SMiRT-12 / K. Kussmaul (Editor)

K05/3

BOUNDARY ELEMENT METHOD IN DYNAMIC INTERACTION OF STRUCTURES WITH MULTILAYERS MEDIA

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ABSTRACT

The paper presents the problems of dynamic interaction between the multilayers media and structure by means of B.E.M., using Green's functions. The structure considered by the authors as a particular problem concerns a reinforced concrete shear wall and soil foundation of three layers having different thicks and mechanical characteristics. The authors will present comparatively the stresses and the displacements in static and dynamic regime interaction response of the structure.
Theoretical part of the paper will presents: Green's functions for the multilayers media in dynamic regime, stiffness matrices, stresses and displacements in the multilayers media exprimed by means of the Green's functions induced by the shear and horizontal forces, computer program, consideration for dynamic, structure-foundation-multilayers soil foundation interaction.

1. BOUNDARY ELEMENT METHOD FOR DYNAMIC INTERACTION STRUCTURES WITH MULTILAYERS SOILS

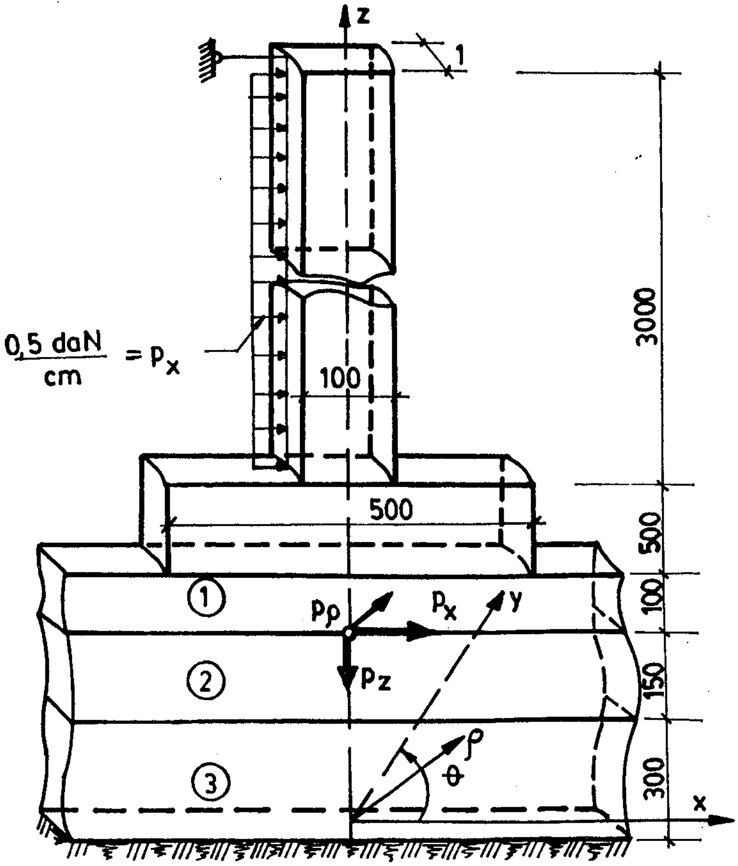
Consider a layered stratum of finite width, as shown in Fig. 1. From the dynamic equilibrium equation

$$\sigma_{ij,j} + b_i = \gamma \ddot{u}_i \qquad (1)$$

we obtain multiplying by the virtual displacements u_i^* and integrating over the volume by parts

$$\int_\Omega \left(\sigma_{ij,j} + b_i - \gamma \ddot{u}_i \right) u_i^* \, d\Omega = -\int_\Omega \sigma_{ij} \epsilon_{ij}^* d\Omega + \int_\Omega \left(b_i - \gamma \ddot{u}_i \right) u_i^* \, d\Omega + \int_S t_i u_i^* \, dS \qquad (2)$$

Considering the principle of the virtual displacements, which is satisfied, the fundamental discrete solution can be written



$$\int_\Omega \delta\varepsilon_{ij}\sigma_{ij}\,d\Omega + \int_\Omega \delta u_i\left(\gamma\,\ddot{u}_i - b_i\right)d\Omega - \int_S \delta u_i t_i\,dS = 0 \qquad (3)$$

Fig. 1

Interchanging the real displacement field and the fundamental solution (*) and introducing the condition

$$u_i\ddot{u}_i^* = u_i^*\ddot{u}_i = -\omega^2 u_i u_i^*, \quad \varepsilon_{ij}^*\,\sigma_{ij} = \varepsilon_{ij}\,\sigma_{ij}^*, \quad b_i = 0 \qquad (4)$$

in (3), we obtain the equations system

$$\sum_{\substack{\text{Exposed}\\ \text{rings}}} \oint_C q_{ij}^* u_i\,ds + \sum_{\substack{\text{Exposed } S\\ \text{hor. surface}}} \int t_{ij}^* u_i\,dS = -u_j(\vec{r}_0) + \qquad (5)$$

$$\sum_{\substack{\text{Exposed}\\ \text{rings}}} \oint_C u_{ij}^* q_i\,ds + \sum_{\substack{\text{Exposed } S\\ \text{hor. surface}}} \int u_{ij}^* t_i\,dS$$

The integrals implied by equation (5) may be evaluated with the assumption that the real boundary displacements and tractions are piecewise uniform in horizontal planes, and vary linearly with the vertical coordinate. Expressions are also needed for the stresses in terms of the displacements, and for the Green's functions associated with the problem of dynamic point loads in the interior of layered stratum.

2. THE GREEN FUNCTIONS FOR DYNAMIC LOADS IN LAYERED MEDIA.

The Green functions for point loads can be obtained from those for disk loads by considering the limit when $R \rightarrow 0$ In the case of loads with intensity q, the coresponding traction are

$$p = q/\pi R^2 \quad \text{(horizontal, vertical)} \qquad (6)$$

The limiting expresions for the displacements when $R \rightarrow 0$, are given in /Kausel, 1981/. Applying now the displacements for N sublayers, Fig. 1, it results a equations system in form

$$K\,U = P \qquad (7)$$

If we isolate a specific layer and preserve equilibrium by application of external loads $P_1 = S$ at the upper interface, and $P_2 = -S$ at the lower interface the relationship between forces and displacements is then

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \qquad (8)$$

In the of thin layers, the layer stiffness matrix can be obtained as Kausel (1981)

$$K_m = A_m k^2 + B_m k + G_m - \omega^2 M_m \qquad (9)$$

where k - wave number, ω - frequency of excitation, and A_m, B_m, G_m, M_m are the matrices given in Kausel, 1981, and which involve only material properties of the layers. For prescribed loadings, P, the displacements U are obtained by formal inversion of the stiffness matrix from the equations system (7) and results

$$U = K^{-1}P \qquad (10)$$

Green's functions for the internal stresses can be calculated with relationship

$$\begin{Bmatrix} \sigma_\rho \\ \tau_{\rho\theta} \\ \tau_{\rho z} \end{Bmatrix} = \left\{ \begin{bmatrix} \lambda+2G & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\partial}{\partial\rho} + \begin{bmatrix} 0 & 0 & \lambda \\ 0 & 0 & 0 \\ G & 0 & 0 \end{bmatrix} \frac{\partial}{\partial z} + \right.$$

$$+ \begin{bmatrix} 0 & \lambda & 0 \\ G & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\partial}{\rho \partial \theta} + \begin{bmatrix} \lambda & 0 & 0 \\ 0 & -G & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{1}{\rho} \Bigg\} \begin{Bmatrix} u_\rho \\ u_\theta \\ u_z \end{Bmatrix} \qquad (11)$$

where the displacement can be calculated by BEM with (4) or (7).

3. TEST EXAMPLE

In the example presented in Fig. 1, in the first time, we determine by FEM the stresses on the contact surface between foundation and soil foundation. These stresses are considered as the loads on the multilayers soil foundation. For the determination of stresses and strain state the multilayers soil we use BEM. The layer characteristics, discretization type by BEM normal stress values σ_x obtained by these methods are presented in Fig.2.

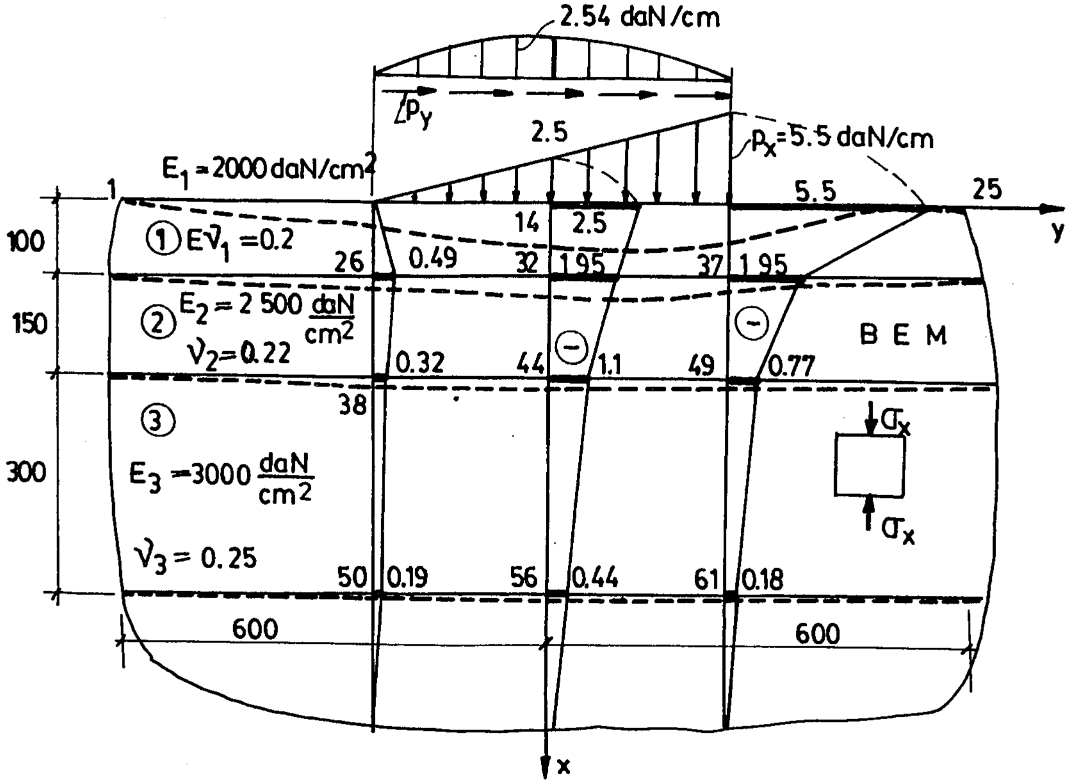


Fig. 2

At 6m from the axis x the displacements towards two directions are null. The normal stress σ_z is null, too, at the distance x = 6m. Great values of the stresses σ_x appear at the foundation-soil foundation contact area in the point A with the coordinates y = 2.50 m, x = 0, where appears plastic domain. For dynamic analysis we use the relation (8), considering for multilayers soil the following characteristics: the period of external load $T = 1$ sec ; $k = 1/T = 1$ sec^{-1}, the freqency $\omega = 2\Pi/T = 6.28$ sec^{-1}. Determining the matrices A_m, B_m, G_m, M_m and for each soil-foundation layer by an assembling processus we arrive to the equation (7) with the form

$$\begin{bmatrix} M & N^T & 0 & 0 \\ N & P & V^T & 0 \\ 0 & V & F & S^T \\ 0 & 0 & S & L \end{bmatrix} \begin{Bmatrix} u_{10} \\ u_{12} \\ u_{23} \\ u_{34} \end{Bmatrix} = \begin{Bmatrix} p_{10} \\ p_{12} \\ p_{23} \\ p_{34} \end{Bmatrix} \qquad (12)$$

The free term in the right hand of the above equation is given by the following vector

$$\begin{Bmatrix} p^x_{12} \\ p^y_{12} \\ p^z_{12} \end{Bmatrix} = \begin{Bmatrix} 1\ 200 \\ -1\ 810 \\ -9\ 750 \end{Bmatrix} \quad \begin{Bmatrix} p^x_{23} \\ p^y_{23} \\ p^z_{23} \end{Bmatrix} = \begin{Bmatrix} 350 \\ -1\ 130 \\ -5\ 540 \end{Bmatrix} \quad \begin{Bmatrix} p^x_{34} \\ p^y_{34} \\ p^z_{34} \end{Bmatrix} = \begin{Bmatrix} 119.5 \\ -440 \\ -2\ 280 \end{Bmatrix}$$

From the equations system (12) we obtain the displacements at the contact surface between the layers.

$$\{u^k_{ij}\}^T = [\ 0.183\ -0.104\ -0.429\ -0.026\ -0.0026\ -0.0388\ 0.0052$$
$$-0.0059\ -0.0242\ -0.0021\ -0.00069\ -0.00678\ ]^T$$

The error between the static and dynamic maximum value of the displacement $u^z_{10} = -0.4289$cm. is approximatively 6.5% . The displacements diminish toward the down part of layer 3.

CONCLUSIONS

Using the conection between FEM and BEM we can obtain a good accuracy as in static as well as in dynamic regime, of the stress and displacements state results for the foundation-multilayers soil foundation interaction action.

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