EFFECTS OF MATERIAL DISINTEGRATION ON STRUCTURAL PERFORMANCE

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SUMMARY

Using a state variable approach it will be shown how the results of appropriately chosen tests can be used to develop constitutive equations which describe the effects of creep deterioration on the deformation of metals operating at elevated temperatures. In this way a systematic phenomenological approach can be developed.

Using this approach in conjunction with the results of multiaxial stress tests a set of constitutive equations are proposed which define the creep deformation rates in terms of applied stress and a material state variable, referred to as damage. In spite of the fact that the equations have been constructed using purely phenomenological procedures, it is possible, nevertheless, to demonstrate the physical interpretation of the damage state.

Using the constitutive equations in conjunction with the virtual work relationships it is possible to obtain bounds on the rupture life of components subjected to both constant and cyclic loading. It is found convenient to express the results in terms of a so-called representative rupture stress, so that the uniaxial creep rupture curves can be used directly, thereby avoiding complex curve fitting procedures.

The bounds are used to predict the rupture life of a number of components subjected to constant and variable load and comparison is made with experimental results. The components tested are:

(1) Plane stress plates with holes and slots, subjected to uniaxial stress.
(2) Plane stress plates with holes and slots subjected to biaxial tension.
(3) Cylindrical bars with round notches.
(4) Reinforced shell intersections subjected to internal pressure.

In all cases it is observed that the effect of stress raisers does not greatly influence the life of the components. This effect is apparently due to the fact that the high stresses relax as a result of local damage. The effect of variable loading is found to be small with time on maximum load acting as the important parameter.

In the cylindrical bars with round notches it is found that the life is greatly influenced by the form of the isochronous rupture curves. If the rupture criterion is maximum stress dependent, then it is found that there is neither strengthening nor weakening when comparison is made on the basis of net section stress. However, when the isochronous rupture surface is of the Von Mises form then considerable strengthening is observed.

The shell component consists of a reinforced radial nozzle in a spherical shell. The reinforcement was designed to be "balanced" so that the limit load of the intersection is equal to that of the parent spherical shell. Theory predicts that in spite of the stress concentration effects it is the parent vessel and not the reinforcement which fails. This result suggests that if the geometry of the reinforcement is selected according to existing procedures used at ambient conditions then it is unnecessary to perform complex creep calculations at the intersection region, since first failure occurs in the membrane. The calculations required to fix main shell thicknesses are relatively simple.

Tests on plane strain specimens with sharp notches are presently in progress. It is hoped to present the results of these tests at the time of the Conference.

1. Introduction

In a previous report Leckie & Wojewódzki [1974] discussed means of obtaining bounds in the life of components which are subjected to constant loads and which operate at temperatures sufficiently high for the effects of creep rupture mechanisms to become the factor which limits component life. The study is now extended to the consideration of components which are subjected to proportional cyclic loading. The operating temperature is assumed to be constant and the study of the additional difficulty when temperatures also vary is left to a later paper.

Because of the process of deterioration which takes place in the material it is to be expected that these portions of components which sustain the higher stress levels shed some of the stress to adjacent material. It has been shown by Leckie & Hayhurst [1973] that the effects of this stress redistribution can be important and that the life of the component can be considerably greater than those estimates of life obtained by using the results of a steady state creep analysis in conjunction with uniaxial stress rupture tests.

In order to measure the effects of stress redistribution it is necessary to use constitutive equations which reflect the material softening introduced by the damage occurring in the materail. Based on the Kachanov concept of damage, Leckie & Hayhurst [1973] suggested constitutive equations which represent in reasonable fashion the macroscopic behaviour of creeping material. Using these equations Leckie & Wojewódzki [1974] were able to obtain an upper bound on the time to rupture of components subjected to constant load. A lower bound has still to be isolated although Leckie & Hayhurst [1973] were able, using the technique suggested by Martin & Leckie [1972], to obtain a lower bound for kinematically determinate components.

The aim of the present study is to obtain upper and lower bounds on the time to rupture of components operating at constant temperatures which are subjected to cyclic proportional loading. As for the constant loading case it is possible in general to find an upper bound on the rupture time.

But efforts to find lower bounds are only partially successful. The effectiveness of the bounds are studied by comparing the time to rupture predictions for a simple component with the results of numerical integration performed by electronic computer.

2. The Constitutive Equations

The strain rate $\dot{\varepsilon}_{ij}$ is assumed to be the sum of the elastic $\dot{e}_{ij}$ and creep $\dot{v}_{ij}$ strain rate components, so that

$$\dot{\varepsilon}_{ij} = \dot{e}_{ij} + \dot{v}_{ij} \; . \tag{2.1}$$

The elastic strains e_{ij} are related to the stress σ_{ij} by the relation

$$e_{ij} = C_{ijk\ell}\, \sigma_{k\ell} \tag{2.2}$$

where $C_{ijk\ell}$ is assumed to be constant.

The expressions used to describe the creep strain rate are those derived by Leckie & Hayhurst [1973] which are a generalisation of those first proposed by Kachanov [1958] suitably modified to fit the results of multi-axial stress experiments. By introducing an internal state variable ψ Kachanov was able to modify the Norton Creep Law so that the tertiary portion of the creep curve can be represented. In the Kachanov equations the functions were selected to fit creep data and there was no apparent attempt made to interpret the physical meaning of ψ other than in vague terms of material damage. On the basis of the studies reported by metal physicists Leckie & Hayhurst [1973] suggested however that the ψ parameter may be used to represent the physical processes of deterioration.

The constitutive equation for the creep rate $\dot{v}_{ij}$ is

$$\dot{v}_{ij}/\dot{v}_o = \frac{1}{\psi^n}\,\phi^n(\sigma_{ij}/\sigma_o)\;\partial\phi(\sigma_{ij}/\sigma_o) \quad . \tag{2.3a}$$

In this expression ϕ is a homogeneous function of degree one and is equal to unity when the applied stress is uniaxial and of magnitude σ_o. The material constants are $\dot{v}_o$, σ_o and n. When the material is undamaged $\psi = 1$ and the expression for $\dot{v}_{ij}$ reduces to that suggested by Calladine & Drucker [1962]. As damage ψ decreases so that the strain rates increase, but the form of (2.3a) implies that the ratio of strain rate components remains constant, which is apparently in accord with experimental observation, Leckie & Hayhurst [1973].

The damage rate equation has the form

$$\frac{d\psi}{dt} = -\frac{A}{\psi^\nu}\,\Delta^\nu(\sigma_{ij}/\sigma_o) \;, \tag{2.4}$$

where A and ν are material constants and Δ is a homogeneous function of degree one in (σ_{ij}/σ_o).

Integrating this equation and using the rupture condition $\psi = 0$ gives the expression for rupture time t_R as

$$t_R = 1/A(1+\nu)\;\Delta^\nu(\sigma_{ij}/\sigma_o) \;. \tag{2.5}$$

For uniaxial states of stress this relationship gives the form of the stress/time to rupture observed experimentally over a considerable stress range. Isochronous surfaces are given by $\Delta(\sigma_{ij}/\sigma_o)$ = constant. The function $\Delta(\sigma_{ij}/\sigma_o)$ can take various forms. Multi-axial rupture experiments show the isochronous rupture surface of some important materials, such as stainless steel and aluminium, satisfy a Huber-Mises shear criterion. Other metals such as copper satisfy a maximum stress criterion. The rupture criterion for most metals Hayhurst [1972] apparently lie between these two extremes.

The integration of equation (2.4) for variable uniaxial stresses gives the rupture condition

$$\Sigma\; t/t_R = 1, \tag{2.6}$$

where t_R is the rupture time associated with the constant uniaxial stress σ_R. The extensive review by Penny & Marriott [1971] of the results of existing experiments supports the view that equation (2.6) is satisfactory for stepped cyclic loading.

For more complex forms of stress history it has been shown by Storåkers [1969] that the equation (2.6) can be substantially in error, and consequently only cyclic loading is considered.

In some important materials such as stainless steels and certain aluminium alloys it appears that the isochronous surfaces $\Delta(\sigma_{ij})$ have the same shape as the constant energy disruption rate surface $\phi(\sigma_{ij})$. The constitutive equations of such materials are then given by

$$\dot{v}_{ij}/\dot{v}_o = \frac{1}{\psi^n}\,\phi^n(\sigma_{ij}/\sigma_o)\;\partial\phi/\partial(\sigma_{ij}/\sigma_o)$$

$$\dot{\psi} = -\frac{A}{\psi^\nu}\,\phi^\nu(\sigma_{ij}/\sigma_o) \;. \tag{2.7a,b}$$

For convenience metals whose behaviour are described by the above equations are referred to as ϕ/ϕ materials, while other materials whose behaviour is described by equations (2.3, 2.4) are referred to as ϕ/Δ materials.

3. The Steady State and Steady State Cyclic Stress Distributions

It proves convenient for the development of subsequent sections to discuss the steady state and steady state cyclic stress distributions in an undamaged component for which $\psi = 1$ throughout.

Consider a structure subjected to constant loads P_i over the surface A_p, the displacements u over the remaining surface A_u being everywhere zero. On the application of the loads the response of the structure is elastic and subsequently with the passage of time the effect of creep deformation is to cause stress redistribution. As time approaches infinity the stress distribution approaches a steady state stress distribution σ^s_{ij} which minimises the function

$$\Omega = \int_V \phi^{n+1}(\sigma_{ij}/\sigma_o)\ dV. \tag{3.1}$$

It has been shown by Leckie & Martin [1967] that the total energy dissipation from the time of load application is

$$\int_{A_p} P_i\ u_i\ dA = \sigma_o \dot{v}_o \int_o^t d\tau \int_V \phi^{n+1}(\sigma^s_{ij}/\sigma_o)\ dV + \delta(t)\ , \tag{3.2}$$

where $\delta(t)$ is an additional energy due to the process of stress redistribution. Bounds have been obtained on $\delta(t)$ from which it can be deduced that the effects of stress redistribution are small and can normally be neglected.

Similar features can be identified when the structure is subjected to cyclic loading $P_i(t)$ of period ΔT so that $P_i(t) = P_i(t+\Delta T)$. Ponter [1974] has shown that as time approaches infinity the stress field approaches a cyclic state σ^{sc}_{ij} so that

$$\sigma^{sc}_{ij}(t) = \sigma^{sc}_{ij}(t+\Delta T)\ , \tag{3.3}$$

and that the total energy dissipation is given by

$$\int_o^t d\tau \int_{A_p} P_i \dot{u}_i dA = \sigma_o \dot{v}_o \int_V \int_o^t \phi^{n+1}(\sigma^{sc}_{ij}/\sigma_o) d\tau dV + \delta(t), \tag{3.4}$$

where $\delta(t)$ is again the additional energy dissipation caused by the stress redistribution as the stress state approaches σ^{sc}_{ij}. Bounds on $\delta(t)$ have again been determined and it can be shown that in most circumstances that this term may again be neglected. The first term on the right hand side of (3.4) may be bounded from above according to the relationship

$$\int_V \int_o^t \phi^{n+1}(\sigma^*_{ij}/\sigma_o)\ d\tau dV \geq \int_V \int_o^t \phi^{n+1}(\sigma^{sc}_{ij}/\sigma_o)\ d\tau dV\ , \tag{3.5}$$

where
$$\sigma^*_{ij}(t) = \sigma^e_{ij}(t) + \rho_{ij}\ . \tag{3.6}$$

In this equation $\sigma^e_{ij}(t)$ is the elastic stress distribution in equilibrium with the loads $P_i(t)$, and ρ_{ij} is a self-equilibrating stress distribution which is constant in time. Furthermore Ponter [1974] has shown that as the cycle time approaches zero

$$\sigma^{sc}_{ij} \to \sigma^e_{ij}(t) + \rho_{ij}\ , \tag{3.7}$$

with the stress distribution ρ_{ij} minimising the expression on the left hand side of (3.5). In situations when the cycle time is ΔT is small in relation to the working life of the component the above criterion provides a direct method of obtaining the steady cyclic stress state σ_{ij}^{sc}, avoiding the need to compute a full time dependent analysis.

4. An Upper Bound on Rupture Time

A body of volume V subjected to proportional cyclic loads $P_i\gamma(t)$ where $\gamma(t) = \gamma(t+\Delta T)$ consists of material whose constitutive equations are those described in Section 2. After the initial elastic response, interaction between the elastic and creep strains results in stress redistributions. Further stress redistribution can be expected as a result of the softening resulting from material deterioration. At time t_1 local rupture will occur at some point in the body when the value of ψ at that point is zero. A damage front will then spread out from this point so that only a portion $\bar{V}$ of the body can sustain stress. Final rupture occurs at time t_R when the body cannot sustain the applied load.

At time $t < t_1$ let the stress and damage distributions be represented by σ_{ij} and ψ respectively. Equation (2.4) can be expressed in the form

$$- \frac{1}{(1+\nu)A} d(\psi^{\nu+1})/dt = \Delta^\nu(\sigma_{ij}/\sigma_o) , \tag{4.1}$$

and integrating over the volume V gives the relationship

$$- \frac{1}{(1+\nu)A} \frac{d}{dt} \int_V \psi^{\nu+1} dV = \int_V \Delta^\nu(\sigma_{ij}/\sigma_o) \, dV. \tag{4.2}$$

Consider now an imaginary body identical in shape to that under consideration and subjected to the same loads $P_i(t)$. The material of the body is an elastic creeping material with constitutive equations.

$$\begin{aligned} e_{ij} &= C_{ijk\ell} \, \sigma_{k\ell} \\ \dot{v}_{ij}/\dot{v}_o &= \Delta^{\nu-1}(\sigma_{ij}/\sigma_o) \, \partial\Delta/\partial \, (\sigma_{ij}/\sigma_o) . \end{aligned} \tag{4.3a,b}$$

It should be noted that this material does not suffer material deterioration. The creep energy dissipation rate is,

$$\dot{D}/\sigma_o\dot{v}_o = \Delta^\nu(\sigma_{ij}/\sigma_o) ,$$

so that the surfaces of constant energy dissipation rate are given by $\Delta(\sigma_{ij}/\sigma_o)$ = constant and coincide with the isochronous surface of the rupturing material. In the analysis which follows it is necessary to a-sume that the isochronous surfaces are convex so that use can be made of certain established relationships. According to Hayhurst [1972] existing experimental evidence supports the validity of the convexity assumption.

Suppose now that σ_{ij}^s is the steady state stress for the imaginary body when subjected to loads P_i. From the complementary energy theorem

$$\int_V \Delta^\nu(\sigma_{ij}^s/\sigma_o) \, \gamma^\nu(t) \, dV \leq \int \Delta^\nu(\sigma_{ij}/\sigma_o) \, dV .$$

Using this result in the global damage rate equation (3.2) gives the inequality

$$- \frac{1}{(1+\nu)A} \frac{d}{dt} \int_V \psi^{\nu+1} dV = \int_V \Delta^\nu(\sigma_{ij}/\sigma_o) \, dV \geq \int_V \gamma^\nu(t) \, \Delta^\nu(\sigma_{ij}^s/\sigma_o) . \tag{4.4}$$

When $t > t_1$, then $\psi = 0$ for that part of the structure which has ruptured. Consequently

$$\int_{\bar{V}} \psi^{\nu+1}\, dV = \int_{V} \psi^{\nu+1}\, dV,$$

so that the inequality (4.4) is still satisfied as the rupture surface moves through the body. Replacing the inequality by an equality and using the condition $\int_V \psi^{\nu+1}\, dV = 0$ at rupture times gives a time to rupture t_u which is an upper bound on the rupture time t_R, according to the equation

$$\int_0^{t_u} \gamma^{\nu}(\tau)\, d\tau = V/A(1+\nu) \int_V \Delta^{\nu}(\sigma_{ij}^{s}/\sigma_o)\, dV\ , \tag{4.5}$$

from which

$$t_u = V/A(1+\nu) \int_0^1 \gamma^{\nu}(x)dx \int_V \Delta^{\nu}(\sigma_{ij}/\sigma_o)\, dV\ , \tag{4.6}$$

where $t = x\ \Delta T$.

It is coinvenient to express this result in terms of a uniaxial representative rupture stress $\sigma_u\gamma(t)$ which ruptures in time t_u. Integrating equation (2.4) for a uniaxial stress state and applying the rupture condition $\psi = 0$ gives the expression for rupture time t_u.

$$\int_0^{t_u} \gamma^{\nu}(\tau)\, d\tau = \frac{1}{(1+\nu)\ A\ (\sigma_u/\sigma_o)^{\nu}}\ .$$

Comparing this equation with equation (4.5) gives the expression for σ_u,

$$\sigma_u/\sigma_o = \left\{ \int_V \Delta^{\nu}(\sigma_{ij}^{s}/\sigma_o)\, dV/V \right\}^{1/\nu} . \tag{4.7}$$

This means that the time to rupture of a uniaxial specimen subjected to the stress $\sigma_u\gamma(t)$ will give a rupture time in excess of the rupture time of the component.

5. A Lower Bound on Rupture Time of Kinematically Determinate Components

Attempts to find a lower bound on the time to rupture of components have not been successful and only results of limited validity have been obtained. The bound is limited to kinematically determinate components composed of ϕ/ϕ materials and it is assumed that the cycle time is short so that in the undamaged state the cyclic steady state stress distribution has the form given by equation (3.7). However, Leckie & Hayhurst [1973] have demonstrated experimentally that a similar bound obtained for components subjected to constant load satisfactorily predicts the life of components which are not strictly kinematically determinate. A similar bound can be obtained for components of ϕ/Δ material which are in a state of plane stress.

If the cycle time ΔT is small then the energy dissipated per cycle in the undamaged component is given by the relationship [equations (3.5),(3.7)]

$$\int_0^1 \int_A P_i \dot{u}_i^{sc}\, dA\, dx = \sigma_o \dot{v}_o \int_V \int_0 \phi^{n+1}(\sigma_{ij}^{e}+\rho_{ij})\, dx\, dV\ , \tag{5.1}$$

where $\tau = x\Delta T$.

The distribution of the self-equilibrating stress field is obtained by finding the stress field which minimises the expression on the right hand side. Having performed this calculation the expression for $\dot{u}_i^{sc}$ and $\dot{v}_{ij}^{sc}$ can then be determined. It is assumed that this process has been completed.

As deterioration occurs in the component the cyclic stress distribution alters with time. Since $T \to 0$ the stress distribution is of the form

$$\sigma_{ij} = \sigma_{ij}^{e} + \rho_{ij}$$

and since the loading is cyclic the slow redistribution can be defined in terms of the time varying self-equilibrating stress distribution $\rho_{ij}(t)$. Consequently if the component is kinematically determinate the displacement and strain rates can be expressed in the form

$$\dot{u}_i = \dot{u}_i^{sc}(\tau)\, f(t) \qquad \text{and} \qquad \dot{v}_{ij} = \dot{v}_{ij}^{sc}(\tau)\, f(t) \tag{5.2}$$

where $f(t)$ is measured at the end of each cycle and is unity when $t = 0$.

If the damage parameter is ψ and since ϕ is homogeneous the energy dissipation rate is

$$\dot{D}/\sigma_o\dot{v}_o = \sigma_{ij}\, \dot{v}_{ij}/\sigma_o\dot{v}_o = \psi\, \phi^{n+1}(\sigma_{ij}/\psi\sigma_o) \ .$$

The total energy dissipation over a cycle is then

$$\int_o^1\!\int_A P_i\, \dot{u}_i\, dA\, dx = \int_o\!\int_V \psi\, \phi^{n+1}(\sigma_{ij}/\psi\sigma_o)\, dV\, dx \ . \tag{5.3}$$

At $t = 0$ when ψ is everywhere unity $f(t) = 1$ and the equation reduces to the form of equation (5.1). At time t and using the expressions (5.2) the energy balance equation becomes

$$f(t)\int_o^1\!\int_A P_i\, \dot{u}_i^{sc}\, dA\, dx = f(t)^{(n+1)/n}\int_o\!\int_V \psi\, \phi^{n+1}(\sigma_{ij}^{sc}/\sigma_o)\, dV\, dx,$$

since the factor $f(t)$ in strain rate causes an $f(t)^{(n+1)/n}$ increase in energy dissipation. Consequently the expression for $f(t)$ is

$$f^{1/n}(t) = \frac{\int_o^1\!\int_V \phi^{n+1}(\sigma_{ij}^{sc}/\sigma_o)\, dv\, dx}{\int_o^1\!\int_V \psi\, \phi^{n+1}(\sigma_{ij}^{sc}/\sigma_o)\, dV} \ .$$

Since the cycle time $\Delta T \to 0$, the value of ψ may, in this calculation, be regarded as constant over the cycle and the expression for $f(t)$ then becomes

$$1/f^{1/n}(t) = \Psi = \frac{\int_V \psi\, \bar{\phi}}{\int_V \bar{\phi}\, dV} \tag{5.4}$$

where $\bar{\phi} = \int_0^1 \phi^{n+1}(\sigma^s_{ij}/\sigma_o)\ dx$. (5.5)

The function Ψ is a normalised measure of global damage weighted by expressions obtained from the cyclic steady state solution. When time is zero $\Psi = 1$ and final rupture occurs when $\Psi = 0$.

Again, making use of the homogeneity of ϕ the damage rate equation (2.7b) can be written in the form,

$$\frac{d\psi}{dt} = - A\ \phi^\nu(\sigma_{ij}/\psi\sigma_o) = - A\ f^{\nu/n}(t)\ \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)$$

$$= - \frac{A}{\Psi^\nu}\ \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ .$$

Assuming that Ψ remains constant over the cycle the increment of Ψ in time ΔT is then

$$\frac{\Delta\psi}{\Delta T} = - \frac{1}{\Psi^\nu} \int_0^1 \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ dx\ . \qquad (5.6)$$

This equation indicates that since Ψ is a global quantity the maximum damage occurs at the point for which $\int_0^1 \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ dx$ is a maximum. Local rupture therefore occurs at this point at time t_1 when $\psi = 0$.

An equation can be obtained for Ψ for times less than t_1. Both sides of equation (5.6) are multiplied by $\phi/\int_V \bar{\phi}\ dV$ and integration performed over the volume to give

$$\frac{\Delta\Psi}{\Delta T} = - \frac{A}{\Psi^\nu} \frac{\int_V \bar{\phi} \int_0^1 \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ dx\ dV}{\int_V \bar{\phi}\ dV}\ .$$

Assuming ΔT is small enough to justify the relation

$$\Delta\Psi/\Delta T = d\Psi/dt\ ,$$

the differential equation in Ψ becomes

$$\frac{d}{dt}\Psi^{\nu+1} = - A(\nu+1) \int_V \bar{\phi} \int_0^1 \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ dx\ dV \Big/ \int_V \bar{\phi}\ dV\ . \qquad (5.7)$$

For times in excess of t_1 integrate over that volume $\bar{v}$ for which $\psi > 0$. Since Ψ is defined over the time $t < t_1$ with $\psi = 0$ in those parts of the structure which have been ruptured then

$$\frac{d}{dt}\Psi^{\nu+1} = - A(\nu+1) \int_{\bar{V}} \bar{\phi} \int_0^1 \phi^\nu(\sigma^{sc}_{ij}/\sigma_o)\ dx\ dV \int_V \bar{\phi}\ dV\ . \qquad (5.8)$$

To determine the location of the boundary of $\bar{V}$ requires a full computer analysis. However it can be seen that the right hand side of equation (5.8) is less than that of (5.7). Consequently equation (5.7) gives a global damage rate which is exact for times less than t_1

and greater than the exact values for times greater than t_1. By using the condition $\Psi = 0$ in the solution of equation (5.7) gives a lower bound on the failure time.

Applying the condition $\Psi = 1$ at $t = 0$, and $\Psi = 0$ at t_L, the solution of equation (5.7) gives

$$t_R > t_L = \int_V \bar{\phi}\, dV \Big/ A(1+\nu) \int_V \bar{\Phi} \int_o^1 \phi^\nu(\sigma_{ij}^{sc}/\sigma_o)\, dx\, dV \,. \tag{5.9}$$

If the time t_L is expressed in terms of a representative stress $\sigma_L \gamma(t)$, then following the procedure given in section 4 the expression for σ_L is

$$\sigma_L/\sigma_o = \left\{ \int_V \bar{\Phi} \int_o^1 \phi^\nu(\sigma_{ij}^s/\sigma_o)\, dx\, dV \Big/ \int_o^1 \gamma^\nu(x)\, dX \int \bar{\Phi}\, dx \right\}^{1/\nu} .$$

5.2 Lower Bound for ϕ/Δ Materials

In general it is not possible using an expression [such as (5.2)] for the viscous strain rate distribution to determine the stress distribution directly. If the viscous strains are incompressible only stress deviators can be determined in terms of the strain field and it is necessary to solve a differential equation before the hydrostatic component of stress can be found. This difficulty does not arise for ϕ/ϕ materials, but it means that the previous procedure falls down for ϕ/Δ materials. However in plane stress situations the stresses can be determined in terms of the given strain field and the previous procedure can be successfully adopted. The damage rate equation (2.4) can be written in the form

$$\frac{d\psi}{dt} = -\frac{A}{\psi^\nu} \Delta^\nu(\sigma_{ij}^{sc}/\sigma_o) \,,$$

and the expression for the lower bound time is

$$t_L = \frac{\int_V \bar{\Phi}\, dV}{A(\nu+1) \int_V \bar{\Phi} \int_o^1 \Delta^\nu(\sigma_{ij}^{sc}/\sigma_o)\, dx\, dV}$$

The corresponding representative rupture stress is $_L(t)$ where

$$\frac{\sigma_L}{\sigma_o} = \left\{ \int_V \bar{\Phi} \int_o^1 \Delta^\nu(\sigma_{ij}^{sc}/\sigma_o)\, dx\, dV \Big/ \int_o^1 \gamma^\nu(x)\, dx \int \bar{\Phi}\, dV \right\}^{1/\nu} . \tag{5.12}$$

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