A Solution of Vibrational Response of Reactor Components to Random Exciting Forces Due to Coolant Flow

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This paper presents the derivation and computer implementation of the mathematical model for a numerical solution of the reactor component responses to the effects of random forces induced by a turbulence of the coolant flow. The model assumes that the reactor component behaves like an r-degree-of-freedom system. However, an i-th element of the system is regarded a continuum. The equations of motion are being solved using the modal analysis and Fourier transforms. The relations for the numerical solution of the response are expressed in matrix form. The computer implementation of the model, applied to cohesive fuel bundles, has been demonstrated in the system with eight degrees of freedom. The velocity vector constitutes the basis for the description of fluidodynamic excitation through the coherence function of the fluctuating surface pressure. The obtained results of the generalized spectral compliances (frequency response function) indicate that the share of fluidodynamic excitation in the bundles response is small within the entire range of the exciting frequencies, with the only exception of the vicinity of the eigenfrequencies of the system. This share reaches its maximum for the basic mode and it is lower for higher modes. It can be judged from the courses of the generalized spectral loads (acceptance integral) that with the increasing intensity of the turbulent excitation, the intensity of the spectral load shifts into the domains of higher exciting frequencies. The values of the mean square radial displacement, induced by so-called joint terms, are of a higher order in comparison with the values resulting from cross terms. The calculated r.m.s. values of the radial displacement obtained for peak frequencies within the defined frequency band are in a good agreement with the measured linear spectral density of the displacement. Since the wear of the moving components depends on the amplitudes thereof, the obtained values of displacement, expressed for instance in terms of the r.m.s. values, can be related to the wear of the fuel bundles and the pressure tube. Using the model, e.g. the operational safety of the reactor core component can be evaluated.

1. Introduction

From practice we know of numerous cases of failure of the structures exposed to the surrounding flow. The failures have been caused by vibration of an aero-hydrodynamic origin. Even nuclear reactors [1, 2] have not been exempted from experiencing such failures, and therefore the reliability and safety of the reactors have currently been given special attention.

The vibrations induced by a fluid flow are generally very complex phenomena. In this paper a comprehensive mathematical model for a numerical solution of the structure stochastic response, excited by a turbulent flow, is formulated.

2. The mathematical model of the response

The solution of the problem of the structure response to the effects of random forces induced by the surrounding flow is generally based on our knowledge of the frequency modal properties of the structure under consideration, and that of exciting forces with a statistic characteristic, expressed in terms of the fluctuating prėssure of the fluid flow in place of its contact with the structure [3]. In the following, we will assume that the frequency and modal properties of the investigated structure are known. For a theoretical solution of these properties (eigenfrequencies and eigénfunctions above others) we usually replace the considered dynamic problem by a problem with a finite number of degrees of freedom, e.g. in such a way that the structure is substituted by a number of elements, which are mutually linked with a finite number of nodes. In these nodal points we will concentrate the element mass and will apply the equilibrium and compatibility requirements. We will keep to this idea even while formulating the mathematical model, based on [4], for the considered structure response to the random force effects, induced by the surrounding flow. Instead of a system with an infinite number of the degrees of freedom, which is represented by the structure with continuously distributed mass and stiffness, we thus get a system of an r-degree freedom.

Let us assume that this system is subject to exciting forces $\{f(t)\}$ of a stochastic nature, which arised due to the fluctuating pressure $p(t)$ of the fluid flow, perpendicular to the surface of the structure. The vector of the stochastic displacement $\{y(t)\}$ of the system is governed by the equation

$$[m]\{\ddot{y}(t)\}+[b]\{\dot{y}(t)\}+[k]\{y(t)\} = \{f(t)\}, \qquad (1)$$

where $[m]$ is the mass matrix including the added mass of the fluid; $[b]$ is the equivalent viscous damping matrix, including the total damping of the system, i.e. the structural damping - and the hydraulic damping - in case the fluid is viscous; $[k]$ is the stiffness matrix including the fluid influence, which represents the system in terms of stiffness distribution.

To simplify the problem, we assume the coupling between the exciting forces and the response as well as the coupling between particular vibration modes, as non-existent. As for the random load, we assume the exciting fluctuating pressure field to be stationary and ergodic. Consequently, $y(t)$ can be expressed as a linear combination of the displacement w_α due to the natural vibrations of the system. The particular integral of eq. (1) is then sought in the form

$$\{y(t)\} = [w]\{q(t)\} , \tag{2}$$

where $[w]$ is the modal matrix containing r modal vectors - $\{w_\alpha\}$ and is the column vector of r rows with generalized Lagrange coordinates $q_\alpha(t)$ for the modes. For α-th generalized coordinate we thus get an equation of motion

$$\ddot{q}_\alpha(t) + 2\xi_\alpha \omega_\alpha q_\alpha(t) + \omega_\alpha^2 q_\alpha(t) = f_\alpha(t) m_\alpha^{-1} , \tag{3}$$

where $\xi_\alpha = b_\alpha (2m_\alpha \omega_\alpha)^{-1}$ is the generalized (equivalent viscous) damping ratio; ω_α is the angular frequency of the α-th mode of considered structure in a surrounding medium of a generalized mass m_α, given

$$m_\alpha = \{w_\alpha\}^T [m] \{w_\alpha\} . \tag{4}$$

The generalized random force $f_\alpha(t)$ for the α-th mode in eq. (3) is given by the relation

$$f_\alpha(t) = \{w_\alpha\}^T \{f(t)\} , \tag{5}$$

which, applying the principle of virtual work in the case of small displacements, we can rewrite

$$f_\alpha(t) = \{w_\alpha\}^T \{\int_{S_i} p(A,t)\, dA\} , \tag{6}$$

where $p(A,t)$ is the fluctuating pressure in a place A on the contact surface S_i of the i-th element of the structure[x)].

Applying Fourier transform to eq. (3) we can expressed the generalized frequency response function of the α-th mode $H_\alpha(j\omega)$, which we will call the generalized spectral complex compliance of the α-th mode[xx)]

$$H_\alpha(j\omega) = (m_\alpha(\omega_\alpha^2 - \omega^2 + j2\xi_\alpha \omega_\alpha \omega))^{-1} . \tag{7}$$

Transforming eq. (2) in terms of Fourier function we get

$$\{Y(j\omega)\} = [w]\{H_\alpha(j\omega) F_\alpha(j\omega)\} , \tag{8}$$

where $Y(j\omega)$, $F_\alpha(j\omega)$ are Fourier images of $y(t)$, $f_\alpha(t)$. The vector of the power spectral density functions of the displacement $\{S_y(\omega)\}$ is related to $\{Y(j\omega)\}$ through the expression

$$\{S_y(\omega)\} = \lim_{T\to\infty} \frac{1}{T} \{Y(j\omega) Y^*(j\omega)\} , \tag{9}$$

where the asterisk signifies the conjugated complex function and T is a half of the realization time of the considered random process. Having inserted into (9), while omitting the phase (for $\alpha \neq \beta$), we get

$$\{S_y(\omega)\} = [w_\alpha w_\beta]\{|H_\alpha(j\omega)||H_\beta^*(j\omega)||L_{\alpha\beta}^{(i)}(j\omega)|\} , \tag{10}$$

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x) The i-th element of the structure is regarded as a continuum.

xx) The absolute value $|H_\alpha(j\omega)|$ is " merely" the generalized spectral compliance.

where $L^{(i)}_{\alpha\beta}(j\omega)$ is the generalized spectral load[x)], given for an i-th element of the structure

$$L^{(i)}_{\alpha\beta}(j\omega) = \iint_{S_i S_i} w_\alpha(A_1)\, w_\beta(A_2)\, S^{(i)}_{p_1p_2}(A_1,A_2,j\omega)\, dA_1 dA_2 \,. \qquad (11)$$

New symbol $S^{(i)}_{p_1p_2}$ means the cross (spatial) power spectral density of the fluctuating pressure field on the contact surface of the i-th element. The relation (11) represents one of the possible forms of so-called acceptance integral, first introduced by A. Powell [5].

If we introduce the coherence function of the exciting pressure field $G^{(i)}_{p_1p_2}(A_1,A_2 j\omega)$ for the i-th element

$$G^{(i)}_{p_1p_2}(A_1A_2,j\omega) = S^{(i)}_{p_1p_2}(A_1,A_2,j\omega)\left(S^{(i)}_p(A_1,\omega)\, S^{(i)}_p(A_2,\omega)\right)^{-1/2}_{;} \qquad (12)$$

where $S^{(i)}_p(A_1,\omega)$, $S_p(A_2,\omega)$ are power spectral densities of the exciting fluctuating pressure at points A_1 and A_2, then the generalized dimensionless spectral loads $l^{(i)}_{\alpha\beta}(j\omega)$ can be expressed

$$l^{(i)}_{\alpha\beta}(j\omega) = S_i^{-2} \iint_{S_i S_i} w_\alpha(A_1)\, w_\beta(A_2)\, G^{(i)}_{p_1p_2}(A_1,A_2,j\omega)\, dA_1\, dA_2 \,. \qquad (13)$$

If $p(t)$ is due to a turbulent homogeneous flow, then eq. (12) can be written

$$G^{(i)}_{p_1p_2}(\Delta_{12},j\omega) = \left(\left(S^{(i)}_p(\omega)\right)\right)^{-1} S^{(i)}_{p_1p_2}(\Delta_{12},j\omega) \,, \qquad (14)$$

where $\Delta_{12} = x_{A_1} - x_{A_2}$ is the difference of the coordinates of the considered points A_1, A_2.

It follows from the properties of the cross spectral density

$$L_{\alpha\beta}(j\omega) = L^*_{\beta\alpha}(j\omega) \qquad (15)$$

and therefore, for $\alpha = \beta$ the generalized spectral load $L_{\alpha\alpha}(\omega)$ is a real function. Eq. (10) can be rewritten

$$\{S_y(\omega)\} = [w_\alpha^2]\{H^*_\alpha(j\omega)\, H_\alpha(j\omega)\, L^{(i)}_{\alpha\alpha}(\omega)\} + 2[w_\alpha w_\beta]\{|H_\alpha(j\omega)||\, H^*_\beta(j\omega)||\, L^{(i)}_{\alpha\beta}(j\omega)|\}, \qquad (16)$$

from which it is already obvious that the phases of the generalized spectral complex compliances and generalized spectral loads have been omitted only in the second term of the symbolic eq. (16), for which it holds $\alpha \neq \beta$. Considering the properties of the modal vectors $\{W_\alpha\}$ and those of the functions of the spectral compliances $H_\alpha(j\omega)$ and the functions $L^{(i)}_{\alpha\beta}(j\omega)$, the major contribution to the values of the vector of the functions $\{S^{(i)}_y(\omega)\}$ can be expected from the first term in eq. (16), i.e. from the joined factors, for which $\alpha = \beta$.

The vector of the mean square displacement $\{\bar{y}^2\}$ can be obtained from the power spectral density of the displacement through integration over all the angular frequencies ω

$$\{\bar{y}^2\} = \left\{\frac{1}{2\pi}\int_0^\infty S_y(\omega)\, d\omega\right\}, \qquad (17)$$

To evaluate the response of the stochastically excited structure we

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x) In terms of physics we interpret this load as a measure of the efficiency of the fluctuating pressure forces to excite the considered mode of vibration.

find it rewarding to use so-called r.m.s. values of the displacement $\bar{y}$, obtained for peak frequencies ω_q within a defined frequency band $\Delta\omega_q$. The vector of these $\{\bar{y}(\omega_q)\}$, which we will call the vector of the linear spectral density of the displacement, is computed from the following relation

$$\{\bar{y}(\omega_q)\} = \{(\frac{1}{2\pi}\int_{\omega_1}^{\omega_2} S_y(\omega_q)\,d\omega)^{1/2}\}, \tag{18}$$

where $S_y(\omega_q)$ is given by eq. (16).

3. Computer implementation of the model

In the computer implementation of the mathematical model of the response, we will simplify the problem choosing a one-dimensional structure, e.g. the CIRENE fuel string [6], which can be substituted by a system of r = 8 degrees of freedom. We assume that the frequency modal characteristics of the system are known. Thus the following is given: 1) the modal matrix $[w]$; 2) ω_α for $\alpha = 1,2,\dots 8$; 3) ζ_α for $\alpha = 1,\dots 8$; 4) the mass matrix $[m]$.

The exciting medium, the flow of which is the structure subject to, is characterized primarily by the mean velocity of the flow V_i, given in places i = 1,2,...8; further, by the coherence function of the fluctuating surface pressure $p(t)$, given by Bessel function of the first kind of the zero order

$$G^{(i)}_{p_1p_2} = J_0(\frac{\omega}{u_i}(x_{A_1} - x_{A_2})), \tag{19}$$

where u_i is the velocity of turbulent eddies at the circular frequency ω, and may it hold for

$$u_i = a_1 V_i \omega^{1/4}, \qquad \text{for } \omega \neq 0 \tag{20}$$

At it is evident from eq. (19), we take the simplifying assumption that the factor characterizing a coherence phase is omitted. The coherence function is therefore given, as it were, only by its magnitude. Further, we need to express the power spectral density of $p(t)$. Let it hold for it

$$S_p^{(i)} = a_2 V_i^4 \omega^{-1}, \text{ for } \omega \geq 1, \quad S_p^{(i)} = a_2 V_i^4, \qquad \text{for } \omega \in <0,1. \tag{21}$$

Computing the integral (13) necessitates finer division of W_α, i.e. in such a way that we know the coefficients of the distribution of the displacement amplitudes of the α-th mode W_α at more places of the i-th element of the system. In achieving this, the relation $W_\alpha(A) = \sin(\alpha\pi x_A h_i^{-1})$ will prove helpful. With regard to this we then compute $l^{(i)}_{(\alpha\beta)}$ from the following equation

$$l^{(i)}_{\alpha\beta} = h_i^{-2}\int_{x_{A_1}=0}^{h_i}\int_{x_{A_2}=0}^{h_i} \sin(\alpha\pi x_{A_1} h_i^{-1})\sin(\beta\pi x_{A_2} h_i^{-1})\, J_0(\frac{\omega}{u_i}(x_{A_1}-x_{A_2}))\,dx_{A_1}dx_{A_2}, \tag{22}$$

where h_i = 0,5 m is the length of the i-th element of the considered structure, applying to all i´s [6].

We have developed the ODEZ 1 code serving to compute the random excitation response, which enables us to determine the statistic quantities of the response according to the relation presented in the foregoing paragraphs.

The code was applied to the cohesive fuel bundles CIRENE ant its input represented by the ω_α , W_α , ξ_α , m was taken or estimated according to [6].

4. Numerical results and their discussion

The obtained values of the generalized spectral compliances $|H_\alpha(j\omega)|$ have been plotted in Fig. 1. It is primarily evident from them that the spectral compliances of the considered system reaches sharp peaks in the near vicinity of $\Delta\omega$ of the natural frequencies of the system ω_α . The maximum values depend upon the mode order. With the mode α increasing, the value $|H_\alpha(j\omega)|$ decreases. Consequently, we can say that within the considered system, the share of the aero-hydrodynamic excitation in the response throughout the total frequency range will be small, except for the vicinity of the natural frequencies of the system in the surrounding medium. This share reaches its maximum for the case of basic mode, and decreases in the case of higher modes. It is also apparent from the graph that the products $|H_\alpha(j\omega)||H_\beta(j\omega)|$ for $\alpha \neq \beta$ are small when compared with the products for $\alpha = \beta$. This is perfectly consistent with the statement given in Section 2 referring to small contribution of the cross factors (see eqs (16).

The computation of the generalized spectral dimensionless load $l^{(i)}_{\alpha\beta}$ has confirmed numerically that $l^{(i)}_{\alpha\beta} = l^{(i)}_{\beta\alpha}$, which followed theoretically from the property of the cross power spectral density of the fluctuating pressure (see eq. (15). The obtained values of $l^{(i)}_{\alpha\beta}$ for some velocities have been processed in the graphic form in Figs. 2 to 10. Apart from other data, it can be seen from them that the cross generalized spectral loads $l^{(i)}_{\alpha\beta}$ in case $\alpha \neq \beta$, are considerably less if compared with the loads $l^{(i)}_{\alpha\alpha}$. This again confirms our assertion from Section 2 stating that the major contribution to the random displacement amplitudes can be ascribed only to the joined factors. Therefore, our consideration will now be given only to $l^{(i)}_{\alpha\alpha}$, as to which it can be said that after reaching the maximum, $l^{(i)}_{\alpha\alpha}$ decreases with growing ω . In dependence on α increasing, the values $\max l^{(i)}_{\alpha\alpha}$ decrease gradually, and appear for larger values of the angular frequencies of the turbulent eddies ω . In the case of higher ω , the function $l^{(i)}_{\alpha\alpha}$ becomes independent of ω. Interesting properties can be observed for $\omega = 0$. In case α is odd, the value of $l^{(i)}_{\alpha\alpha}(\omega=0) \neq 0$, whereas for even α is $l^{(i)}_{\alpha\alpha} = 0$. This is closely connected with the chosen forms of the eigen functions and the coherence function.

So far we have been paying our attention to the appreciation of the course of $l^{(i)}_{\alpha\alpha}$ regardless of the flow velocity V . Note here that with V increasing also increases, u (see eq. (20). Comparing $l^{(i)}_{\alpha\alpha}(\omega)$ in dependence on V (Fig. 2, 5, 8), we shall find out the maximum values $l^{(i)}_{\alpha\alpha}$ to be independent of V , however, they appear for V increasing in case ω is higher. Hence it can be said that if the turbulent excitation intensity increases, the generalized load intenzity shifts into the region of higher exciting frequencies.

As noted above, the r.m.s. values of the stochastic displacement, obtained for peak frequencies in a defined frequency band, are useful to appreciate the response of the structure exposed to random excitation. These values have been computed according to (18), while the narrow vicinity of $\omega_q = \omega_\alpha$ was taken as $\Delta\omega_q$, and they have been plotted by the square marks in the graph in Fig. 11 which was taken from [6]. The full line means the measured linear spectral density of the fuel string relative motion. Comparing the both results we can see a reasonable agreement.

5. Conclusion

The derived mathematical model for solving the vibrational response of the structure due to the stochastic force effects, induced by the surrouding flow, is not limited only to a response of a particular component; it has been designed in such a way as to cover even structures different in shape, providing modern computing techniques are employed.

The computer implementation of the model, demonstrated on a numerical solution of the cohesive fuel bundles excited by a flowing surrounding medium, has confirmed the antecedent theoretical conclusions, particurarly those referring to the main contribution of the joined terms to the amplitudes of random displacement. This finding anables us to omit the cross terms, which leads to a substantial cut in the computing time.

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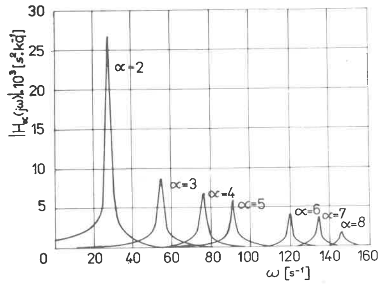

Fig. 1
Generalized spectral compliances $|H(j\omega)|$

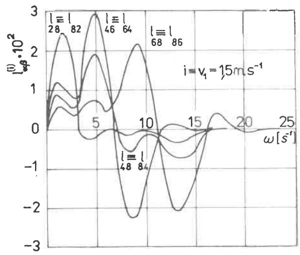

Fig. 3
Generalized spectral dimensionless cross loads $l^{(i)}_{\alpha\beta}$ at velocity v_1

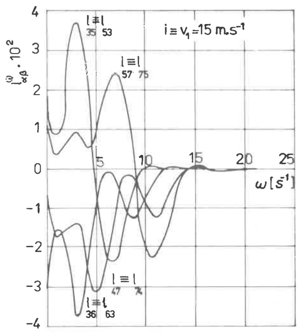

Fig. 4
Generalized spectral dimensionless cross loads $l^{(i)}_{\alpha\beta}(\omega)$ at velocity v_1

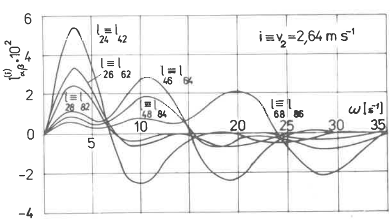

Fig. 6
Generalized spectral dimensionless cross loads $l^{(i)}_{\alpha\beta}(\omega)$ at velocity v_2

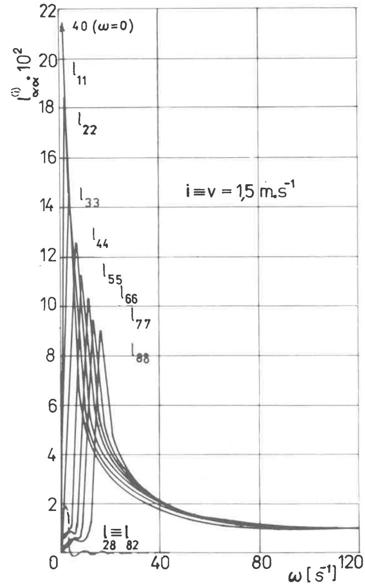

Fig. 2
Generalized spectral dimensionless joined loads $l^{(i)}_{\alpha\alpha}(\omega)$ at velocity v_1

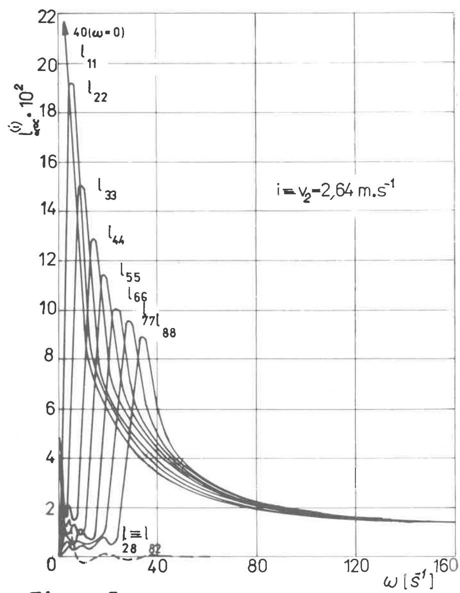

Fig. 5
Generalized spectral dimensionless joined loads $l^{(i)}_{\alpha\alpha}(\omega)$ at velocity v_2

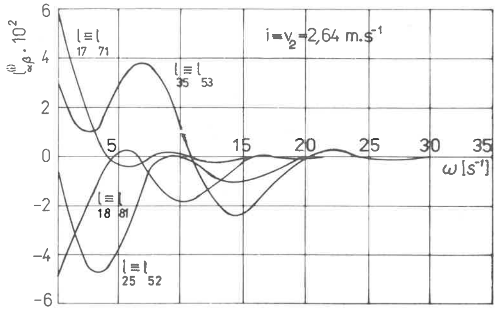

Fig. 7
Generalized spectral dimensionless cross loads $l_{\alpha\beta}^{(i)}(\omega)$ at velocity V_2

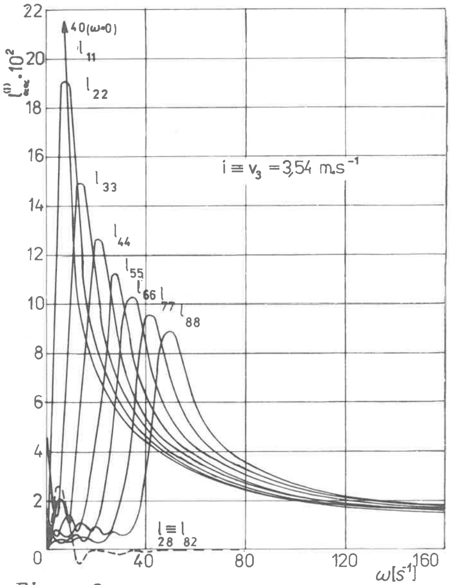

Fig. 8
Generalized spectral dimensionless joined loads $l_{\alpha\alpha}^{(i)}(\omega)$ at velocity V_3

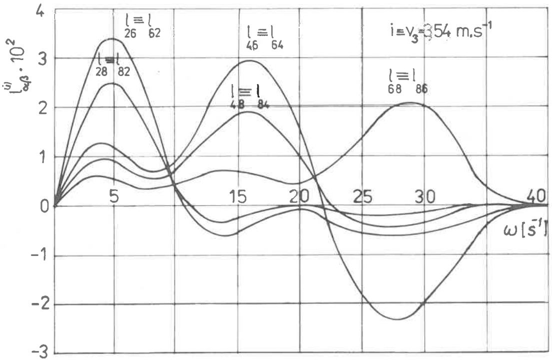

Fig. 9
Generalized spectral dimensionless cross loads $l_{\alpha\beta}^{(i)}(\omega)$ at velocity V_3

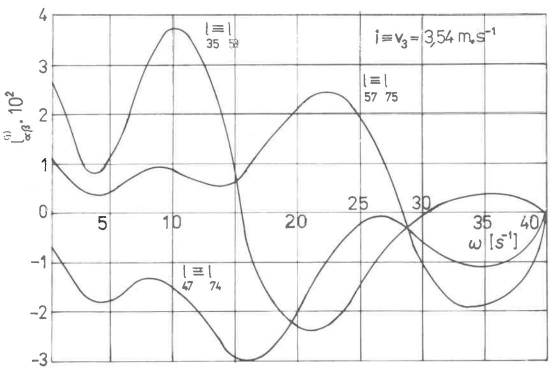

Fig. 10
Generalized spectral dimensionless cross loads $l_{\alpha\beta}^{(i)}(\omega)$ at velocity V_3

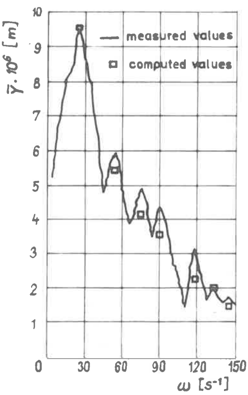

Fig. 11
Linear spectral density of the fuel string relative displacement $\bar{y}(\omega_q)$