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# PROBABILISTIC ASSESSMENT OF THE LIQUEFACTION POTENTIAL OF WATERSATURATED SANDS

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## ABSTRACT

Deterministic models of soil liquefaction give a yes or no answer as to whether liquefaction will occur or not, or an answer in the form of a factor of safety. In either case, some consideration of probabilities must be made, to answer questions such as: Is the risk of liquefaction high enough to justify a large monetary expenditure to improve the ground at a project site, or should the investments already made at the project site be abandoned? Deterministic answers by themselves do not generally provide clear-cut decisions in cases where potential failure must be weighed against potential cost.

Due to high variation of characteristics of soils and seismic motion and because of difficulties in determining these characteristics with high degree of confidence, probabilistic methods appear to be basic in solving this problem.

Simplified procedure of Seed (1979) was used for evaluation of liquefaction potential of soils.

This paper introduces an opportunity of applying probabilistic methods for liquefaction risk analyses. The base problem in that kind of analyses is estimation of conditional probability of liquefaction. The estimation of total probability of liquefaction induced by cycling loading is the main purpose of the analyses.

## INTRODUCTION

The term “liquefaction” describes a condition in which a cohesionless soil undergo deformations caused by increasing of pore water pressure or generally it describes condition in which cohesionless soil loses strength during earthquakes and acquires a degree of mobility sufficient to permit movements ranging form several meters to several hundreds meters.

The liquefaction potential of any given soil deposit is determined by combination of the soil properties (dynamic shear modulus, damping characteristics, unit weight, grain characteristics, relative density), environmental factors (method of soil formation, seismic history, depth of water table), earthquake characteristics (intensity of ground shaking, duration of ground shaking) [7].

The purpose of this paper is to illustrate a probabilistic liquefaction risk analysis for saturated sands.

## GENERAL POINTS IN PROBABILISTIC LIQUEFACTION RISK ASSESSMENT

Deterministic models of soil liquefaction give a yes or no answer as to whether liquefaction will occur or not, or an answer in the form of factor of safety. In either case, it is difficult to answer questions such as: Is the risk of liquefaction high enough to justify a large monetary expenditure to improve the ground at project site, or should the investment already made at that project site be abandoned? Deterministic answers by themselves do not generally provide clear-cut decisions in cases where potential failure must be weighed against potential cost.

Probabilistic methods can be introduced at various stages of liquefaction risk assessment. Depending on the objectives, consideration have focused on one of the following sources of uncertainty:

- Uncertainty in the magnitude and location of earthquakes that can potentially affect the site;
- Uncertainty of the acceleration and duration of ground motion at a site, resulting from earthquake but attenuated by distance and filtered by the site response;
- Uncertainty in the basic physical models of soil liquefaction behavior;
- Uncertainty in the soil resistance parameters input in the physical model

The general steps in liquefaction risk analysis are indicated in Figure 1. There are two essential parts. One deals with the probability that earthquakes occur and second deals with the probability that there is liquefaction. The probabilities from these two parts must be combined and summed over all possible earthquakes to give the probability of liquefaction.

To begin any liquefaction analyses it is first asked: What is the probability of liquefaction resulting from an earthquake at a specific point and having a specific magnitude?

To answer this question requires answering the following sequence of questions:

- What is the probability that an earthquake will occur along fault for given time period?
- If this earthquake occurs, what is the probability that the earthquake will have magnitude greater than given value?

- If an earthquake of this magnitude occurs on fault what is the probability that its effects at a point of investigations are of engineering interest.?

To obtain the combined probability of occurrence of the specific earthquake, the probability estimates that answer the above questions are multiplied together. Than it is asked: If earthquake with defined magnitude (converted in annual probability of exceedance of given parameter of seismic motion) occurs on fault, what is the probability that liquefaction will occur at the project site?

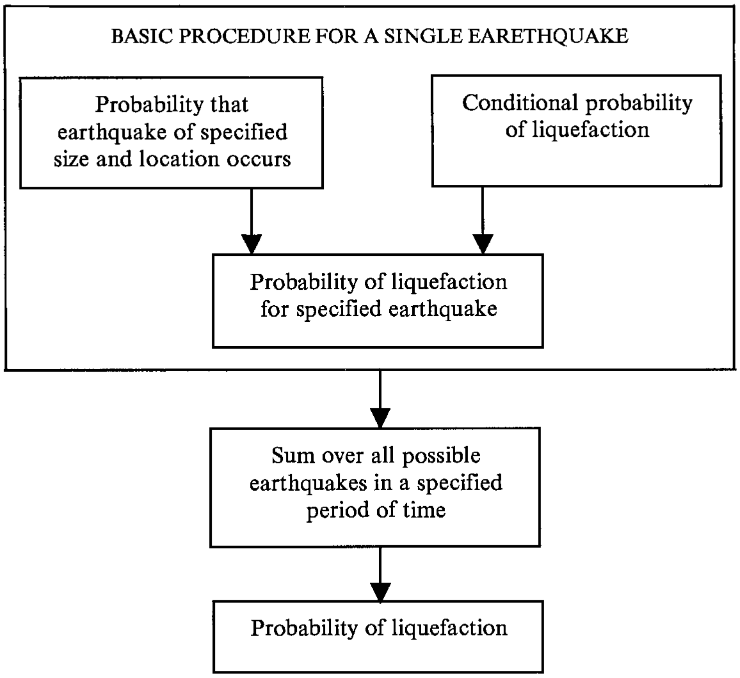

**Figure 1. Schematic of steps in liquefaction risk analyses** [5]

The estimate of probability that answers the last question is termed "conditional probability of liquefaction". Multiplying this conditional probability of liquefaction by the annual probability of exceedance gives the overall probability of liquefaction being caused by the specified earthquake.

It is important to consider all possible earthquakes in the vicinity of the project site. Thus the following possible situations must also be examined:

- Earthquakes with magnitudes other than defined can occur.
- Earthquakes can occur not only at this point, but at several points of fault.
- Earthquakes can occur not only on this fault but also on faults from other source zones.

The probability of liquefaction associated with each and every combination of the above situations must be evaluated and summed to yield the total probability of liquefaction at the project site.

## SEISMIC FRASGILITY ANALYSES

The basic steps of procedure for evaluation of liquefaction risk as we mentioned above consist on: assessment of seismic hazard and uncertainties; definition of the failure criteria; evaluation of the probability of failure.

The general formulation of the procedure applied in this investigation is presented in (Franzini, et al., 1984).

The probability of failure for an expected time period can be obtain from the annual frequency of failure, $\beta_E$ , determined by the relation [2]:

$$\beta_E = \int [d[\beta(x)/dx]] P(f,x)dx \tag{1}$$

$\beta(x)$ is the annual frequency of exceedance of a load level x (for example, the variable x may be peak ground acceleration); P(f,x) is the conditional probability of failure at a given seismic load level x. The problem leads to the assessment of the seismic hazard $\beta(x)$ and the fragility P(f,x) [2].

The fragility of soils is defined as conditional probability of liquefaction at a given value of seismic response parameter as maximum acceleration, velocity, displacement, spectral acceleration, Arias intensity, etc.

Generally there are two ways of determine seismic fragilities, i.e. in terms of global ground motion parameter or in terms of local response parameter. The first step in generation fragility curve is a clear-cut definition of what constitutes failure for the analyzed object. This definition may differ significantly depending on the purposes of the analyses.

The fragility could be derived either based on scaling procedure or on the base of generation.

Most frequently the objective of the fragility analyses is to estimate the peak ground motion acceleration value for which the seismic response exceeds the capacity of soils against liquefaction resulting in failure. The estimation of peak ground acceleration could be performed on the base of calculations or based on experience data. Because there are many sources of variability the fragility is expressed usually by family of fragility curves. A probability value is assigned to each curve to reflect the uncertainty in the fragility estimation (fig.2).

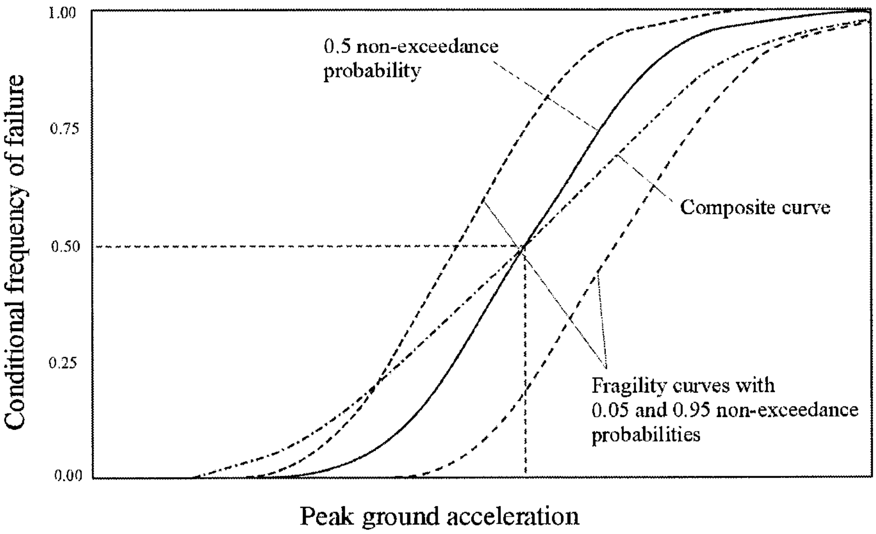

**Figure 2. Seismic fragility family curves** [4]

Scaling method for determination of fragility curves may be exposed with one simple but effective fragility model that supposes that the entire family of curves can be expressed by median ground acceleration $A_m$ and two random variables $\varepsilon_R$ and $\varepsilon_U$, thus the ground acceleration capacity A is given by:

$$A=A_m.\ \varepsilon_R.\ \varepsilon_U \tag{2}$$

$\varepsilon_R$ and $\varepsilon_U$ are log-normally distributed with unit medians and standard deviations $\beta_R$ and $\beta_U$ respectively. They represent the inherent randomness about the median and the uncertainty of the median value respectively. In some cases the composite variability $\beta_C$ is used, defined as:

$$\beta_C = \sqrt{(\beta_R^2 + \beta_U^2)} \tag{3}$$

The use of $A_m$ and $\beta_C$ provide a single best estimate fragility curve which does not explicitly separate randomness from uncertainty.

In estimating the fragility parameters it is convenient to use an intermediate random variable, called factor of safety, F. The factor of safety is defined as:

$$F = \frac{\text{Actual seismic capacity}}{\text{Actual response due to RE}} \tag{4}$$

where RE is the referent earthquake.
Further more the factor of safety can be expressed by:

$$F=F_S.F_\beta.F_{RS} \tag{5}$$

$F_S$ is called stress factor, representing the ratio of the ultimate strength to the stress, calculated for design earthquake; $F_\beta$ is inelastic energy absorption factor; $F_{RS}$ is the structural response factor.

In this paper we used generation as a method for obtaining fragility curves. The fragility curves are obtained from the computed discrete values of the conditional probabilities for define seismic level. For this purpose the function that passes through those discrete values is approximated with cumulative lognormal distribution functions. The approximation is done by the least square method.

The main steps of the computation procedure are:

1. Preparation a set of input variables by Latin Hypercube Experimental Design procedure.
2. Computation of stresses due to seismic loads.
3. Computation of soil resistance.
4. Definition of failure
5. Statistic of results.

Conditional probability of liquefaction could be determined by the expression [1]:

$$P_f = \beta FR(x) fL(x) dx \tag{6}$$

where FR(x) is the distribution function of the resistance and fL(x) is the density function of the seismic loading distribution

The Seismic hazard curves represent annual probability of exceedance of a given parameter of seismic motion. They are result from the application of probabilistic models of the site region defined on the basis of complex analyses including description of regional tectonic, review of historical seismicity, identification of seismic source zones, development of earthquake recurrence relationships. Determining the uncertainties is basic problem in seismic hazard analyses.

**Case Study**

There is big number of liquefaction methods. In the presented case some of the methods were assessing the site as potentially liquefable, some of them says that it is not. The characteristics of the site are presented in table 1.

**Table 1. Site Characteristics**

| Layer No | Material | Thickness | Wight Density | Relative Density | SPT (N) | Permeability |
|---|---|---|---|---|---|---|
| | | m | t/m$^3$ | % | N | cm/sec |
| 1 | Sandy loess | 8.6-1.5 | 1.9-2.1 | - | - | - |
| 2 | Loes-clay | 1.7-3.5 | 1.9-2.1 | - | - | - |
| 3 | Fine sand | 0.5-3.0 | 1.6-1.7 | 53 | 9/21 | 4.E$^{-4}$ |
| 4 | Fine sand | 0.5-4.7 | 1.6-1.7 | 44 | 9/21 | 4.E$^{-4}$ |

For the liquefaction analyses the Seed's method was used. It defines a factor of safety for liquefaction as:

$$FS = \frac{SR}{L} \tag{7}$$

SR is the cyclic ratio for equivalent number of cycles and L is the stress ratio caused by the equivalent number cycles by an earthquake [5].

The experimentally determined cyclic stress ratio to generated liquefaction for the site is shown in figure 3.

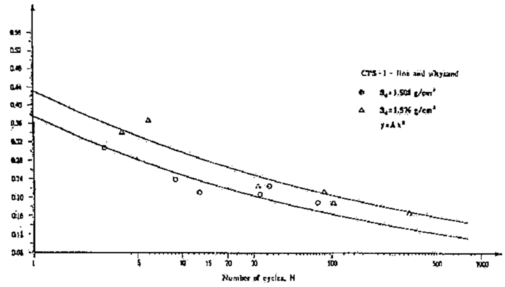

**Figure 3. Experimentally determined cyclic stress ratio** [4]

For each set of variables the Seed's procedure for evaluating liquefaction potential is applied deterministically for three levels of seismic excitation. The seismic levels correspond to annual probability of exceedance $10^{-3}$, $10^{-4}$ and $10^{-5}$.

The seismic hazard is governed by two types of seismic sources: local shallow sources with maximum magnitude up to 5.5 and fare field intermediate depth source with maximum magnitude up to 7.5. The seismic hazard curve is presented in figure 4.

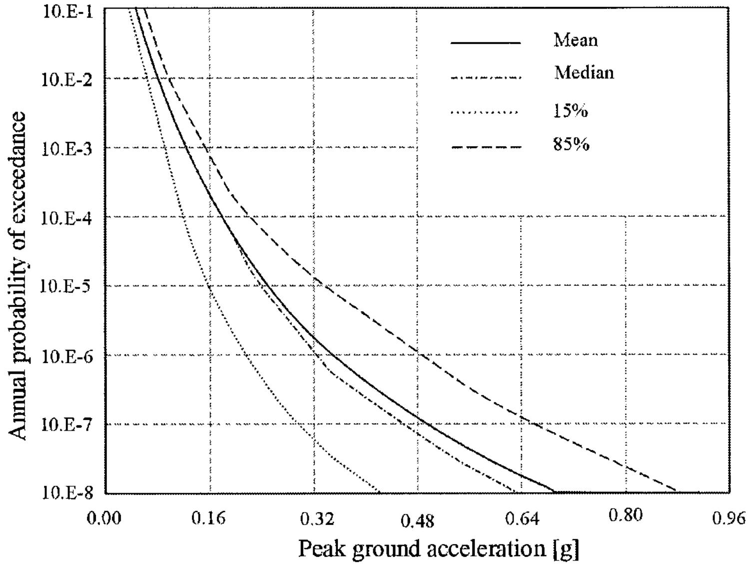

**Figure 4.Seismic hazard curves mean, median, 15 and 85 percentiles.**

The equivalent number of cycles is determined according to Seed's procedure (1975).

The deterministic analysis is based on the LHCED procedure. There are 10 samples generated and analyzed for three seismic levels ($10^{-3}$, $10^{-4}$, $10^{-5}$). The generation is done on the following parameters (table 2).

**Table 2. LHCED generated parameters**

| Parameter/Generation | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Thickness of layer 1 | 8.98 | 10.03 | 9.76 | 8.69 | 10.43 | 9.39 | 9.1 | 9.15 | 9.66 | 9.31 |
| Thickness of layer 2 | 2.89 | 2.68 | 3.12 | 2.96 | 3.25 | 3.09 | 3.15 | 3.31 | 3.03 | 3.60 |
| Thickness of layer 3 | 1.34 | 3.00 | 2.36 | 1.77 | 1.93 | 1.65 | 0.90 | 2.29 | 1.57 | 1.30 |
| Thickness of layer 4 | 2.02 | 3.45 | 1.66 | 2.38 | 2.51 | 2.78 | 1.97 | 2.94 | 5.05 | 3.29 |
| Weight density of layer 1 | 2.02 | 2.07 | 2.06 | 2.06 | 2.02 | 2.04 | 2.00 | 2.05 | 2.04 | 2.03 |
| Weight density of layer 2 | 2.09 | 2.12 | 2.11 | 2.11 | 2.09 | 2.10 | 2.08 | 2.10 | 2.10 | 2.10 |
| Weight density of layer 3 | 1.64 | 1.62 | 1.64 | 1.66 | 1.63 | 1.65 | 1.67 | 1.69 | 1.66 | 1.65 |
| Weight density of layer 4 | 1.60 | 1.67 | 1.65 | 1.71 | 1.67 | 1.68 | 1.59 | 1.63 | 1.66 | 1.63 |
| GWT* | 28.95 | 28.60 | 29.81 | 29.14 | 29.21 | 29.34 | 29.00 | 28.82 | 29.57 | 30.02 |
| Equivalent cycles N ($10^{-3}$) | 2.44 | 3.58 | 3.25 | 2.18 | 4.10 | 2.85 | 2.98 | 2.62 | 3.15 | 2.77 |
| Equivalent cycles N ($10^{-4}$) | 7.48 | 5.59 | 9.99 | 8.25 | 11.69 | 9.68 | 10.38 | 12.52 | 8.95 | 17.3 |
| Equivalent cycles N ($10^{-5}$) | 10.75 | 16.79 | 8.67 | 12.46 | 14.63 | 23.24 | 20.49 | 719.15 | 13.40 | 27.63 |
| Max. acceleration $10^{-3}$ | 0.10 | 0.12 | 0.10 | 0.11 | 0.12 | 0.14 | 0.13 | 0.13 | 0.11 | 0.15 |
| Max. acceleration $10^{-4}$ | 0.15 | 0.19 | 0.14 | 0.16 | 0.17 | 0.21 | 0.20 | 0.23 | 0.24 | 0.17 |
| Max. acceleration $10^{-5}$ | 0.20 | 0.27 | 0.22 | 0.18 | 0.26 | 0.33 | 0.28 | 0.35 | 0.31 | 0.25 |
| Relative density of layer 3 | 42.02 | 54.22 | 39.05 | 46.59 | 49.92 | 60.22 | 57.04 | 55.62 | 70.85 | 47.69 |
| Relative density of layer 4 | 32.92 | 44.00 | 36.37 | 28.92 | 41.80 | 54.77 | 45.47 | 58.20 | 50.45 | 40.32 |
| *- Underground water level is relative to the sea level.The site level is +35.00m | | | | | | | | | | |

In order to achieve better accuracy layers 3 and 4 are subdivided in thinner layers. For each layer a central factor of safety and the corresponding standard deviation is computed for each seismic level.

The conditional probability of liquefaction is determined assuming that the resistance and load are log-normally distributed. The computed results are presented in table 3.

**Table 3. Conditional probability of liquefaction of layers 3 and 4**

| Layer No | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ |
|---|---|---|---|
| 3.1 | 1.78E-10 | 5.07E-02 | 5.96E-01 |
| 3.2 | 2.24E-10 | 5.33E-02 | 6.05E-01 |
| 3.3 | 2.06E-10 | 5.51E-02 | 6.11E-01 |
| 3.4 | 2.34E-10 | 5.71E-02 | 6.17E-01 |
| 4.1 | 2.62E-10 | 5.81E-02 | 6.21E-01 |
| 4.2 | 2.59E-10 | 5.69E-02 | 6.23E-01 |

Base on those conditional probabilities a fragility curve is generated by interpolation and extrapolation of the results for each layer (sub-layer). They are presented in figure 5.

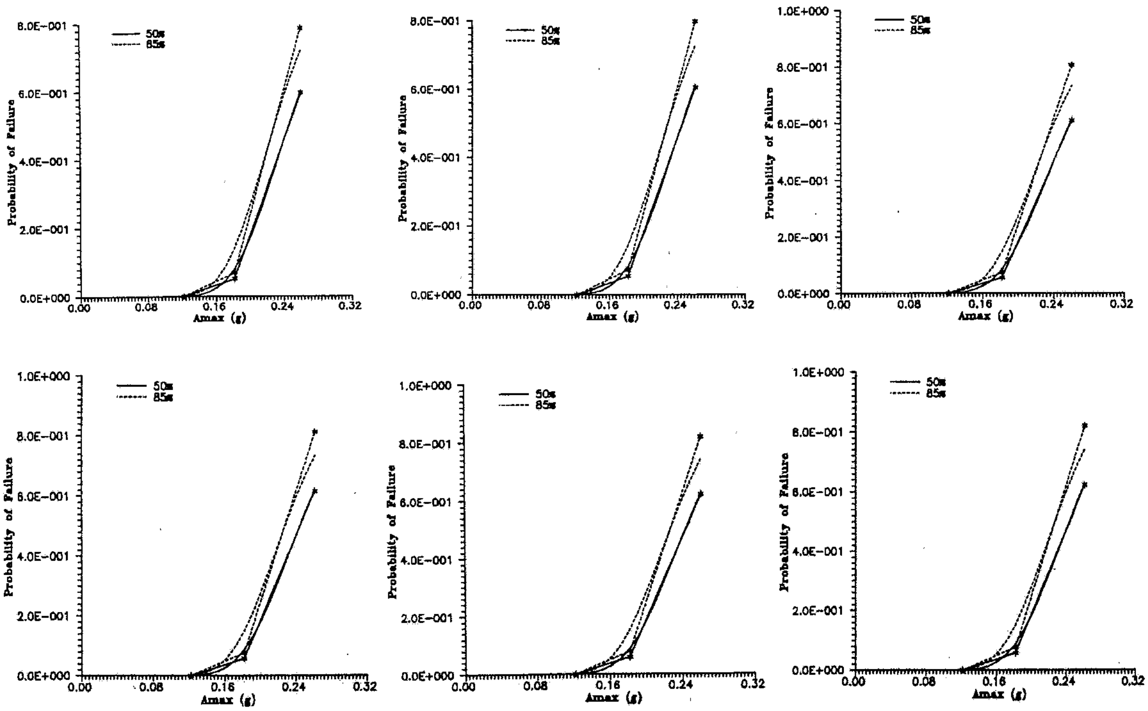

**Figure 5. Fragility curve generated by interpolation and extrapolation of the results for each layer**

The fragility then is convoluted by the hazard curve and the probability for liquefaction of each layer is obtained (table 4).

**Table 4. Probability of liquefaction for layers 3 and 4**

| Layer No | Probability for liquefaction |
|---|---|
| 3.1 | 6.71E-05 |
| 3.2 | 6.89E-05 |
| 3.3 | 6.98E-05 |
| 3.4 | 7.07E-05 |
| 4.1 | 7.18E-05 |
| 4.2 | 7.19E-05 |

## CONCLUSIONS

There are some significant advantages in using probabilistic methods to assess the potential for liquefaction. The risk of liquefaction can be compared in equivalent terms to the other risks to which a structure is exposed. Uncertainties in the different inputs can be treated systematically and uniformly and those factors having the most influence of the risk can be readily identified. Using probabilistic methods for evaluation of liquefaction risk we can define the influence of any of countermeasures against liquefaction at probability of liquefaction and we can choose the most efficient countermeasure for reducing the liquefaction risk.

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