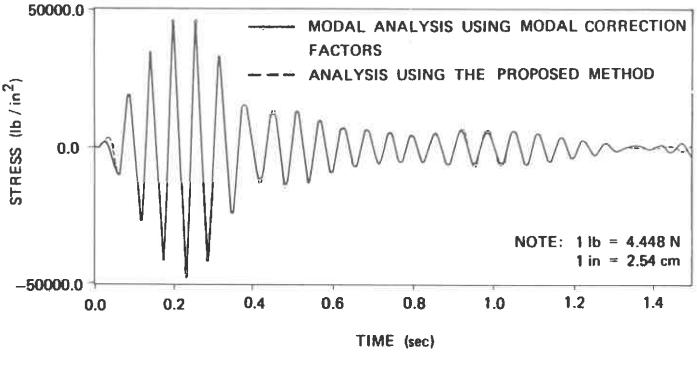

Figure 6. Time History Comparison

The transfer function of the tip displacement of the beam is obtained using α-β damping and is shown in Figure 2. A damping value of 2 percent is used in the analysis. The constants α and β are computed by matching damping values at 2 and 6 radians per second. The transfer function for a uniform damping of 10 percent is computed using the proposed method (Equation 5) as well as the modal superposition method with a uniform modal damping of 10 percent and is shown in Figure 2. An excellent agreement is found between these two transfer functions, thus verifying the proposed method.

3.2 Toroidal Shell

To demonstrate the effectiveness of the method when analyzing a large practical problem, a shell structure with 3957 d.o.f. is analyzed. The toroid suppression pool chamber is an integral part of the General Electric Boiling Water Reactor (BWR) Mark I power generation system (GE Report 1978). In the event of a system accident of safety relief valve actuation, large amounts of steam are generated. This steam is fed into a suppression chamber for condensation. During the condensation process, various types of loads are generated on the pool surface. A typical load time history is shown in Figure 3. The finite element model of the torus shell consists of a 1/32 sector of the toroid suppression chamber with columns, and is shown in Figure 4. The purpose of this analysis is to update modal response by applying modal correction factors to the existing responses using Equation 7. The existing transfer function of a typical membrane stress component is shown in Figure 5. The modal correction factors to be applied to the existing analysis were obtained from an empirical approach and are given in Table 1. The existing transfer function is used to recover all important modal variables using Equation 1. The transfer function is regenerated by applying modal correction factors using Equation 7. The corrected transfer function is then used along with the load time-history to obtain the response time-history using Equation 5, and is shown in Figure 6. To validate these results, a modal analysis is also performed. A total of 72 modes are extracted for obtaining frequencies up to 50 hertz. The modal correction factors are applied directly to each modal response before they are combined to obtain the summed response time-history. This response time-history is compared with the result obtained using the proposed method in Figure 6. An excellent agreement is found. It should be noted that few modes are considered in recovering the stress component modal characteristics using Equation 1. These modal characteristics can also be utilized, if necessary, to obtain responses for a new set of modal damping values.

4 SUMMARY

The conventional direct integration time-history method in dynamic analysis uses mass and stiffness proportional damping. However, it would be useful to employ modal damping in this method for a class of practical problems. Presently, there is no direct method available for such analysis. This paper demonstrates how to use modal damping in the direct integration time-history method of dynamic analysis.

Table 1 - Modal Correction Factors

$\bar{f}$ = 18.87 Cycles per Second	
$\bar{f}$/Modal Frequency	Correction Factors
0.0	1.0
0.65	1.0
0.7	0.9
0.8	0.7
0.9	0.5
1.0	0.3
1.1	0.5
1.2	0.6
1.3	0.9
1.35	1.0
1.4	1.0

REFERENCES

Bathe, K.J., and Wilson, E.L., Numerical Methods in Finite Element Analysis, Prentice-Hall, Englewood Cliffs, N.J., 1976.

Singh, A.K., Holmes, N.A., Khatua, T.P., and Chu, S.L., "Response Using Dynamic Influence Coefficients," Seventh Conference on Electronic Computation, ASCE, St. Louis, Missouri, Aug. 1979.

General Electric Report NEDO-21888, "Mark I Containment Program Load Definition Report," Rev. 0, December 1978.