Combined Dynamic Effects of Correlated Load Processes

Y.-K. Wen
Department of Civil Engineering, University of Illinois, 3106 Civil Engineering Bldg., Urbana, Illinois 61801, U.S.A.

Abstract

A large number of loadings on structures, particularly nuclear structures, are random and dynamic in nature and may be generated from common or related sources (e.g., storms, accidents). The combination of load effects which may be nonlinear, dynamic and statistically dependent is therefore an important issue in the assessment of safety and performance of a nuclear structure over its lifetime. This paper summarizes the latest developments in this area with emphasis on the versatile method of analysis based on a consideration of load coincidence. The validity and accuracy of this method is established by comparison with other methods and Monte Carlo simulations. Parametric studies are carried out and effects of dynamic response and load dependencies are examined. The accuracies of some of load combination rules are also studied and ranges of validity of each rule are identified such that some risk-consistency can be achieved if such rules are to be used.

1. Introduction

Most loadings on structures fluctuate in time and space and are stochastic in nature. In view of the uncertainties involved, the satisfactory performance of a structure can be assured only in terms of the probability of prescribed limit states not being exceeded over its lifetime under all loadings, including the possible combined actions of different loads. This "load combination" problem has been traditionally handled on an empirical and judgmental basis. Recently, researchers began to re-examine this problem more rigorously by treating the time varying loads and load effects as random processes rather than as random variables or deterministic quantities. Most probabilistic studies of the load combination problem have been concentrating on the linear combination of independent static loads and load effects.

As many loadings capable of causing adverse effects in a nuclear structure are dynamic and may be correlated in occurrence time, duration and intensity, a methodology is clearly needed to include these factors into consideration in the evaluation of the safety of the structure. In the following, a brief account of the latest developments of such a methodology is given including (1) modeling of micro- and macro-scale time varying random loads and load effects, (2) load coincidence method for combination of dynamic responses, (3) modeling and effects of load dependencies, and (4) errors in load combination rules currently in use.

2. Modeling of Micro- and Macro-Scale Time Varying Load Effects

Time varying loads can be described in terms of their scales of fluctuation being in the vicinity of the natural periods of the structure (micro-scale) or much larger (macro-scale), or both. While live load, snow load, etc. vary primarily with a macro-scale; wind, wave, earthquake loads, etc., in addition have important micro-scale variation causing significant dynamic effects. A simple and flexible model for macro-time variabilities is the pulse process in which the loading is characterized by a random occurrence time, random duration and intensity. Most load modelers assume these parameters to be statistically independent. In this case, the process can be specified by an occurrence rate λ, mean duration μ_d and a random intensity; e.g., Wen [1], Larrabee and Cornell [2]. However, this restriction can be relaxed, i.e., occurrence times may be correlated within each load as well as between loads, and the same is true for duration and intensity. The modeling and effect of such dependencies has been studied extensively by Wen and Pearce [3] and Wen [4]. When, in addition, the within-occurrence micro-scale variability is also important, it can be modeled by a continuous random process conditional on a given set of the macro-time parameters described in the pulse process. Such a composite random process is referred to hereafter as "Intermittent Continuous Process (I.C.P.)." Sample functions are shown in Fig. 1. A quantity of interest is the maximum value R over a time period T. If the occurrence of the event (storm, earthquake, accident loading, etc.), is modeled by a Poisson process, one can show that the probability distribution of R is

$$F_R(r) \simeq \exp\{-\lambda T[1 - F_{X_m}(r)]\} \quad (1)$$

in which X_m = the within-occurrence maximum value. For example, if the within-occurrence process is assumed to be a piece-wise stationary Gaussian Process based on a Poisson approximation,

$$F_{X_m}(r) \approx \int_0^\infty \exp\{-\nu\mu_d \exp[-\frac{1}{2}(\frac{r - \mu_x}{\sigma_x})^2]\}\, f_Y(y)\, dy \quad (2)$$

in which μ_x, σ_x and ν are mean response, r.m.s. response and apparent frequency, conditional on the excitation intensity $Y = y$. Note that the excitation intensity varies from pulse to pulse and is treated as a random variable, e.g., the maximum mean wind velocity in a storm, the maximum ground acceleration in an earthquake.

3. Load Coincidence (L.C.) Method

When time-varying load effects are combined, the contribution from the part that the process coincide plays a significant role in the probability of maximum load effects. A method with a consideration of this coincidence effect has been developed for linear combination of independent processes [1] and extended to nonlinear combination as well as combination of dynamic and dependent load effects. See e.g., Pearce and Wen [5]. For linear combination of two I.C.P.'s, the lifetime combined maximum R_m, has the following distribution function,

$$F_{R_m}(r) \approx \exp[-\lambda_1 T F^*_{X_1}(r) - \lambda_2 T F^*_{X_2}(r) - \lambda_{12} T F^*_{X_{12}}(r)] \tag{3}$$

in which $F^*_{X_i} = 1 - F_{X_i}$. The first two terms are contributions from individual I.C.P. (eqs. (1) and (2)). The third term is the contribution from the coincidence of events, and $\lambda_{12} \approx \lambda_1\lambda_2(\mu_{d_1} + \mu_{d_2})$, the mean rate of coincidence.

$$F^*_{X_{12}}(r) \approx \int_0^\infty \int_0^\infty [1 - \exp\{-\nu_{12}\mu_{d_{12}} \exp[-\tfrac{1}{2}(\frac{r - \mu_{X_{12}}}{\sigma_{X_{12}}})^2]\}]\ f_{Y_1}(y_1)\ f_{Y_2}(y_2)\ dy_1 dy_2 \tag{4}$$

in which ν_{12}, $\mu_{X_{12}}$ and $\sigma_{X_{12}}$ are respectively the apparent frequency, mean response and r.m.s. response of the combined process given the coincidence and the excitation intensities being $Y_1 = y_1$ and $Y_2 = y_2$. Equations (2) and (4) can be modified to include the within-occurrence intensity variation with time. $\mu_{d_{12}}$ = the mean duration of the coincidence of two loads. $\approx \mu_{d_1}\mu_{d_2}/(\mu_{d_1} + \mu_{d_2})$.

For combination of multi-(n) load effects,

$$F_{R_m}(r) \approx \exp\{-[\sum_{i=1}^{n} \lambda_i F^*_{X_i}(r) + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \lambda_{ij} F^*_{X_{ij}}(r) + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n} \lambda_{ijk} F^*_{X_{ijk}}(r) + \text{---}]\ T\} \tag{5}$$

in which λ_{ij} = mean rate of coincidence of load $S_i(t)$ and $S_j(t)$, λ_{ijk} = mean rate of coincidence of load $S_i(t)$, $S_j(t)$ and $S_k(t)$, etc. $\approx \lambda_i\lambda_j\lambda_k(\mu_{d_i}\mu_{d_j} + \mu_{d_j}\mu_{d_1} + \mu_{d_i}\mu_{d_k})$, $F^*_{X_{ij}}(r)$ = conditional probability of r being exceeded given the coincidence of $S_i(t)$ and $S_j(t)$, and ----, etc.

A major advantage of the L.C. method is the simplicity in its formulation and required calculation. Also, it can be easily extended to consider the effect of load dependencies.

The accuracy of the L.C. method for pulse processes has been established by comparison with other methods and extensive Monte Carlo simulations. For I.C.P.'s, the accuracy depends mainly on those of eqs. (2) and (4). It is found that the errors introduced by using the simple Poisson approximation in eqs. (2) and (4) are quite insignificant because of the usually much larger macro-scale uncertainties [5], and in the study of structural safety only high threshold levels are of concern. Equation (5) generally gives good, conservative results.

A method based on the upcrossing rate (U.R.) has also been proposed for the combination of I.C.P.'s by Winterstein [6] and Shinozuka and Tan [7]. One can show that the two methods coincide as $r \to \infty$. However, for finite r, long T and long occurrence or coincidence durations (e.g., hours instead of seconds), the U.R. method has a tendency to give unduly conservative results. This is due to the fact that in the L.C. method, through eqs. (2) and (4), failure (upcrossings) is counted at most once in each occurrence (pulse), whereas in a U.R. method they may be counted a large number of times even at very high threshold levels. A comparison of the results for such a case based on these two methods is shown in Fig. 2. Also, conceptually, it is difficult to include the variation of intensity from occurrence to occurrence. The method has been recently modified to correct some of the preceding shortcomings by Cornell and Winterstein [8].

4. Modeling and Effects of Load Dependencies

The L.C. method has been generalized to include the dependencies between load parameters into consideration for pulse processes [3,4]. The same approach is used here for the combination of dependent I.C.P., i.e., the macro-time parameters (occurrence time, duration and intensity) are modeled as correlated random variables. The within-occurrence r.m.s. intensity is assumed to be proportional to the intensity random variable (e.g., Y_i in eq. (4)). Examples are the r.m.s. fluctuation of wind velocity being approximately proportional to the mean wind in a storm and the r.m.s. acceleration to the maximum acceleration in an earthquake. The micro-scale variabilities, therefore, are also correlated in an indirect way. It is a reasonable model for the real physical process. Details of the model and analysis procedures can be found in reference 5. The findings are summarized in the following.

The effects of dependencies in general follow the trends observed for combination of static loads in references 3 and 4. No significant departure is found because of the dynamic effect. Comparisons of results for the dependent and independent cases are shown in Figs. 3 to 6. The structural parameters are: natural frequency $\omega = 3$ rad/sec, damping ratio $\zeta = 0.05$. The within-occurrence load intensity is assumed to be Gaussian with a coefficient of variation = 0.3. The time period considered is 50 years. The dependencies are described in terms of whether they are within or between loads.

(a) Within Load Dependencies. The correlation between duration and intensity (e.g., in storms) causes a higher maximum response. However, this difference is small enough that it would not significantly affect any design decision (Figs. 3 to 5). Correlation between intensities (e.g., SRV load in a nuclear structure may be correlated in intensity from occurrence to occurrence) is important only if the correlation is almost perfect ($\rho = 1.0$). Also, neglecting the dependence would err on the safe side (Figs. 3 to 5). The same is true for occurrence dependence such as clustering (e.g., SRV load is known to arrive in clusters, so are hazards like tornadoes). Note that clustering affects the coincidence rate, but not the dynamic response, the preceding conclusion being drawn from reference 3.

(b) Between Load Dependencies. When I.C.P.'s are correlated in intensity both within load and between loads, two counteracting effects can be seen. For extremely sparse process ($\lambda\mu_d \ll 1$), the coincidence effect is negligible, the main effect is that of the within load correlation which causes a lower combined maximum at the medium and low levels (Fig. 3);

whereas when $\lambda\mu_d$ increases, the coincidence term takes effect and the between load correlation becomes important, causing a much higher response (Fig. 5).

Between load occurrence clustering (e.g., wind and wave loads due to storms and S.R.V. or LOCA load triggered by earthquakes or accidents) can cause tremendous increase in the coincidence rate [4]. Therefore, it gives much higher combined response (Fig. 6).

When both intensity and occurrence dependence exist between loads, the effects are multiplicative which can cause a much higher probability of exceedance at the high threshold levels.

5. Error of Load Combination Rules

Simple rules for load combination are useful and perhaps necessary in routine safety checking and code formulation. However, since such rules are mostly judgment-based, structures designed according to such procedures may not have the intended safety level. It is therefore important to examine the accuracies of such rules. This is done by comparing the probability of the combined load effect implied by such rules with analytical results. For the combination of dynamic responses, the rules examined here are the Square Root of the Sum of the Squares (SRSS) and Turkstra's Rule (TR).

The popular SRSS rule for combination of modal responses in structural dynamics has been suggested for combination of loadings on nuclear structures, e.g., by Mattu [9]. The maximum combined load effect is calculated according to

$$\mathrm{Max}[S_1(t) + S_2(t)] \simeq \sqrt{R_1^2 + R_2^2} \tag{6}$$

in which $S_i(t)$ are load effect processes and R_i are their maximum value over a time period T.

Turkstra [10] suggested a simple method for combining loads, i.e.,

$$\mathrm{Max}[S_1(t) + S_2(t)] \simeq \mathrm{Max}[R_1 + S_2,\ R_2 + S_1] \tag{7}$$

in which S_i are the arbitrary-point-in-time values. This procedure has been used in recent code studies. See e.g., Ellingwood, et al. [11]. In general, the accuracy of the rule depends on load and structural system parameters as well as probability level. A typical comparison of results is shown in Figs. 2 and 7, for a system with natural frequency $\omega = 5$ rad/sec and damping factor $\zeta = 0.05$. The load intensities are modeled by gamma variates with $\mu_Y = 1.0$ and $\sigma_Y = 0.3$. The r.m.s. value of the dynamic oscillation is proportional to the intensity. Because most stochastic dynamic loadings are infrequent and brief, only small values of $\lambda\mu_d$ are considered. T = 50 yrs. It is seen that the SRSS rule is generally very conservative, perhaps too much so when $\lambda\mu_d$ is very small (loads which are extremely infrequent and brief). But, in general, it gives comparatively better results than in the static load effect combination. Systems of different natural frequencies are also investigated. As structural stiffness increases, SRSS is less conservative and may become unconservative at low risk level, say 10^{-4}. The TR, on the other hand, consistently gives a lower bound estimate which is good only when $\lambda\mu_d$ is extremely small and does not seem sensitive to changes in structural system parameters.

The conclusions given in the foregoing are valid for combination of load or load effect processes which are statistically independent of one another. For dependent loads, as shown in the foregoing, inter-(between) load dependencies may significantly increase the probability of threshold level being exceeded by the combined load. Since all load combination rules do not consider effects of such dependencies, they would give results more on the unconservative side when such dependencies exist.

Acknowledgment

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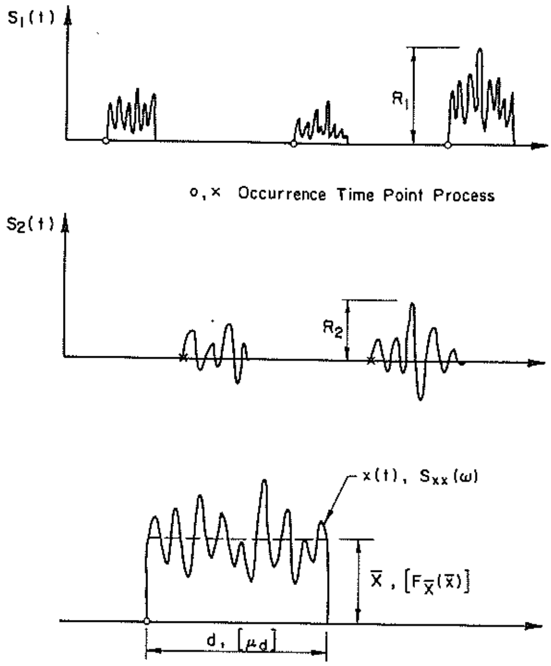

Fig. 1 Intermittent Continuous Process (I.C.P.) Model for Macro- and Micro-Scale Load Effect Variabilities

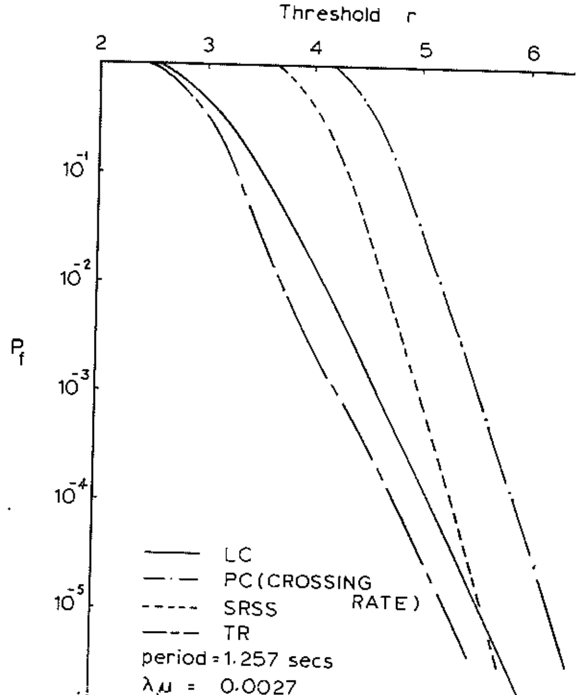

Fig. 2 Comparison of Probabilities of Lifetime Maximum Combined Response Being Exceeded

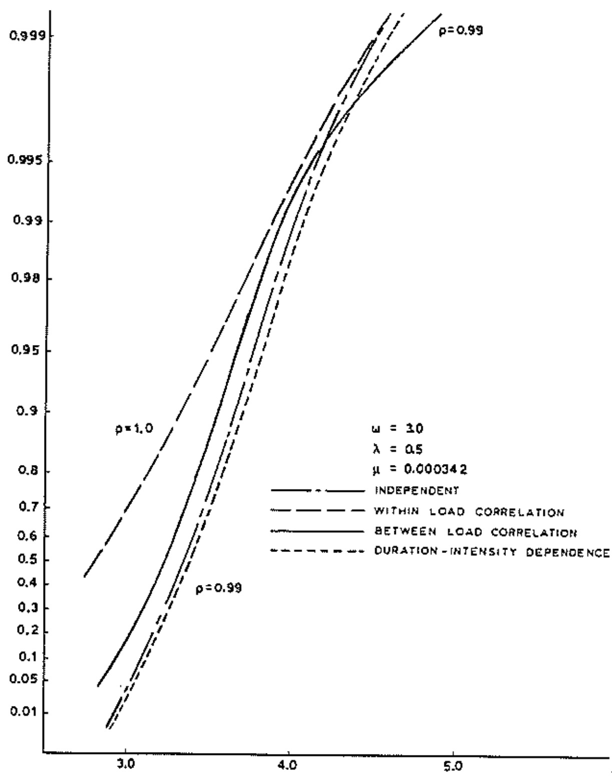

Fig. 3 Effect of Dependencies in Intensity and Duration on Probability of Combined Dynamic Response ($\lambda\mu_d$=0.00017)

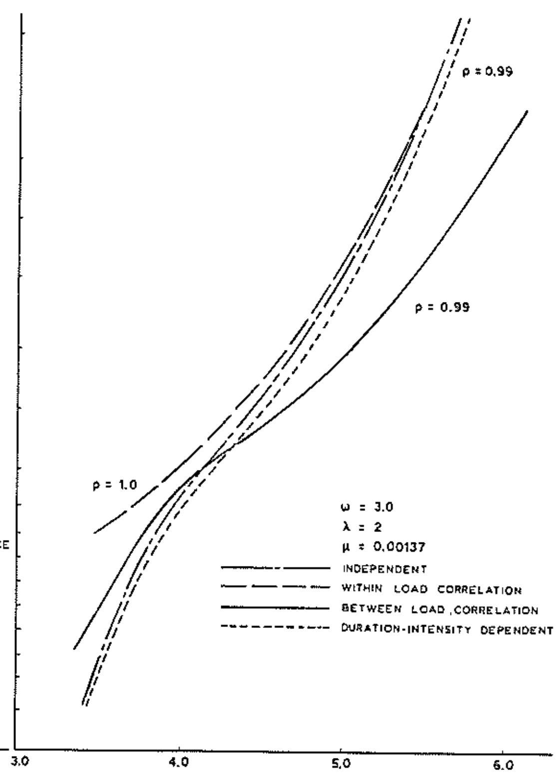

Fig. 4 Effect of Dependencies in Intensity and Duration on Probability of Combined Dynamic Response ($\lambda\mu_d$=0.0027)

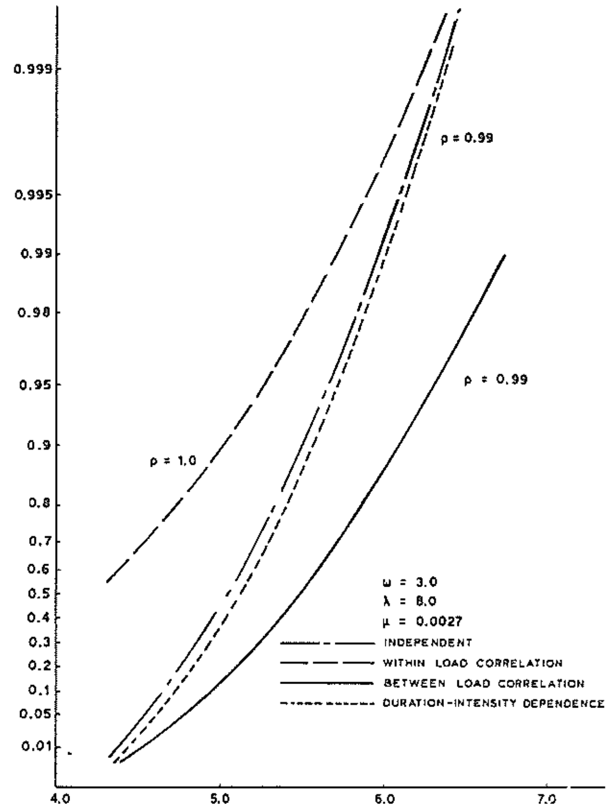

Fig. 5 Effect of Dependencies in Intensity and Duration on Probability of Combined Dynamic Response ($\lambda\mu_d$=0.022)

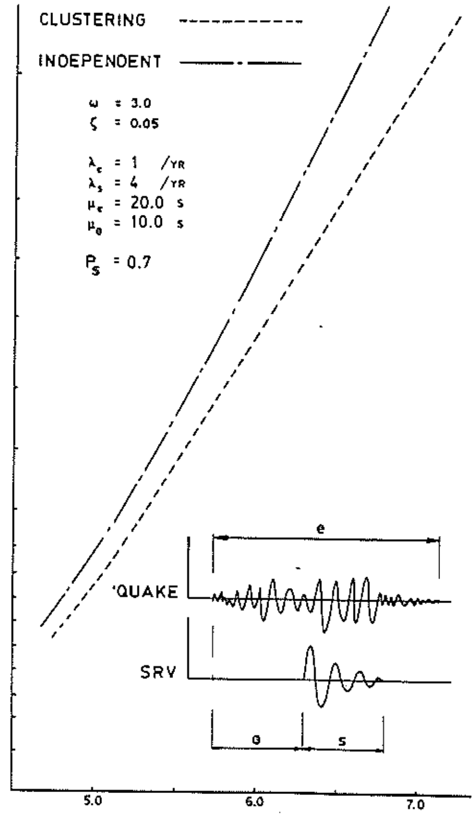

Fig. 6 Effect of Between Load Occurrence Clustering on Probability of Combined Earthquake and SRV Load Response

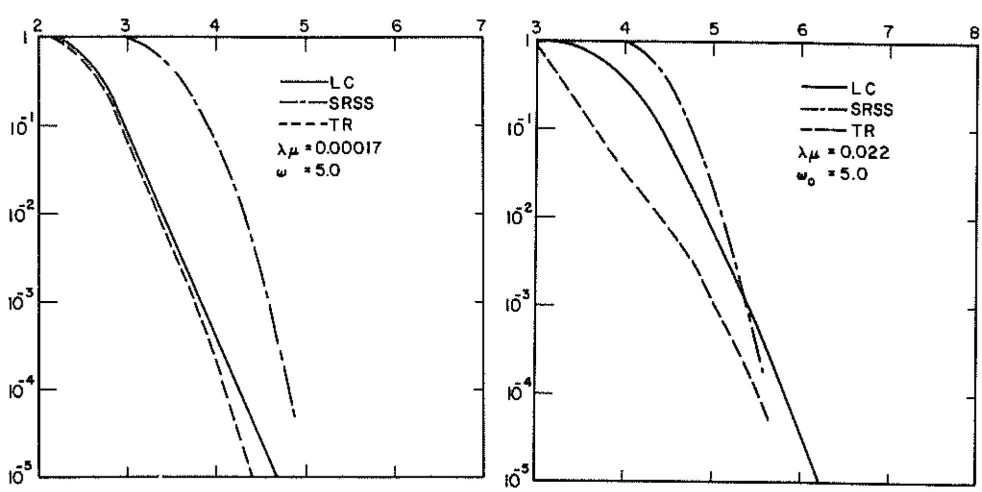

Fig. 7 Error of Load Combination Rule for Dynamic Effects