Verification of the two-criteria approach for crack growth initiation of an axially cracked cylinder

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1 INTRODUCTION

An application of elastic-plastic fracture mechanics is desired to evaluate the integrity of nuclear pressure vessels and piping accurately and realistically. However, analyses of the elastic-plastic fracture mechanics parameters such as the J-integral require a lot of numerical calculations. Therefore, development of a simple engineering method is necessary to apply the elastic-plastic fracture mechanics widely for integrity evaluations.

Kumar et al. (1984) have proposed simple J-estimation formulas combining the elastic and fully plastic solutions. The simplified method based on tearing instability concepts has been developed to evaluate the integrity of irradiated nuclear pressure vessels (Jhonson 1981). Also, the two-criteria approach known as the R-6 failure assessment diagram (Milne et al. 1986) and deformation plasticity failure assessment curve (Bloom and Malki 1982) have been proposed.

In this study, the two-criteria approach was used to predict the pressures at the initiation of crack growth of an axially cracked cylinder under various crack depths and temperature distributions. Also, the $\hat{J}$-integral, proposed by Aoki et al. (1981) to consider the effects of the thermal stress field, was calculated by the finite element method. The results obtained by the two-criteria approach and the J_{IC}-criterion were compared to verify the accuracy of the two-criteria approach.

2 EVALUATION METHOD

2.1 Two-criteria approach

The two-criteria approach is a simplified procedure that combines linear elastic fracture mechanics with the plastic collapse. The plastic collapse parameter Lr and the linear elastic fracture mechanics parameter K_r are defined as follows, respectively :

(1) $$Lr = \frac{\sigma}{\sigma_{lim}} ,$$ (2) $$Kr = K_r^p + K_r^s ,$$

where, σ is the applied stress and σ_{lim} is the plastic collapse stress, and, K_r^p and K_r^s are expressed as follows :

$$(3) \qquad K_r^p = \frac{K_I^p}{K_{IC}}, \qquad (4) \qquad K_r^s = \frac{K_I^s}{K_{IC}} + \rho,$$

where, K_I^p is the stress intensity factor due to pressure loading, K_I^s is the stress intensity factor due to thermal stress and K_{IC} is the fracture toughness. Influence of thermal stress to plastic collapse is introduced by the correction factor ρ (Milne et al. 1986). In this method, two parameters Lr and Kr are evaluated independently and plotted on the failure assessment diagram. Crack growth initiates when an assessment point (Lr, Kr) lies on the assessment line. Airsworth (1984) proposed the assessment line considering material strain hardening based on the systematic J-integral analyses. The assessment line is expressed by :

$$(5) \qquad Kr = \left(\frac{E\varepsilon_{ref}}{L_r\sigma_y} + \frac{L_r^3\sigma_y}{2E\varepsilon_{ref}}\right)^{-\frac{1}{2}},$$

where, σ_y is the yield stress, ε_{ref} is the true strain when stress is $L_r\sigma_y$ and E is Young's modulus.

2.2 $\hat{J}$-integral

The parameter $\hat{J}$-integral, that has been proposed to consider the effects of the thermal stress field, was used to determine the critical pressure for crack growth initiation. The definition of the $\hat{J}$-integral is as follows :

$$(6) \qquad \hat{J} = \int_\Gamma (W^e dy - T_i\frac{\partial u_i}{\partial \chi})d\Gamma + \iint_A \sigma_{ij}\frac{\varepsilon_{ij}^*}{\partial \chi}\, dA,$$

where,

$$(7) \qquad \varepsilon_{ij}^* = \varepsilon_{ij}^t + \varepsilon_{ij}^p,$$

ε_{ij}^t is the thermal strain tensor, ε_{ij}^p is the plastic strain tensor, W^e is elastic strain energy density, u_i denotes displacement, σ_{ij} is the stress tensor, Γ denotes an arbitrary contour surrounding the crack tip, and A is an area surrounded by contour Γ and the crack surfaces. Crack growth initiates when the $\hat{J}$-integral is equal to the fracture toughness J_{IC}.

3 MODEL FOR EVALUATION

The accuracy of the two-criteria approach was examined on the axially cracked cylinder shown in Fig. 1. Inner radius R is 3250 mm and the wall thickness t is 160 mm. An infinitely long axial crack was postulated on the inside surface of the cylinder as shown in Fig. 1. Crack depths of 20 mm (1/8 t), 40 mm (1/4 t) and 60 mm (3/8 t) were assumed to evaluate the critical pressures for the crack growth initiation.

The material properties used for the evaluation were as folllows :

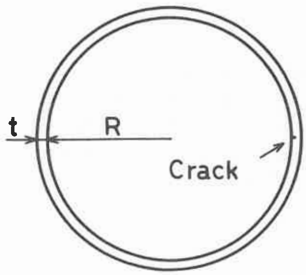


Fig. 1 Axially cracked cylinder

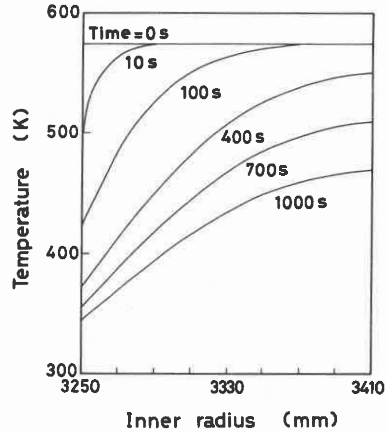


Fig. 2 Temperature distribution through the wall thickness

Yield stress	σ_y	:	412 MPa
Ultimate stress	σ_u	:	549 MPa
Flow stress	$\bar{\sigma}$	:	481 MPa
Fracture toughness	J_{IC}	:	200 kN/m.

The stress-strain relationship for the material was expressed by the following Ramberg-Osgood power raw :

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha \left(\frac{\sigma}{\sigma_y}\right)^n, \tag{8}$$

where, $\varepsilon_y = \sigma_y/E$, and parameters α and n are 1.115 and 9.7, respectively.

4. TEMPERATURE ANALYSIS

The initial temperature of the cylinder was assumed to be 573K and the inside was continuously cooled by water of 293K. The temperature distributions through the wall thickness of the cylinder were calculated by the finite element method. The obtained temperature distributions through the wall thickness are shown in Fig. 2. In this study, four kinds of temperature distributions at times 0, 10, 100 and 400s were selected for the evaluations. The temperature distribution through the wall thickness is uniform at the time of 0s and the cylinder is subjected only to pressure. The maximum temperature difference between the inside and outside surface was added to pressure loading at 400s.

5. EVALUATION OF CRACK GROWTH INITIATION

5.1 Two-criteria approach

The parameter L_r is defined as equation (1), and L_r for the internal axially cracked cylinder (Chell 1979) is given by :

$$(9) \qquad L_r = \frac{P(R - a)}{\bar{\sigma}(t - a)},$$

where, P is the applied pressure. Stress intensity factors, K_I^p and K_I^s, were determind from elastic $\hat{J}$ value calculated by the finite element method described in paragraph 5.2. Fracture toughness K_{IC} of 212 $MN/m^{3/2}$ was also calculated from the J_{IC} value.

An assessment point (L_r, K_r) moves along the loading line with an increase in pressure. Critical pressure $(Pc)_{Two}$, for crack growth initiation is given when an assessment point lies on the assessment line. Evaluation results of critical pressures under various crack depths and thermal distributions are shown in Table 1.

5.2 J_{IC}-criterion

The $\hat{J}$-integral was calculated by the finite element method with 8 node isoparametric elements. The plane strain condition was assumed and the $\hat{J}$-integral was calculated by path integration. Critical pressure $(Pc)J_{IC}$, for crack growth initiation is given when the $\hat{J}$-integral is equal to J_{IC}. Critical pressures obtained by the J_{IC}-criterion under various crack depths and temperature distributions are summarized in Table 2.

Table 1 Results of evaluation for crack growth initiation obtained by the Two-criteria approach

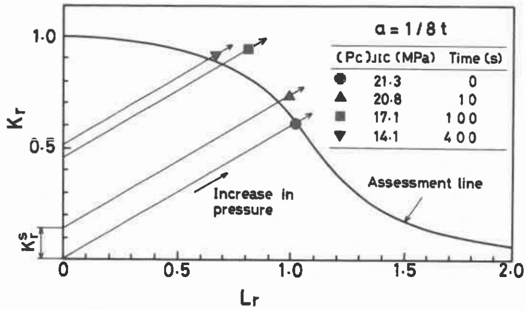
a (mm)	time (s)	$(Pc)_{Two}$ (MPa)	Kr^p	Kr^s	Kr	Lr
20.0	0	21.4	0.615	0.0	0.615	1.026
	10	19.8	0.569	0.120	0.689	0.951
	100	14.7	0.423	0.455	0.878	0.706
	400	13.6	0.392	0.509	0.901	0.654
40.0	0	15.8	0.775	0.0	0.775	0.879
	10	14.6	0.718	0.111	0.829	0.813
	100	8.7	0.429	0.512	0.941	0.486
	400	6.5	0.318	0.651	0.969	0.360
60.0	0	11.3	0.868	0.0	0.868	0.749
	10	10.0	0.769	0.127	0.896	0.664
	100	5.3	0.407	0.561	0.968	0.352
	400	2.6	0.204	0.788	0.992	0.176

Table 2 Critical pressure for crack growth initiation obtained by the J_{IC}-criterion

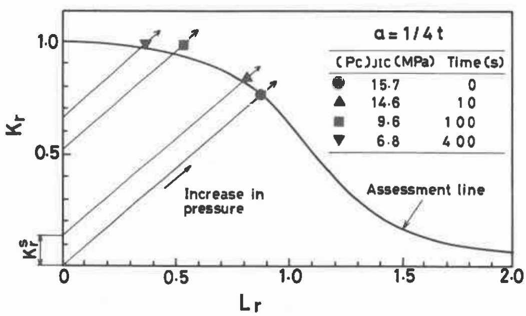
a (mm)	time (s)	$(Pc)_{JIC}$ (MPa)
20.0	0	21.3
	10	20.8
	100	17.1
	400	14.1
40.0	0	15.7
	10	14.6
	100	9.6
	400	6.8
60.0	0	11.0
	10	9.9
	100	5.6
	400	2.9

6 DISCUSSION

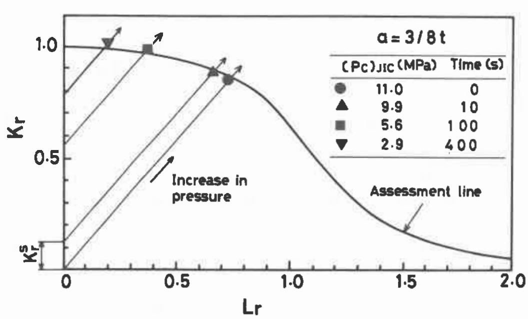
The crack initiation points obtained by the J_{IC}-criterion were plotted on the failure assessment diagram to compare with the assessment line. Parameter L_r and K_r for critical pressures obtained by the J_{IC}-criterion were calculated to plot on the failure assessment diagram. Comparison of crack initiation points by the J_{IC}-criterion with the assessment line are shown in Fig. 3. From this Fig., it is found that the initiation points by the J_{IC}-criterion and the assessment line



(a) a = 1/8t



(b) a = 1/4t



(c) a = 3/8t

Fig. 3 Comparison of the initiation points by the J_{IC}-criterion with the assessment line

Table 3 Ratio of critical pressure obtained by the Two-criteria approach and the J_{IC}-criterion

a (mm)	time(s)	$(Pc)_{Two}/(Pc)_{JIC}$
20.0	0	1.005
	10	0.951
	100	0.860
	400	0.965
40.0	0	1.006
	10	1.000
	100	0.906
	400	0.956
60.0	0	1.027
	10	1.010
	100	0.946
	400	0.897

almost agree in the case when there are no thermal stresses at the time of 0s. When the cylinder was subjected to thermal stresses at times of 10, 100 and 400s, the assessment line is conservative in comparison with the initiation points determined by the J_{IC}-criterion. The initiation points by the J_{IC}-criterion approach the assessment line more closely with the increase in crack depth. Also, the ratio of critical pressure $(Pc)_{Two}/(Pc)J_{IC}$, is shown in Table 3. Critical pressures obtained by both methods show good agreement when there are no thermal stresses at the time of 0s. When thermal stresses exist at 100 and 400s, critical pressures $(Pc)_{Two}$ are slightly less than $(Pc)J_{IC}$.

7 CONCLUSIONS

The results of this study are summarized as the follows.

(1) In the case of only pressure loading, the two-criteria approach can evaluate crack initiation pressure with the same accuracy as the J_{IC}-criterion.

(2) When thermal stresses exist, the two-criteria approach give slightly conservative initiation pressures in comparison with the J_{IC}-criterion.

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