Capacity Aspects of Bit Interleaved Coded Modulation

by

Nishant Jacob

A thesis submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Master of Science

Department of Electrical and Computer Engineering

Raleigh, NC
May 14, 2001

Approved By:

Dr. Brian Hughes Chair of Advisory Committee	Dr. J. Keith Townsend
Dr. Gianluca Lazzi	

ABSTRACT

JACOB, NISHANT. Capacity Aspects of Bit Interleaved Coded Modulation.
(Under the direction of Dr Brian .L. Hughes.)

Space-time block codes are a recently discovered, attractive modulation scheme for multiple antenna wireless channels. They are capable of providing full diversity over fading channels, at the same time requiring only low computational costs. Over the last few years, several distinct approaches to designing space-time codes have been proposed in the literature, including the layered architecture of [9, 22], trellis codes in [3, 17, 23], turbo codes [13, 16] and the orthogonal designs in [1, 19]. Most work on space-time coding has focused on the problem of designing codes to perform well under quasi-static fading conditions. Some examples of codes designed for fast-fading are [17, 6]. In practice, it is not unusual for a cellular handset to experience conditions which range from fast fading to nearly static within seconds (e.g., a vehicle suddenly braking). It is therefore of interest to design codes that are robust in the sense that they perform well under a wide variety of channel fading conditions. A robust coding architecture called *Bit-Interleaved Space-Time Coded Modulation* is proposed in [10] for channels with multiple transmit and receive antennas. It is designed to perform well under a wide variety of channel fading conditions and which (when differentially encoded) does not require accurate channel estimates at the receiver. The architecture combines serial concatenation of short, full-diversity space-time block codes (inner code) with bit-interleaved coded modulation.

This thesis examines the capacity aspects of this architecture namely, the ergodic capacity, the outage capacity and the ergodic and outage cut-off rates in Rayleigh flat-fading channels. It is shown that, if the inner block codes are chosen properly, not only is the capacity close to the information-theoretic limits, but also a better tradeoff between performance and coding complexity can be obtained.

Biography

Nishant Jacob was born in Gujarat, India on Oct. 11th, 1977. He received his B.S degree in Electrical Engineering from Bharathiyar University in Tamil Nadu, India. He is currently working towards the completion of his Master of Science Degree in Electrical and Computer Engineering at North Carolina State University.

Acknowledgements

I would like to extend my thanks, first and foremost to my committee chairman, Dr. Brian Hughes for his patient guidance and support all through my research.My thanks also go out to Dr. Keith Townsend and Dr. Gianluca Lazzi for being on my thesis committee. I owe the deepest gratitude to Zhihong Hong, who was instrumental in this research and its successful completion. Lastly I would like to thank Carmela Cozzo who readily gave me the technical and moral support, I so often needed.

Contents

List of Figures

Chapter 1

Introduction

Fading makes reliable high speed communication over multipath environments quite a challenge. Information-theoretic studies of the fading channel have been reported several times in literature, as is well summarized in [5]. Antenna diversity is an effective and practical means of mitigating the effect of the multipath fading. It is well known, that the trade off between spectral efficiency and power can be significantly improved through antenna diversity. Further to get the most of fading multipath channels, it becomes necessary to fully utilize all the implicit and explicit diversity available. In order to increase the capacity of Rayleigh fading wireless channels space diversity can be used. When multiple transmit and receive antennas are used in conjunction with efficient signal processing, significant improvements in capacity are possible. Space time codes are designed for optimum transmission using multiple transmit antennas and provide diversity as well as coding gain. They provide diversity by introducing space and time correlation between the transmitted signals over different antennas. This leads to a dramatic increase in throughput as well as reliability of such systems. Several approaches to space-time coding have been proposed, like the layered architecture of [9, 22], the block codes of orthogonal designs in [1, 19], the trellis codes of [3, 17, 23], the turbo codes of [13, 16]. A detailed study and generalization of the orthogonal code concept was given in [19]. It concentrated on the extracting full diversity gain from the quasi-static fading channel with

minimum encoding and decoding complexities. A dramatic reduction of encoding and decoding complexities is shown to be possible by partitioning the transmit array into small groups and using individual space-time codes to transmit information from each group of antennas. At the receiver, simple linear processing is used to suppress signals transmitted from other groups of antennas by treating them as interference. Performance is shown to be determined by matrices constructed from pairs of distinct code sequences. The minimum rank among these matrices gives the *Diversity Gain*, and this is quantified by the generalized hamming distance of certain sequences. The minimum determinant gives the *Coding Gain* and this is quantified by the generalized product distance of these sequences. The design criteria for these space-time codes are based on the probability of confusing two distinct codewords. They are two criteria to achieve maximum diversity namely, *Rank Criteria* and *Determinant Criteria*. For the ideal fast-fading channel, the criteria become a vector extension of the product distance of scalar fading channels. Space-time codes are designed so as to achieve the maximum possible diversity $n_R \times n_T$ under the constraint of simple decoding. They are not designed to achieve an additional coding gain. Hence they must be concatenated with outer codes that provide the necessary coding gain.

In a fading channel, channel capacity is measured in different ways, depending on how fast the channel is changing. Under *quasi-static* conditions, where we assume that the fading path gains remain constant for the entire duration of a frame, the capacity is measured by the outage capacity. However under *ideal fast-fading* fading conditions, the fading path gains are modeled as i.i.d. random variables, because of ideal interleaving and motion-induced Doppler spread in the multipath. In this case the sensible measure of capacity is the ergodic capacity. Like channel capacity, the channel cutoff rate is another useful measure, which gives the tradeoff between performance and complexity when considering channel coding with different modulation schemes. In [21], ergodic and outage cutoff rates are analyzed for various transmit diversity schemes like maximum ratio combining (MRC) transmit diversity, switched-time diversity (STD) and orthogonal transmit diversity (OTD). Most of the work so far, focus on the design of codes that are optimum in quasi-static fading conditions.

Some examples of space-time codes designed for fast-fading can be found in [17, 8]. Robust space-time codes that can perform well in both quasi-static and fast-fading conditions have been reported in [15, 6]. Performance of any coded system in a fading channel depends on its diversity order. It is well known [24] and later [7] that, for single antenna systems, *Bit Interleaved Coded Modulation* (BICM) provides good performance over both the Rayleigh fading and the additive white Gaussian noise channel. BICM improves reliability over the Rayleigh fading channel as compared to symbol (block) interleaving. In symbol interleaving the order of diversity and hence the fade margin is dependent on the minimum number of distinct symbols between two code words. This can only be improved by increasing the constraint length of the code and by preventing parallel transitions. BICM maximizes the code diversity order to the minimum hamming distance of a code, and increases the fade margin, while preserving the free Euclidian distance of the underlying constellation to a large extent. Most of the coding gain is achieved by the increased diversity through bit interleaving. This gain is larger than the loss associated with the random modulation and suboptimal decoding. Random modulation is caused due to the destruction of the direct mapping between code symbols and constellation points. Suboptimal decoding is a practical necessity due to the complexity of joint demodulation and convolutional decoding, as is required by optimal ML decoding. BICM is achieved by bit interleaving each bit output of a convolutional encoder. Binary convolution codes have a larger free distance than non-binary codes, and BICM offers a much better trade-off between code diversity and trellis complexity than plain trellis-coded modulation. With Gray labeling, BICM provides a free Euclidian distance $d_E^2 = d_H d_0^2$, where d_H is the free Hamming distance of the code and d_0 is the smallest Euclidian distance of the constellation. Based on the robustness and relevance of the BICM approach on fading single antenna channels, BICM for multiple-antenna systems seems an attractive concept. Further, since BICM isolates the design of the encoder from that of the modulator, the design of such space-time codes is simpler than those for regular coded modulation(CM) systems.

In [6], a space-time code is proposed which combines BICM with a full constella-

tion(FC), made up of all the code matrices over some scalar constellation (denoted here, as BI-FC for convenience). The output of an encoder is cycled between each of the t transmitters. Each transmitter has an ideal bit interleaver, modulator and transmit antenna. Each receiver computes the necessary bit metrics, deinterleaves them to the original order and then uses them for the decoding stage. Information-theoretic analysis of BI-FC for different number of transmit and receive antennas in terms of ergodic capacity and outage capacity is also done in [6]. BI-FC is compared to space-time coding using standard coded modulation(CM) and it is shown that with two transmit antennas, BI-FC incurs a capacity loss to the CM-FC in both the ergodic and outage sense. More recently, a robust coding architecture called *Bit-interleaved Space-Time Coded Modulation* (BI-STC) is proposed in [10] for channels with multiple transmit and receive antennas. This is based on the serial concatenation of short, full-diversity, space-time block codes that serve as inner codes, with BICM. Performance analysis and computer simulations in [10] show these space-time codes can perform very well in fast-fading conditions. In quasi-static fading conditions, it performs comparably to the best known space-time codes designed for quasi-static fading conditions.

The important capacity aspects of this new architecture, namely, the ergodic capacity, the outage capacity, and the ergodic and outage cutoff rates in the Rayleigh flat-fading channel, now need to be invesitgated.

Chapter 2

Background

2.1 Channel Model

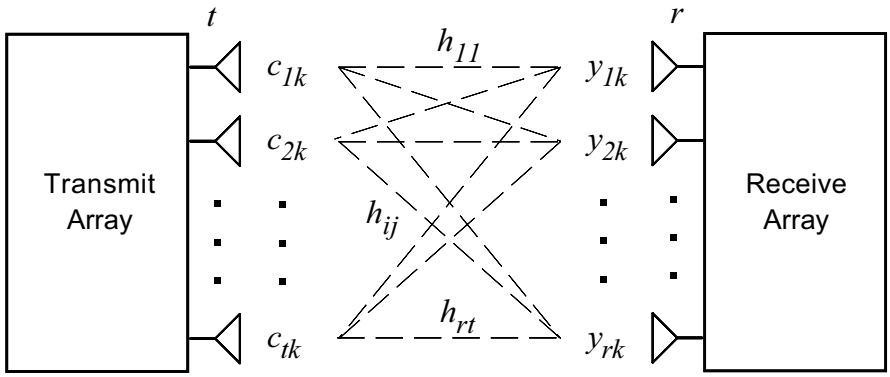

Figure 2.1: Channel model

We consider the channel with t transmit antennas and r receive antennas illustrated in Fig. 2.1. Under Rayleigh flat-fading conditions, this channel can be described by

$$Y_k = \sqrt{\rho} H_k X_k + N_k \, , \; k = 1, \ldots, K \tag{2.1}$$

where Y_k is the $r \times n$ matrix of received samples, H_k is the $r \times t$ matrix of fading path gains during block k, X_k is the $t \times n$ transmitted matrix, and N_k is an $r \times n$

matrix of noise. We assume that the elements of N_k and H_k are independent and identically distributed (i.i.d.) complex Gaussian random variables with probability density function (pdf)

$$p(h) = 1/\pi \exp(-|h|^2) \ . \tag{2.2}$$

We further assume that the transmitted signals are normalized to unit power, $\mathrm{Tr}[X_k X_k^\dagger] \leq nt$, so that ρ represents the signal-to-noise ratio (SNR) per transmission path. We consider two idealized models of the behavior of H_k: under *quasi-static* conditions, H_k is fixed for the duration of a frame, so $H_k = H$, $k = 1, \ldots, K$; under *fast-fading* conditions, we assume that ideal interleaving and time-selective fading renders $H_1, \ldots, H_K$ an i.i.d. sequence of random matrices.

2.2 Review of Space-Time codes

To get the most of fading multipath channels, it becomes necessary to fully utilize all the implicit and explicit diversity available. When multiple transmit and receive antennas are used in conjunction with efficient signal processing, significant improvements in capacity are possible. Space time codes are designed for optimum transmission using multiple transmit antennas. Many methods of transmit and receiver processing can be used to achieve good diversity at reasonable complexity. Space time codes introduce space and time correlation between the transmitted signals over different antennas, and thereby provide diversity at the receiver.

Orthogonal space-time block codes

Orthogonal space-time codes were introduced in [1]. A detailed study and generalization of the concept was given in [19], and concentrated on the extracting full diversity gain from the channel with minimum encoding and decoding complexities. The Alamouti scheme uses $t = 2$ transmit diversity with a simple code matrix given

by

$$X = \begin{bmatrix} x1 & -x2^* \\ x2 & x1^* \end{bmatrix}$$

where $x1, x2 \in \mathcal{C}$ are the transmitted symbols. This can be extended to $r = m$ receive antennas trivially to provide an overall diversity of $2m$. Following [10], for any binary sequence say, $\{b_i\}$, let b_s^p denote the subsequence $(b_{s+1}, \ldots, b_{s+p})$, where the subscript is generally omitted for $s = 0$. For $|\mathcal{C}| = 2^p$, let $x(b^p)$ denote a one-to-one mapping of binary labels to points in $\mathcal{C}$. Then the code matrix becomes,

$$X(b^{2p}) \;=\; \begin{bmatrix} x(b^p) & -x(b_p^p)^* \\ x(b_p^p) & x(b^p)^* \end{bmatrix} . \tag{2.3}$$

Further when $x(b^p)$ is a Gray labeling for $\mathcal{C}, X(b^{2p})$ becomes Gray labeling for $\mathcal{X}$.

In [18] space-time trellis codes are designed for standard modulation schemes like 4-PSK,8-PSK and 16-QAM that perform extremely well from a capacity perspective and achieve within 2-3 dB of the outage limits. These trellis codes combine spatial and temporal diversity and are hence called $r - space - time$ trellis codes, where r represents the guaranteed diversity advantage provided by the code. The objective of the code design is to achieve maximum possible rate at a diversity advantage of rm for m transmit antennas. Fundamental tradeoffs exist between transmission rate, diversity advantage, constellation size and trellis decoding complexity. The design criteria for these codes are based on the probability of confusing two distinct code-words. There are two criteria to achieve maximum diversity namely, *Rank Criteria* and *Determinant Criteria*. Several 4-PSK codes of rate 2 bps/Hz and 8-PSK codes at 3 bps/Hz for transmission over two antennas are provided. These trellis codes have 4, 8, 16 or 32 states. When a single receive antenna is used, these codes provide a diversity advantage of 2. Performance is determined by matrices constructed from pairs of distinct code sequences. The minimum rank among these matrices gives the *Diversity Gain*, that is quantified by the generalized Hamming distance of certain sequences. The minimum determinant gives the *Coding Gain* and this is quantified by the generalized product distance of these sequences.

Space-time group codes

A general approach to differential modulation is established in [11]. This is based on space-time block codes that have a group structure, and extends to any number of antennas and any signal constellation. Since the class of group codes is inherently very rich, it is possible to construct codes for any number of antennas and constellations. Further group codes may also be differentially encoded, in a manner similar to PSK. These codes may be used as space-time block codes when CSI is available at the receiver and as differentially encoded codes when CSI is unavailable. The performance degrades by only around 3 dB in the latter case over the former. The group structure also leads to a more simpler modulation and demodulation processes. For a system with t transmit antennas with a constellation $\mathcal{C}$, a *group code* of any length n is given by

$$\mathcal{D}G \equiv \{DG : G \in \mathcal{G}\}$$

where $n \geq t$, $\mathcal{G}$ is any group of $n \times n$ unitary matrices , $\mathcal{D}$ is a $t \times n$ matrix such that $DG \in \mathcal{C}^{t\times\ n}$ for all $G \in \mathcal{G}$ and the code rate is given by $R = (1/n)log_2 \mid \mathcal{G} \mid$ bps/Hz.

For $t = n = 1$ M-ary PSK is a group code with group $\mathcal{G} = \{1, \omega_M, \omega_M^2, ..., \omega_M^{M-1}\}$, with $\omega_M \equiv \exp(2\pi j/M)$ and $D = 1$.

For $t = n = 2$, a group code is the pair,

$$\mathcal{G} = \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \pm \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} \pm \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \pm \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \right\}$$

and $D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ over the QPSK constellation $\mathcal{C} = \{1, j, -1, -j\}$.

For $M = 2^p > 1$, all optimal unitary group codes with $\mid \mathcal{G} \mid = M$ and diversity advantage $v_p = n = t = 2$ are either *cyclic* or *dicyclic*. The (M, k) *cyclic group code* is given by,

$$\mathcal{G} = \left\langle \begin{bmatrix} \omega_M & 0 \\ 0 & \omega_M^k \end{bmatrix} \right\rangle \ , \quad D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

where k is odd and $0 < k < M$. It takes values in the M-PSK constellation with $\mid \mathcal{G} \mid = M$ code matrices, and a diversity advantage $v_p = 2$.

For $M \geq 8$, the *dicyclic group code* is given by

$$\mathcal{G} = \left\langle \begin{bmatrix} \omega_{M/2} & 0 \\ 0 & \omega_{M/2}^* \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle, \; D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

This has M code matrices and takes on values in the $M/2$ -PSK constellation. It has a diversity advantage of $v_p = 2$.When $R = 0.5$ bps/Hz, the only optimal unitary group code is the (2,1) cyclic group code. For $R = 1$, the (4,1) and (4,3) cyclic codes are both optimal. The (4,3) is basically the Alamouti code.

For $R = 1.5$, the optimal dicyclic code is given by,

$$\mathcal{G} = \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \pm \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} \pm \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \pm \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \right\}, \; D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

over the QPSK constellation $\mathcal{C} = \{1, j, -1, -j\}$. This is called the *quaternion code* ,since the group $\mathcal{G}$ is the quaternion group. It has the desirable property of maintaining the minimum product distance of the optimal $R = 1$ code. In addition, with perfect CSI it can achieve 75% the rate of the Alamouti QPSK, with a 3 dB higher coding gain.In the case, where CSI is unavailable at the receiver, the quaternion code may be differentially encoded and detected without any constellation expansion. When $R = 2$, both the best cyclic and dicyclic group codes have the same minimum distance, but dicyclic codes have a larger second-nearest-neighbor spacing. For $R = 2.5$ the $(32, k)$ cyclic group codes are optimal for $k = 7, 9, 23, 25$. Though these codes have 2.3 dB loss in coding advantage over the codes in [8, 19], they are simpler and can be differentially encoded, while preserving the constant modulus property of the constellation.

As in [10], for both the cyclic and dicyclic group codes, a binary labeling $x(b^p)$ of $\mathcal{C}_p$ induces a natural labeling on $\mathcal{X}$. For the cyclic code, this is

$$X(b^p) \; = \; \begin{bmatrix} x(b^p) & 0 \\ 0 & x(b^p)^k \end{bmatrix} . \tag{2.4}$$

while for the dicyclic code is given by

$$X(b^{p+1}) = \begin{bmatrix} x(b^p) & 0 \\ 0 & x^*(b^p) \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{b_{p+1}} . \tag{2.5}$$

Further, when $x(b^p$ is a Gray Labeling for $\mathcal{C}$, $X(b^{2p})$ is a Gray labeling for the Group code (except dicyclic R = 1.5).

2.3 Bit-interleaved Space-time coded modulation

In a single antenna system, BICM is achieved by bit interleaving each bit output of a convolutional encoder. To provide maximum flexibility and symmetry, all the encoder outputs are fed to a single-bit interleaver as in [7]. The interleaved sequence is then split into several subsequences of m bits each. These subsequences are mapped by an N - dimensional memoryless modulator over a signal set $\mathcal{X} \in \mathcal{C}^N$ of size $\mid \mathcal{X} \mid = M = 2^m$, through the binary labeling map $\mu : \{0,1\}^m \rightarrow \chi$. Using common signal sets with Gray labeling provides capacities very close to those of the signal sets themselves. The asymptotic optimality of Gray labeling with BICM is shown in [7]. Gray labeling is best for suboptimal decoding and thus plays a key role in BICM. A Gray coding scheme is defined by the mapping $\mu : \{0,1\}^m \rightarrow \chi$, such that for each $i = 1, ..., m$ and $b \in \{0,1\}$ each $x \in \chi_b^i$ has at most one $z \in \chi_{\overline{b}}^i$ at the minimum Euclidian distance d_{min}. When one assumes ideal interleaving (infinite depth and totally random) and the channel state is iid, the BICM communication channel can be modeled as a set of m parallel binary-input channels connected to the encoder. Each channel represents a bit position in the label of the signal set χ. The encoder randomly and independently uses one of the m binary channels for each of its binary outputs. The receiver which knows the sequence of channels used for transmission, deinterleaves the sequence and computes the relevant branch metrics. Results show that BICM with gray labeling for many common signal sets achieve capacities very close to those of TCM. Since the robustness and relevance of the BICM approach on fading single antenna is well established, BICM for multiple-antenna systems now

needs to be explored.

Full Constellation BICM (FC-BICM)

The application of BICM (Gray Labeling) concept to multi-antenna systems on fading channels, was first described in [6], from an information theoretic view point. In this scheme, the output of an encoder is cycled between each of the t transmitters. Every transmitter has an ideal bit interleaver, modulator ,transmit antenna and transmits a power of $\mathcal{P}/t$. An antenna actually transmits a code that is the punctured version of the encoder output. Since the transmitted symbol is chosen from the entire constellation without restriction, this scheme is called Full Constellation BICM (FC-BICM). The receivers are assumed to have perfect channel state information (CSI). Each receiver computes the relevant bit metrics, deinterleaves them to restore the original order and then uses them for the decoding stage. Biglieri's results indicate that bit interleaved space-time codes achieve ergodic and outage capacities remarkably close to, those of general coded modulation(CM) over multiple transmit and receive antennas.

Serially Concatenated BICM (SC-BICM)

It has been well established that concatenated codes also called multilevel codes, have large coding gains and high efficiency. They obtain low bit error probabilities at rates well beyond the channel cut off rate. Further they may be decoded through the use of a remarkably simple iterative decoding scheme. Serial concatenated codes (SCC) have been to shown to achieve near error-free performance at rates close to the capacity. A serial concatenated code (SCC) and interleaver consists of an *outer* encoder , a bit *interleaver* and an *inner* encoder connected in *series*. The input to the inner encoder is the *permuted* version of the outer code. The extension of the serial concatenation concept to space-time block codes, called *Serially Concatenated BICM*(SC-BICM) is proposed in [10]. This scheme involves the use a *outer* convolutional code and an *inner* space-time block code, cascaded via

a bit interleaver. A block of m input bits are first encoded using a rate m/q outer convolutional encoder. The encoder output, a block of q bits is then bit interleaved and then encoded through the inner the space-time encoder. The space-time encoder maps the block on to space-time block code of length n, say $\mathcal{X}$ of size 2^q. This gives an overall spectral efficiency of m/n bps/Hz. Assuming perfect CSI at the receiver, and using Gray labeling each binary q-tuple output, of the interleaves at time k, b_k^q is mapped on to a space-time block code $\mathcal{X} = \mu(b_k^q), X_k \in \chi$, which are then transmitted. Thus a sequence of information bits $\{i_1^m, i_2^m ... i_K^m\}$ are mapped on to the sequence of block codes $X = \{X_1, X_2, ..., X_K\}$. The space-time demodulator at the receiver calculates the maximum likelihood (ML) metric for each bit. The bit metric is given by

$$\lambda(b_{k+i=b}) = log \sum_{Z \in \chi_b^i} P(Y_k \mid Z, H_k) \approx min_{\chi_b^i} Tr\{(Y_k \sqrt{p} H_k Z)(Y_k - \sqrt{p} H_k Z)\dagger\}$$

where $i = 1, 2, ..., q$, $b = 0, 1$ and $\mathcal{X}_b^i = \{\mu(b_k^q) \mid b_{k+1} = b\}$ is the signal subset. After $metric\ de-interleaving$, the bit metrics are given to a ML decoder, using the decision rule,

$$\hat{X} = arg\ min_{X \in \chi_b^i} \sum_{k=1}^{K} \sum_{i=1}^{q} \lambda(b_{k+1})$$

[10] also gives the design criteria for SC-BICM code design and the performance results. The code performance is significantly dependent on d_{free} and this needs to be maximized. The SC-BICM approach is suboptimal since it optimizes the diversity gain rather than the product distance. SC-BICM is guaranteed to achieve full diversity $v = tr$ in Quasi-static and $v = trd_{free}$ under fast fading conditions, where d_{free} is the free Hamming distance of the outer code.Space-Time Trellis coding (STTC) requires codes with larger constraint length and thus complexity to achieve the same diversity order. Further the use of the high diversity inner codes makes the inner coding stage significantly more reliable and improves the overall code performance for the same trellis complexity. On the other hand TCM concatenated with the Alamouti code (SC-TCM) as in [2] optimizes the product distance. Under Quasi-static fading conditions SC-TCM would thus perform better, while SC-BICM will perform better in fast fading. Performance analysis and computer simulations in [10] show

that concatenatenation with space-time block codes of full diversity, has several advantages: First, under quasi-static fading conditions, the inner code guarantees full diversity. Second, under fast-fading conditions, the same inner code can achieve a much higher code diversity for given hamming distance of the underlying convolutional code. Third, as shown in [10], that Gray labeling often exists for most of the space-time block codes with full diversity. The result indicate that SC-BICM outperforms the codes suggested in [17, 8] under both fading conditions. SC-BICM performs comparably to Smart Robust (SR) codes (the best known code for fast fading) in quasi-static and remarkably better in the fast fading case. Further SC-BICM approaches the performance of turbo code, introduced in [13, 16], even without iterative decoding. Hence the robustness and relevance of the SC-BICM approach under various channel conditions is apparent.

Chapter 3

Capacity Aspects

3.1 Capacity Calculation of BI-STC

3.1.1 Ergodic Capacity

Telatar[20] has shown that the ergodic capacity of channel (2.1) is given by,

$$C = \frac{1}{n} I(X, Y|H) = \mathcal{E}\left[\log_2 \left| I + \rho H H^{\dagger} \right|\right] \tag{3.1}$$

where $I(X, Y|H)$ is the conditional mutual information and $|\cdot|$ is the determinant. Moreover capacity is achieved when the elements of X_k are i.i.d. complex Gaussian random variables. Under the fast-fading conditions, Hong and Hughes show in [10], that BI-STC often achieves a better tradeoff between code diversity and trellis complexity than conventional symbol-interleaved space-time codes. In [10], the sequence of code matrices $X_1, \ldots, X_K$ is constrained to take values in a full-diversity space-time block code $\mathcal{X}$ and decoding is performed with suboptimal metrics, both of which entail a loss of capacity relative to [20], in which $\mathcal{X}$ consists of all matrices over a scalar constellation $\mathcal{C}$.

We now determine the ergodic capacity of BI-STC. To this end, let B_i denote the i-th bit of the binary label of X_k and let $\mathcal{X}_b^i$ be the set of all codewords in $\mathcal{X}$ that

have $b \in \{0,1\}$ in the i-th position of their binary label. Following [7], we can write the ergodic capacity of BI-STC as

$$C = \frac{1}{n}\sum_{i=1}^{q} I(B_i, Y|H) = \frac{1}{n}\left(q - \sum_{i=1}^{q} \mathcal{E}_{Y,H,B_i}\left[\log_2 \frac{\sum_{Z\in\mathcal{X}} p(Y|Z,H)}{\sum_{Z\in\mathcal{X}_{B_i}^i} p(Y|Z,H)}\right]\right) \tag{3.2}$$

where

$$p(Y|X,H) = \frac{1}{\pi^{nr}} \exp\left\{-\|Y - \sqrt{\rho}HX\|^2\right\}$$

and $\|A\|^2 = \mathrm{Tr}[AA^\dagger]$ is the Frobenius norm. Here $B_i, \ldots, B_q$ are i.i.d., uniformly-distributed, binary random variables. If space-time block codes are concatenated with standard coded modulation instead of BICM, the ergodic capacity of CM-STC becomes

$$C = \frac{1}{n} I(X, Y|H) = \frac{1}{n}\left(q - \mathcal{E}_{Y,H,X}\left[\log_2 \frac{\sum_{Z\in\mathcal{X}} p(Y|Z,H)}{p(Y|X,H)}\right]\right) \tag{3.3}$$

3.1.2 Outage Capacity

Under the quasi-static fading condition, the channel capacity is a random variable depending on the instantaneous realization of the fading process h. In such a situation, the outage capacity is used to assess the system performance. For a given rate R, the outage probability is defined as

$$P_{out}(R) = P\left[\frac{1}{n}\mathcal{I}(X, Y|h) < R\right] \tag{3.4}$$

Here $\frac{1}{n}\mathcal{I}(X, Y|h)$ is determined by (3.2) or (3.3) for BI-STC or CM-STC respectively.

3.1.3 Channel Cutoff Rate

Like the ergodic and outage capacities which give ultimate performance limits under fast-fading and quasi-static fading conditions, the channel cutoff rate is a well accepted means of assessing the trade-offs between performance and complexity for various coding schemes, taking the modulation into account. Channel cutoff rates for

various transmit diversity schemes, e.g. MRC transmit diversity, STD and OTD are evaluated in [21]. Now we evaluate the channel cutoff rate for concatenated space-time block codes. From the system model, with perfect CSI, the average bit-error probability for fast fading could be obtained by Bhattacharyya bound [7],

$$P_b \leq \frac{1}{q}\sum_{i=1}^{q}\mathcal{E}_{Y,H,B_i}\left[\sqrt{\frac{\sum_{Z\in\mathcal{X}_{\bar{B}_i}} p(Y|Z,H)}{\sum_{Z\in\mathcal{X}_{B_i}} p(Y|Z,H)}}\right] = Q \tag{3.5}$$

where, by definition Q is the Bhattacharyya factor and $\mathcal{X}_{\bar{B}_i}$ denotes the complementary subset of $\mathcal{X}_B^i$. Then under fast-fading conditions, the ergodic cutoff rate of BI-STC is

$$R_0 = \frac{q}{n}\left[1 - \log_2\left(Q+1\right)\right] \tag{3.6}$$

While for CM-STC, the ergodic cutoff rate with CSI could be written as

$$R_0 = \frac{1}{n}\left(q - \log_2 \sum_{Z\in\mathcal{X}} \mathcal{E}_{Y,H,Z}\left[\sqrt{\frac{p(Y|Z,H)}{p(Y|X,H)}}\right]\right) \ . \tag{3.7}$$

Under quasi-static fading conditions, depending on the instantaneous realization of the fading process h, the channel cutoff rate is also a random variable, denoted as $\mathcal{R}_0$. Following (3.4) the outage cutoff rate is

$$P_{out}(R) = P[\mathcal{R}_0 < R] \tag{3.8}$$

Again, $\mathcal{R}_0$ is evaluated by (3.6) or (3.7) for BI-STC or CM-STC, respectively. The difference between BI-STC and BI-FC lies in the inner code constellation, so accordingly we could change $n = 1$ and $\mathcal{X} = \{[x_1\ x_2]^T\}$ to obtain the ergodic and outage cutoff rate for BI-FC and CM-FC.

Chapter 4

Numerical Results and Performance Analysis

In this section, we use Monte Carlo simulation to calculate the channel capacity of BI-STC and standard CM-STC which is proposed in [4, 12, 15] and compare the results with BI-FC and CM-FC of [6].

4.1 BI-STC Vs. BI-FC

Fig. 4.1, plots the ergodic capacity of BI-STC with $t = 2$ transmit antennas and $r = 1$ receive antenna for five space-time block codes under fast-fading conditions, namely the QPSK, 8PSK, and 16QAM Alamouti codes[1] (denoted by BI-ALAM), the rate $R = 1.5$ quaternion code (BI-QUAT) and $R = 2$ dicyclic code (BI-DICYL) given in [11]. Also shown for reference, is the Gaussian-input capacity (solid) from [20], and the ergodic capacity of BI-FC as proposed in [6], which $\mathcal{X}$ consists of all $t \times n$ matrices over the scalar QPSK constellation $\{\pm 1, \pm j\}$. The 16QAM BI-ALAM outperforms the QPSK BI-FC approach over the entire range considered. For $R \leq 1$ bps/ch use, all of the BI-STC schemes yield comparable capacities, within 1-2 dB of the Gaussian-input capacity. For $R = 2 - 3$ bps/ch use, BI-FC suffers a loss in

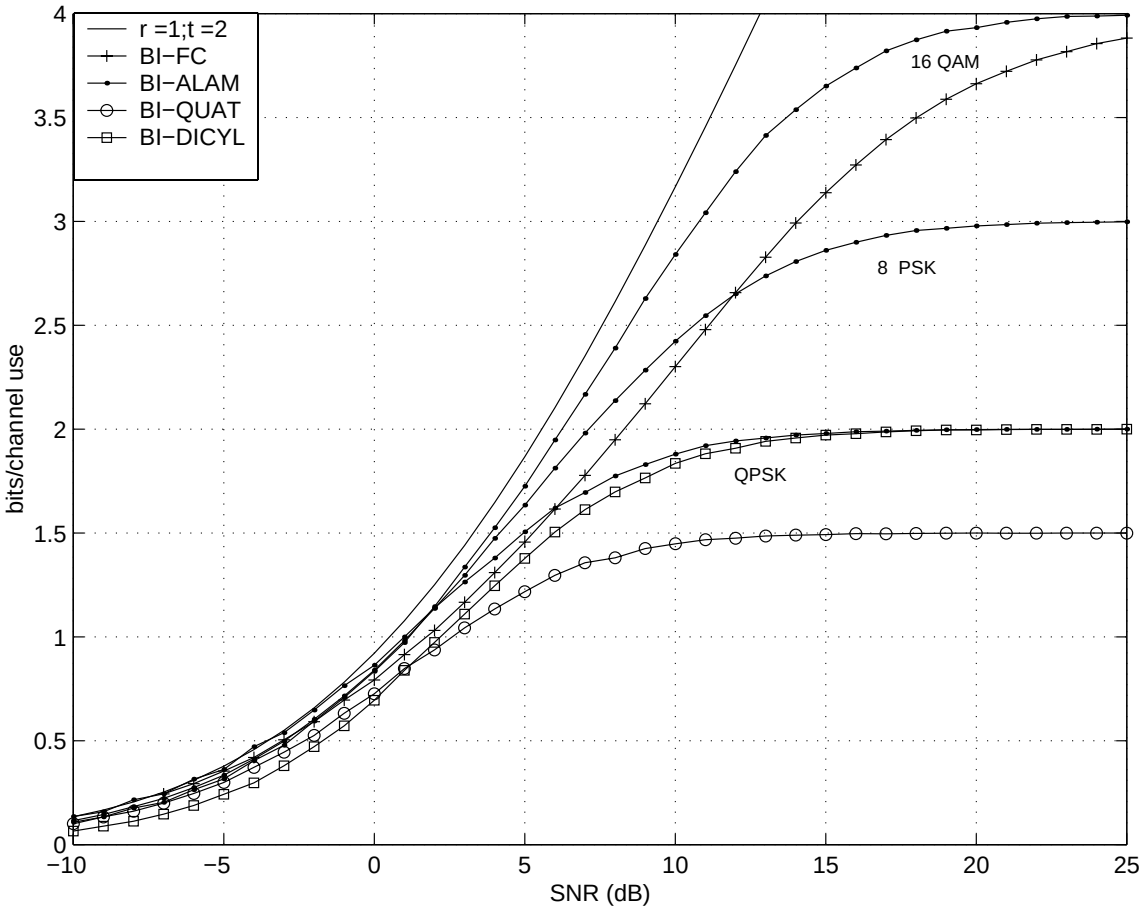

Figure 4.1: Ergodic capacities of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 1$

performance of $2-3$ dB relative to the 16QAM BI-ALAM code, which remains within 1 dB of the Gaussian-input capacity.

Fig. 4.2, plots the ergodic capacities of the same BI-STC codes for $t = 2$ transmit and $r = 2$ receive antennas. The loss in capacity of all BI-STC schemes relative to the Gaussian-input capacity for $r = 2$ receive antennas[20] is greater than the $r = 1$ case, and is most marked at larger rates. However both schemes are within 2 dB of the Gaussian-input capacity for all rates up to 3 bps/ch use. In contrast to the $r = 1$ case, QPSK BI-FC outperforms 16QAM BI-ALAM by up to 0.5 dB over most rates,

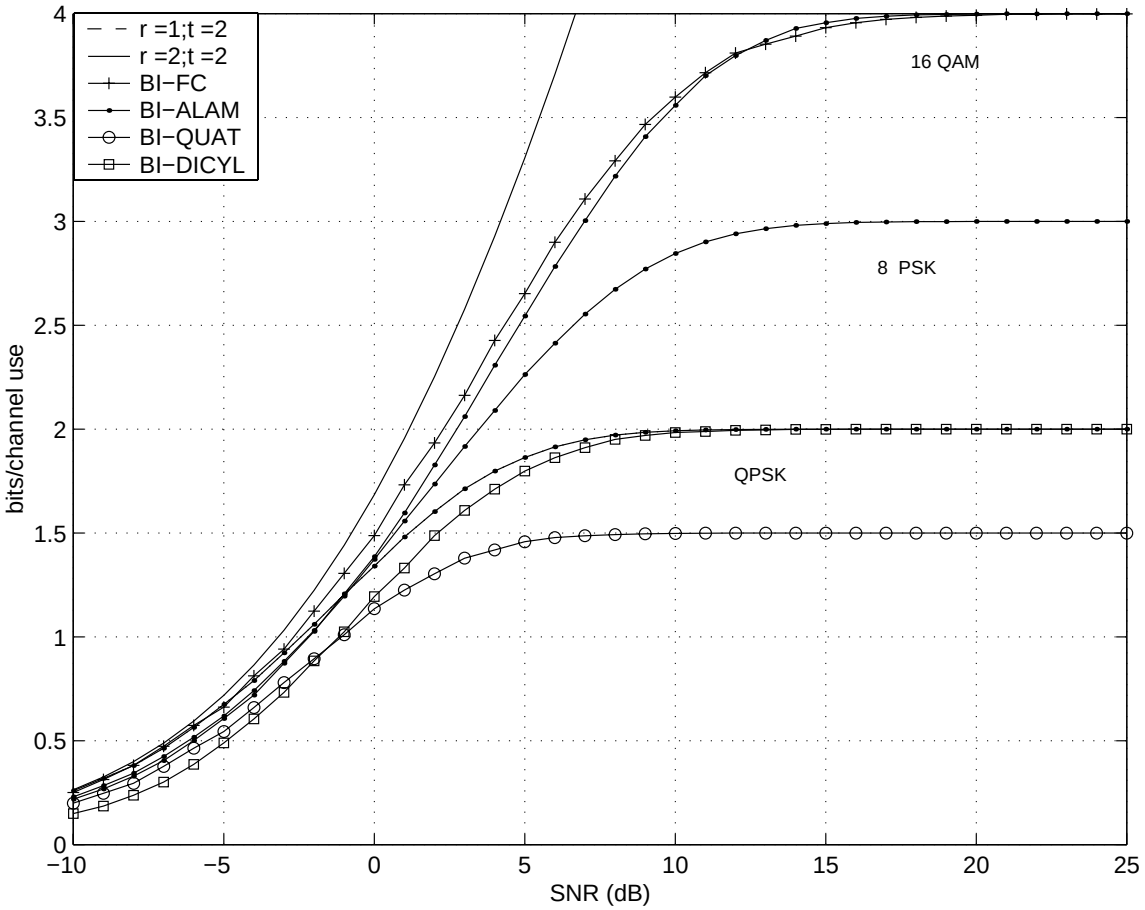

Figure 4.2: Ergodic capacities of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 2$

except very near saturation.

The ergodic cutoff rates of BI-FC with QPSK and 8PSK and BI-ALAM with QPSK, 8PSK and 16QAM with $t = 2, r = 1$ is shown in Fig. 4.3. BI-ALAM has larger cutoff rate than BI-FC over the entire capacity range. This performance gap is even larger than that of ergodic capacity. Specifically, the BI-ALAM 16QAM outperforms BI-FC QPSK 3 dB in terms of the ergodic capacity at 3 bps/Hz. This advantage becomes 4 dB in terms of the ergodic cutoff rate at the same spectral efficiency.

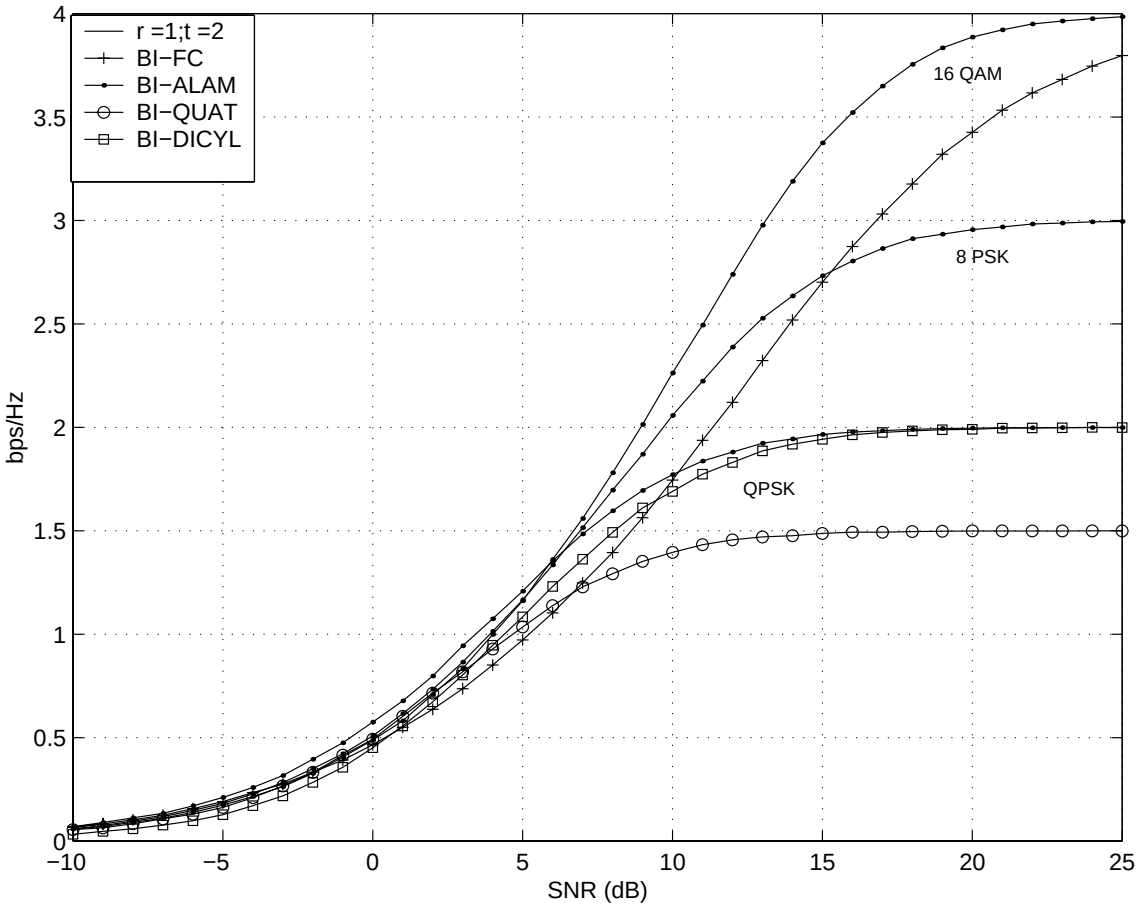

Figure 4.3: Ergodic cut-off rates of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 1$

However for two receive antennas, unlike the one receive antenna case, they perform comparably as illustrated in Fig. 4.4 .

Under quasi-static fading conditions, the 90% outage capacity of BI-STC with various inner codes $\mathcal{X}$ and QPSK BI-FC are plotted in Fig. 4.5 for $t = 2$, $r = 1$. Also shown for reference, is the Gaussian-input capacity. For $r = 1$, we can see that 16QAM BI-ALAM outperform QPSK BI-FC for the entire range of rate considered, especially at high rates. For example, the gap is about 5 dB at $R = 3$ bps/Hz.

Fig. 4.6 illustrates the 90% outage capacity for the $t = 2$, $r = 2$ case . In this

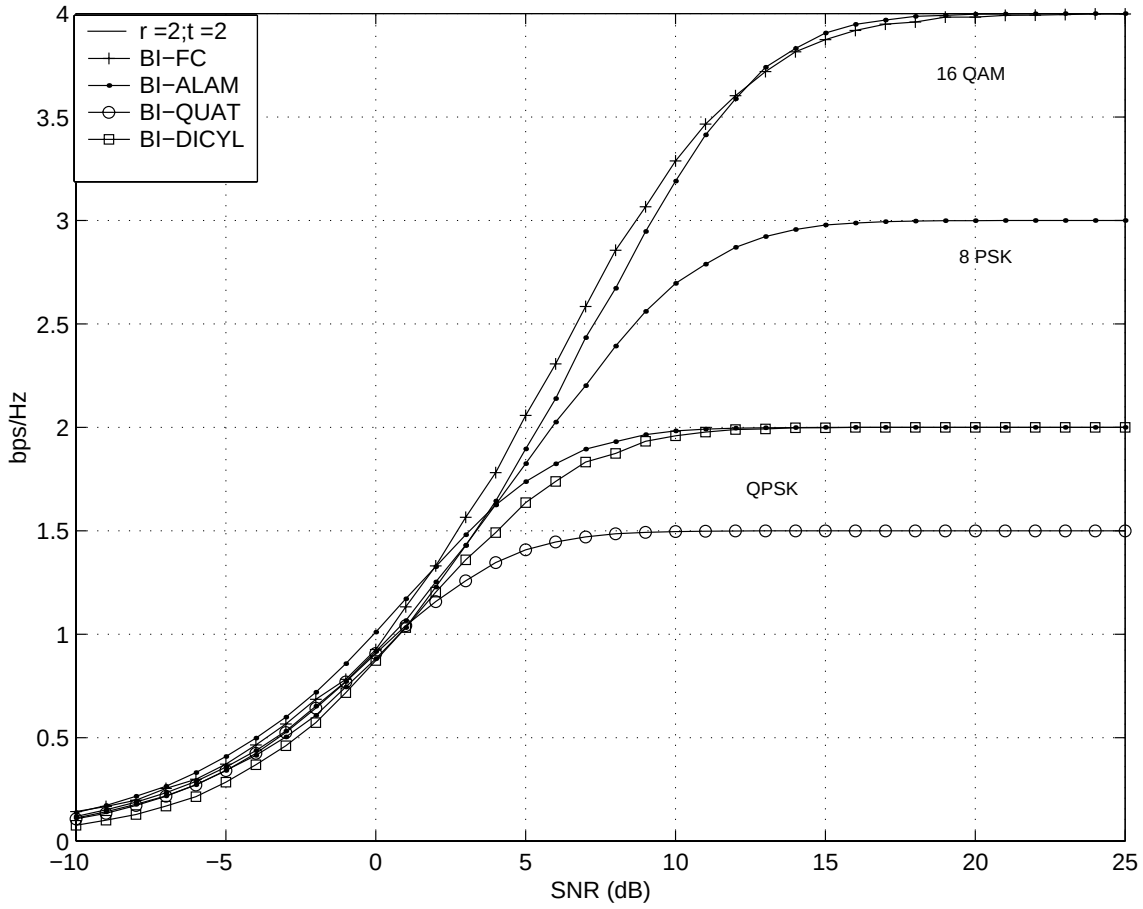

Figure 4.4: Ergodic cut-off rates of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 2$

case, QAM BI-ALAM performs comparable with QPSK BI-FC below $R = 2$ bps/Hz, but outperforms QPSK BI-FC by about 0.5 dB above $R = 2$ bps/Hz.

90% outage cutoff rates of QPSK BI-FC and BI-STC with various $\mathcal{X}$ are plotted in Fig. 4.7 and Fig. 4.8 for $t = 2, r = 1$ and $r = 2$ respectively. A trend similar to the outage capacity is observed.

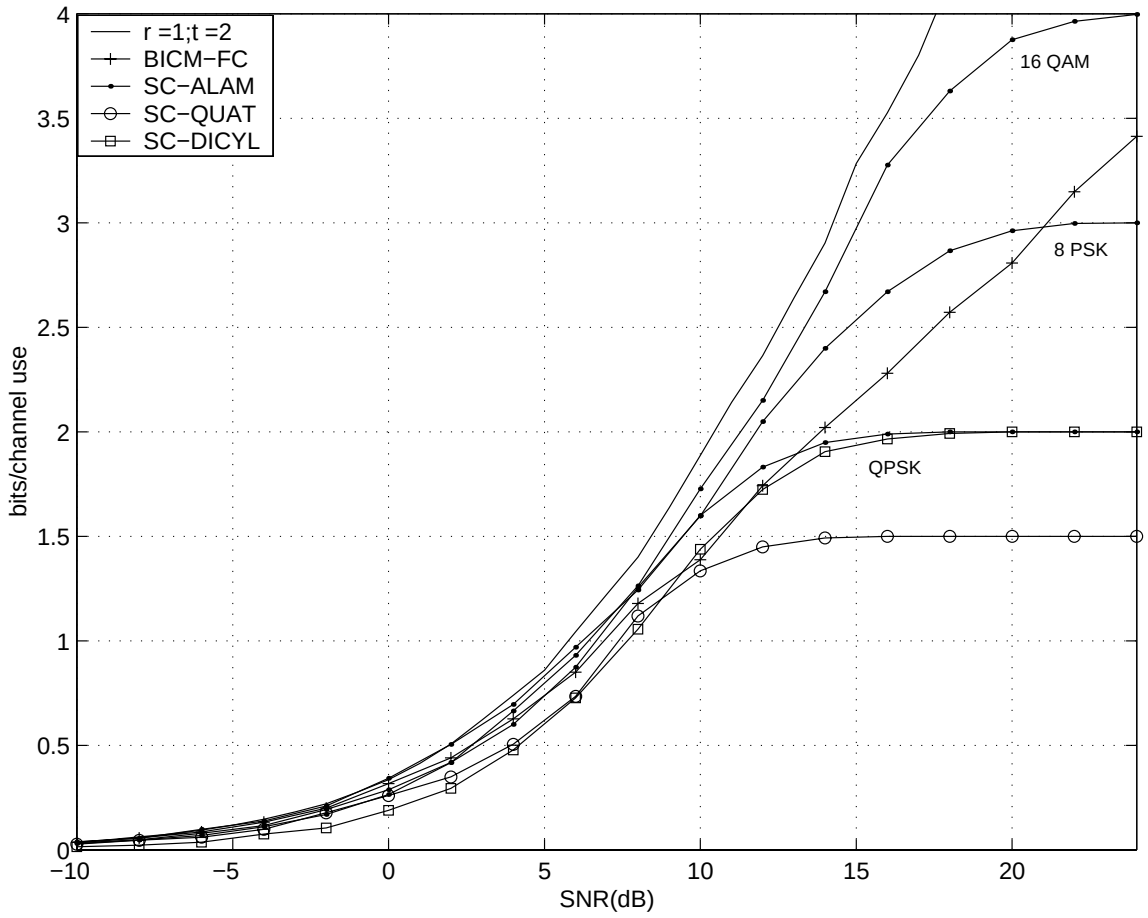

Figure 4.5: 90% Outage capacities of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 1$

4.2 BI-STC Vs. CM-FC

While BI-FC suffers capacity loss to CM-FC, as shown in [6], BI-STC achieves almost the same capacity as CM-FC, for two transmit and one receive antennas. The ergodic capacities of CM-FC with QPSK and BI-ALAM with various $\mathcal{X}$ are plotted in Fig. 4.9. With two receive antennas, BI-ALAM suffers about 1dB loss at high rate, which is shown in Fig. 4.10. This is because, Orthogonal Space-Time Block Codes(OSTBC) are only optimal in the sense of achieving channel capacity with only

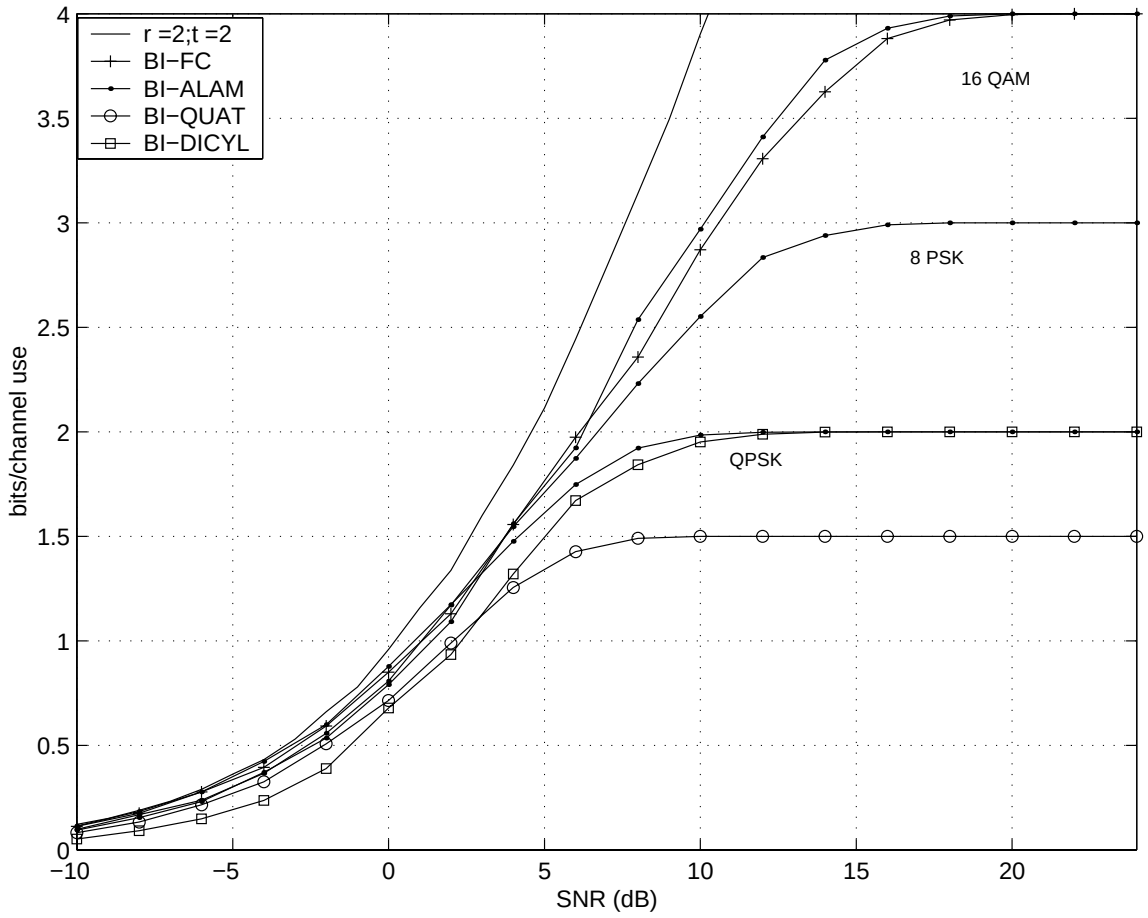

Figure 4.6: 90% Outage capacities of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 2$

one receive antenna [14]. They suffer an inherent capacity loss for more than one receive antenna.

4.3 BI-STC Vs. CM-STC

For a fast-fading scalar channel, the information-theoretic analysis done in [7] shows that, BICM only suffers a fractional capacity loss compared to CM. However BICM does better in terms of ergodic cutoff rate than CM especially at high rates.

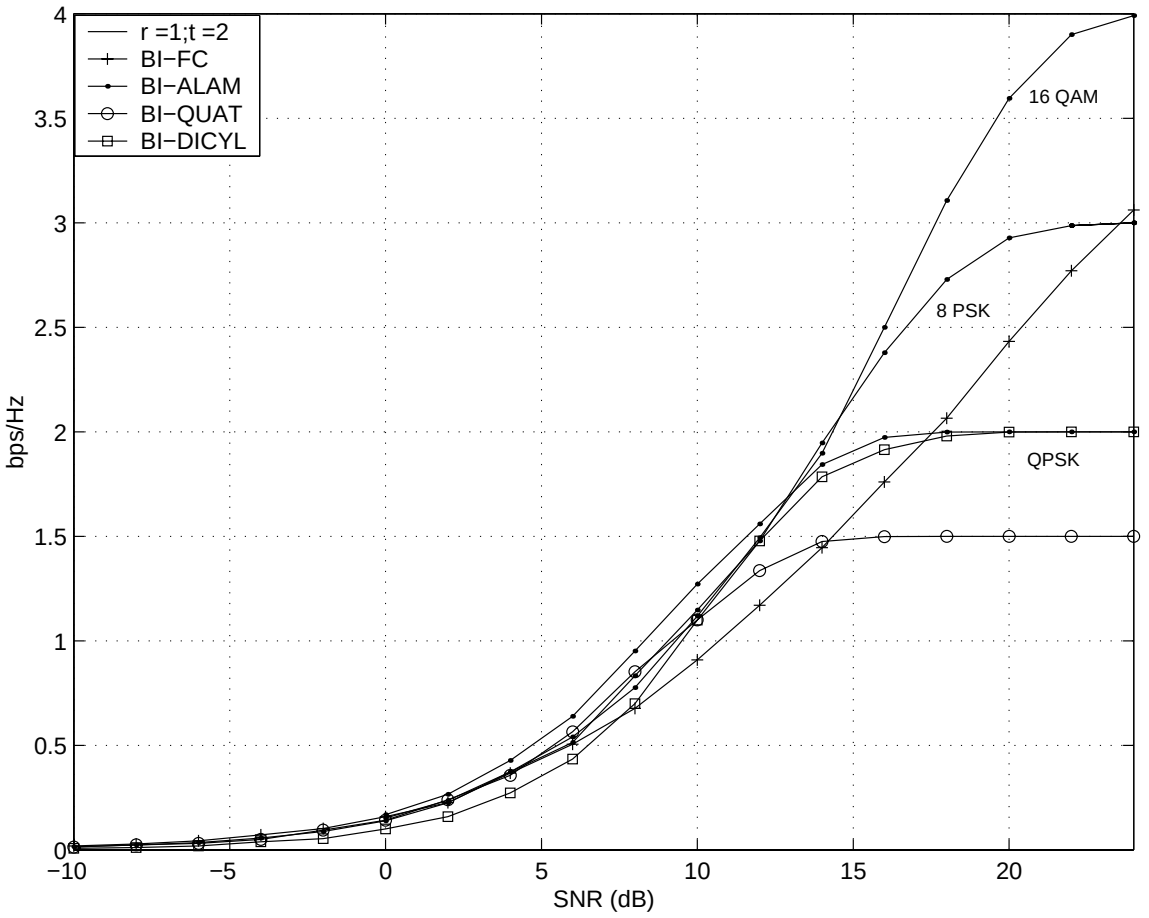

Figure 4.7: 90% Outage cut-off rates of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 2$

This is because, for comparable complexity, BICM achieves much larger diversity (Hamming distance of the underlying convolutional codes) than CM. Therefore BICM generally provides a much better tradeoff between code diversity and complexity. In the quasi-staic additive white Gaussian noise (AWGN) channel, performance is primarily determined by Euclidean distance. Here the situation is reversed, and CM performs better than BICM in terms of cutoff rate. For two transmit antennas, CM-STC with the Alamouti code (denoted as CM-ALAM) and BI-ALAM are compared in terms of their 90% outage and ergodic cutoff rates in Fig. 4.11. We can see that

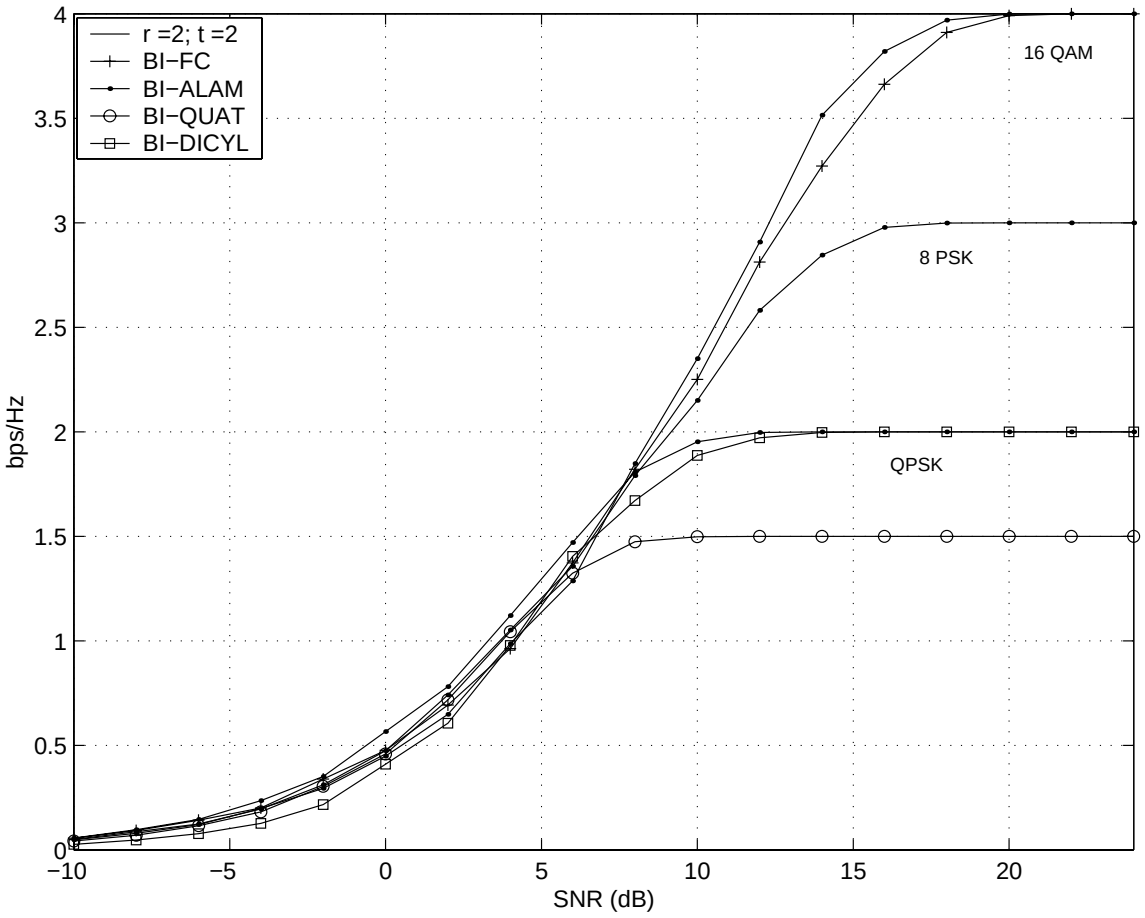

Figure 4.8: 90% Outage cut-off rates of BI-FC and BI-STC with various space-time block codes for $t = 2, r = 2$

BI-STC provides a better ergodic cutoff rate under fast-fading conditions. Under quasi-static fading conditions, BI-STC and CM-STC have the same outage cutoff rate with QPSK constellation, but CM-STC outperforms BI-STC with the 8PSK constellation at rates lower than 2.2 bps/Hz.

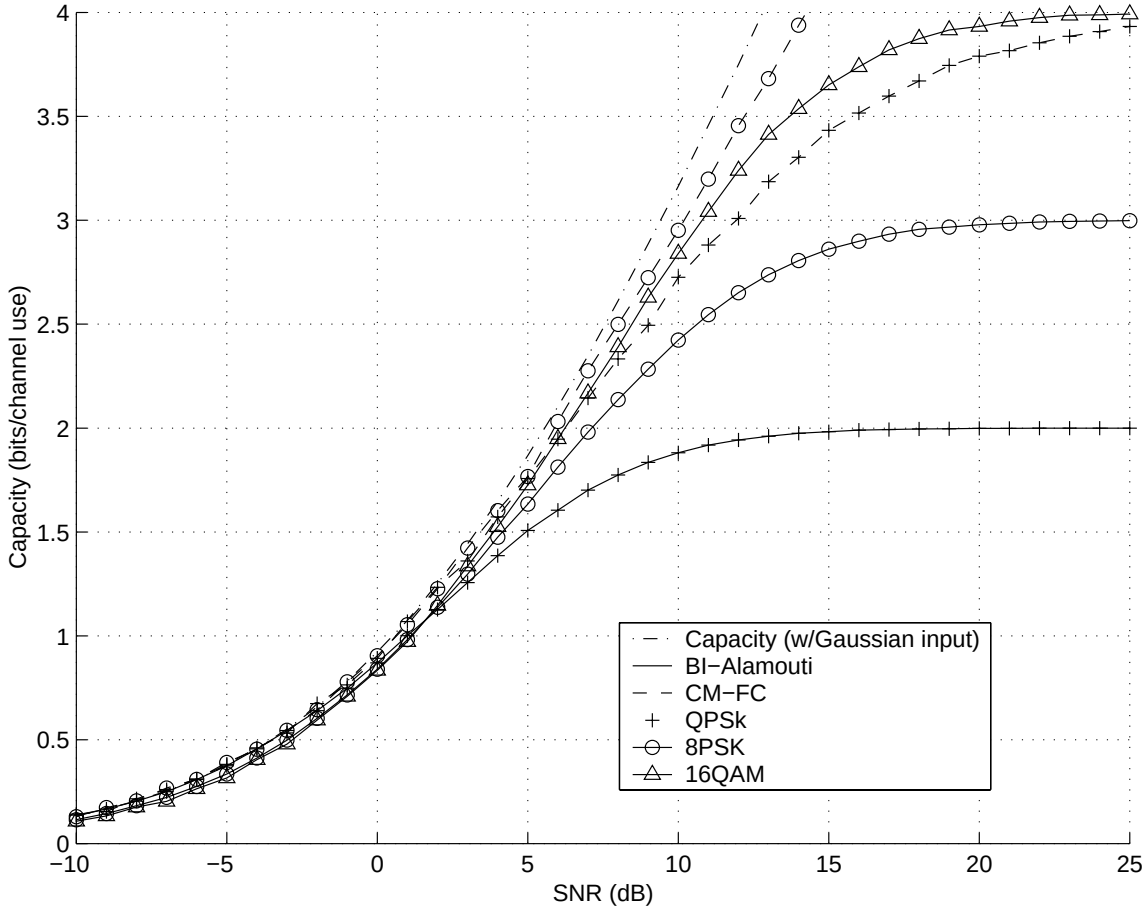

Figure 4.9: Ergodic capacities of CM-FC and BI-ALAM for $t = 2, r = 1$

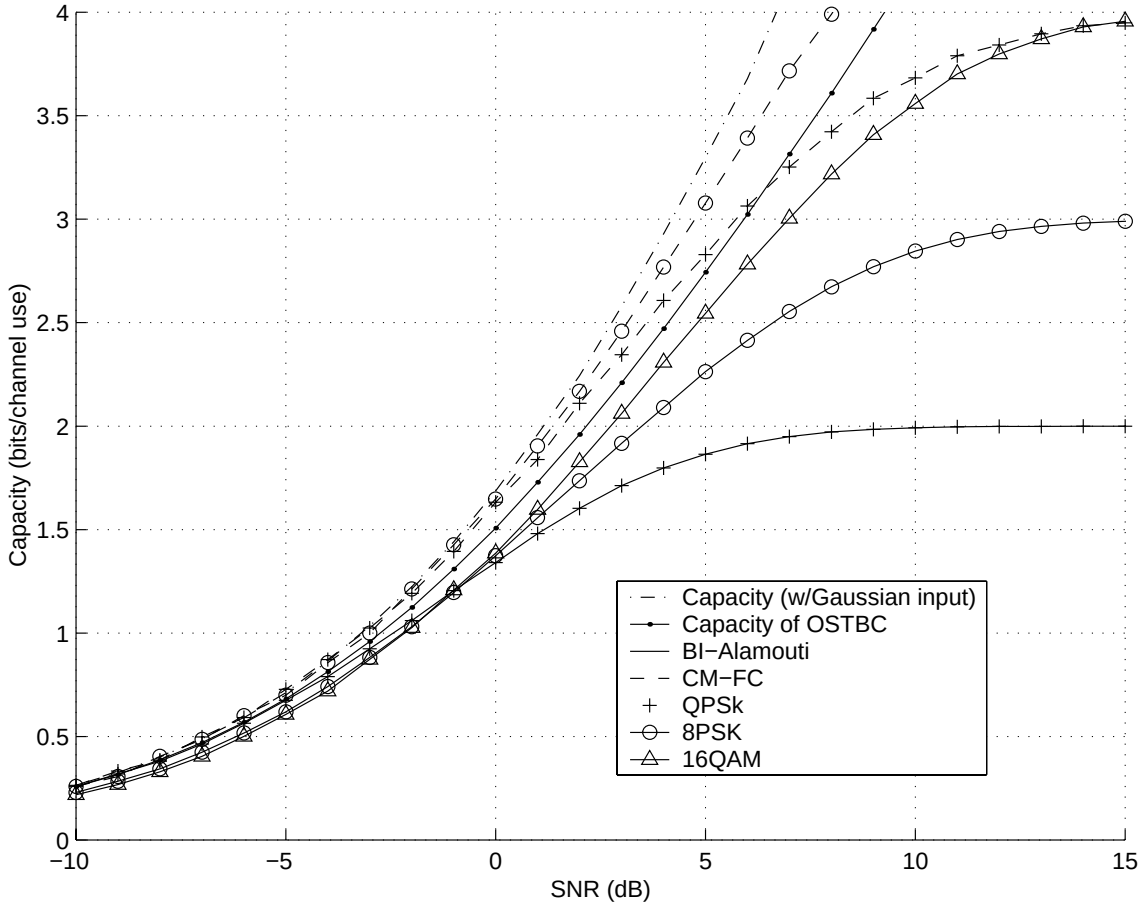

Figure 4.10: Ergodic capacities of CM-FC and BI-ALAM for $t = 2, r = 2$

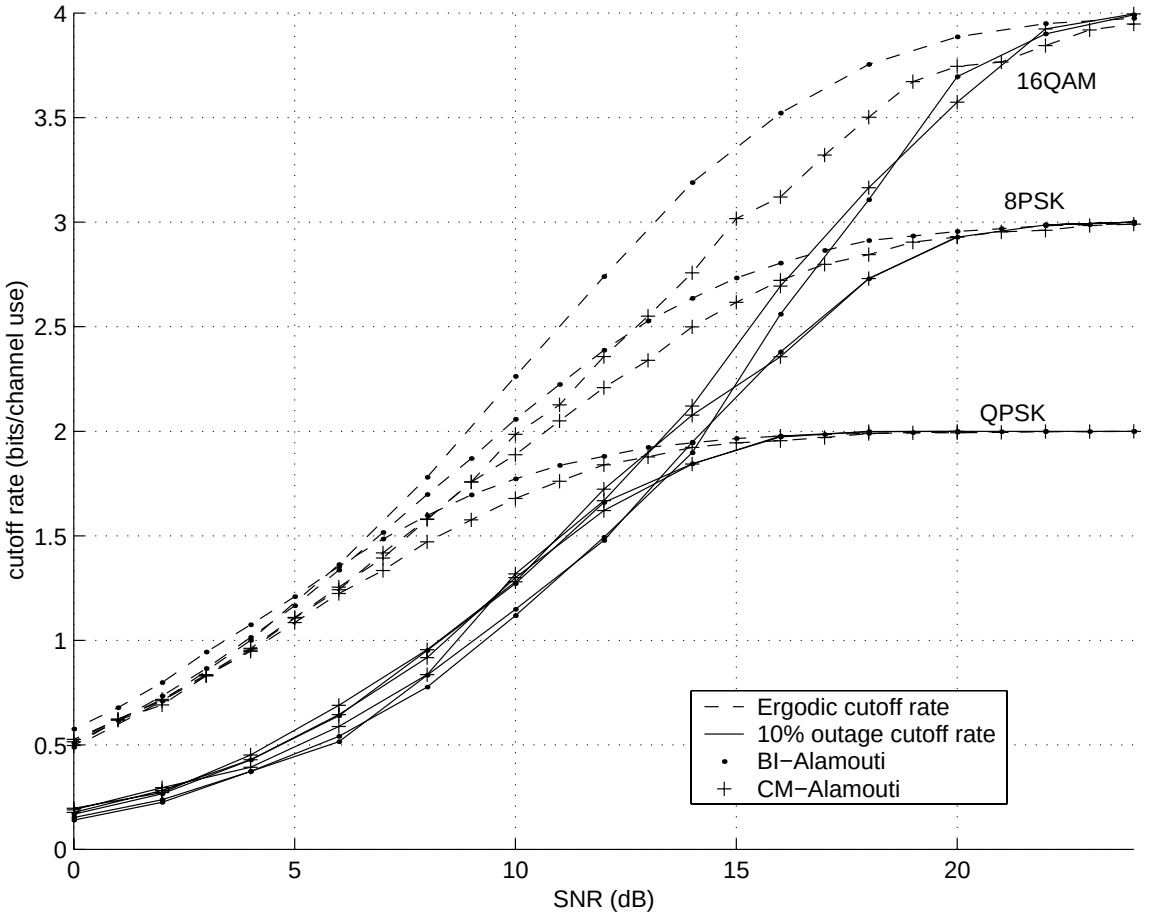

Figure 4.11: Ergodic and 90% Outage cut-off rates of CM-ALAM and BI-ALAM with various space-time block codes for $t = 2, r = 1$

Chapter 5

Conclusions

In conclusion, the limits imposed by information theory on the performance of bit-interleaved space-time coded modulation, that combines short space-time block codes with bit-interleaved coded modulation have been considered. The ergodic, outage capacities and cut-off rates of BI-STC with several possible inner block code are compared. These BI-STC schemes use the suboptimal decoding metric described in [10]. For $t = 2$ transmit antennas and all rates $R \leq 3$ bps/Hz, results show that BI-STC can approach the ergodic capacity of the Rayleigh flat-fading channel within 1 dB for $r = 1$, and within 2 dB for $r = 2$ receive antennas. Thus, when the inner block codes are chosen properly, these results suggest that the ergodic capacity of BI-STC is often close to the information-theoretic limits of the multiple-antenna channel. A similar conclusion can also be drawn for the outage capacity.

In practice, a more fair comparison is the channel cutoff rate for a finite coding complexity. With receive antennas $r \leq 2$, BI-STC can achieve a larger ergodic and outage cutoff rate than BI-FC. Thus BI-STC can offer a better tradeoff between performance and complexity than BI-FC.

Bibliography

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.,* vol. 16, no. 8, pp. 1451-1458, October 1998.

[2] S. M. Alamouti, V. Tarokh, and P. Poon, "Trellis-Coded Modulation and Transmit Diversity: Design Criteria and Performance Evaluation," in *Proc. ICUPC*, 1998,, pp. 703-707.

[3] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time trellis-coded modulation," *IEEE Commun. Letters,* vol. 4, pp. 20-22, Jan. 2000.

[4] G. Bauch, "Concatenation of space-time block codes and "turbo"-TCM," in *Proc. ICC '99,* vol. 2, pp. 1202-1206, 1999.

[5] E. Biglieri, J. Prokis and S. Shamai (Shitz), "Fading channels: information-theoretic and communications aspects," *IEEE Trans, Inform. Theory,* vol. 44, no. 6, pp. 2619-2692, Oct. 1998.

[6] E. Biglieri, G. Taricco and E. Viterbo, "Bit-interleaved time-space codes for fading channels," *Proc. of Conf. Inform. Sciences and Systems (CISS'00)*, pp. WA4-1-6, March 2000.

[7] G. Caire, G. Taricco and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inform. Theory,* vol. 44, no. 3, pp. 927-946, May 1998.

[8] W. Firmanto, B. Vucetic and J. Yuan, "Space-time TCM with improved performance on fast-fading channels," to appear in *IEEE Commun. Letters,* 2001.

[9] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41-59, Aug. 1996.

[10] Z. Hong and B. L. Hughes, "Robust Space-Time Trellis Codes Based on Bit-Interleaved Coded Modulation," *CISS'01*, 2001.

[11] B. L. Hughes, "Differential space-time modulation," *IEEE Trans. Inform. Theory,* vol. 46, no. 7, pp. 2567-2578, Nov. 2000.

[12] T. H. Liew, J. Pliquett, B. L. Yeap, L-L. Yang and L. Hanzo, "Concatenated space time block codes and TCM, turbo TCM, convolutional as well as turbo codes," in *Proc. IEEE Globecomm'00,* vol. 3, pp. 1828-1833, 2000.

[13] Y. Liu, M. P. Fitz and O. Y. Takeshita, "Full rate space-time turbo codes," submitted to *JSAC Turbo Codes*, 2000.

[14] S. Sandhu, and A. Paulraj, "Space-time block codes: A capacity perspective," submitted to *ICC '01.*

[15] S. Siwamogsatham and M. P. Fitz, "Robust space-time coding for correlated Rayleigh fading channels," in *Proc. 38th Annual Allerton Conf. on Communication, Control, and Computing*, October 2000, Monticello, Illinois, USA.

[16] A. Stefanov and T. M. Duman, "Turbo coded modulation for wireless communications with antenna diversity," in *Proc. of IEEE VTC-Fall*, (Amsterdam, Netherlands), pp. 1565-1569, Sept. 1999.

[17] V. Tarokh, A. Naguib, N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory,* vol. 44, no. 2, pp. 744-765, March 1998.

[18] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: performance results," *IEEE J. Select. Areas Commun.* Vol. 17, pp. 451-460, Mar. 1999.

[19] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory,* vol. 45, no. 5, pp. 1456-1467, July 1999.

[20] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585-595, November-December, 1999.

[21] H. Viswanathan, H. Huang and L. Mailaender, "A unified derivation and cutoff rate analysis of transmit diversity," *Proc. IEEE VTC'00,* vol. 2, pp. 869-873, Spring Tokyo, 2000.

[22] P. W. Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," *Proc. ISSSE-98*, pp. 295-300, Sept. 29, 1998.

[23] Q. Yan and R. S. Blum, "Optimum space-time convolutional codes for quasistatic slow fading channels," submitted to *IEEE J. Sel. Areas Commun.,* 2000.

[24] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel," *IEEE Trans. on Commun.,* vol. 40, no. 5, pp. 873-884, May 1992.