Phase characteristics of earthquake ground motions and those applications to synthetic ones

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1 INTRODUCTION

Earthquake ground motions are non-stationary in those amplitude and frequency contents, though stochastically quite random. Through the conversion of these non-stationary earthquake ground motions into frequency domain, these non-stationarities are reflected into phase components, not into amplitude components, because of those constant amplitudes across the entire time history. Therefore, phase components in frequency domain should contain all informations of non-stationarities of earthquake ground motions, such as deterministic intensity function as well as frequency contents.

Taking the difference of phase angles for every two successive values of frequency and plotting their distribution, it is observed that the shape of the distribution normalized by (-2π) is similar to the envelope function of the time history normalized by the total duration of the record. Hence, the information of non-stationarity contained into phase component may be represented by the probability density function of phase differences distribution. In the present research the probability density function of phase differences was studied and by the use of this, simulated earthquake motions were obtained.

2 METHOD OF ANALYSIS

A digitized earthquake ground motion record a(t) can be expanded into Fourier series as follows;

$$a(t) = \sum_{k=0}^{N/2} A_k \cos\left(\frac{2\pi}{T_k} t + \phi_k\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where, A_k is the Fourier amplitude, T_k the period and ϕ_k the phase angle. The characteristics of non-stationarity of recorded earthquake ground motions can be expressed by the distribution of phase differencies $\Delta\phi_k$, which are given by

$$\Delta\phi_k = \phi_{k+1} - \phi_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $\Delta\phi_k$ is defined in the interval $(-2\pi, 0)$. In the fig.1 the phase (ϕ_k) is plotted versus the period T_k. It can be observed that in the

case of very short periods is appeared a kind of "noise", probably owed to the used filter at that range of periods. In order to avoid this kind of "noise" the range of short periods that correspond the cumulative power spectrum values less that 1% have been cut off.

Furthermore, the phase differences range $(-2\pi,0)$ was divided into 36 equal intervals and the relative frequency for each interval was calculated. At Fig.2 is showed the calculated distribution of phase differences of a real earthquake record. The distribution of phase differemces has a quite similar feature to the envelope of the ground motion. A better visualization of this fact may be obtained if the axis of time, from 0 to Td (Td: total duration) and on the other hand the axis of phase differences, from 0 to -2π are scaled to the same length as showed at fig.3.

The phase difference characteristics of horizontal and vertical components of earthquake ground motions obtained at four stations located in the vicinity of Tokyo were examined. These accelerograms correspond to a range of earthquake magnitudes between 4.5 and 7.0 R., to epicentral distances between 10 and 240km and are recorded on rock sites. The shear wave velocity at the three stations is ranging from 0.6 to 0.7 km/sec and at the fourth one is approximately 1.4 km/sec. Only the records with maximum acceleration greater than 10 gal were used.

In order to derive the phase difference characteristics of the part of the earthquake ground motion that contributes significantly to the seismic energy the low amplitude parts of the records have not been taken into account. The beginning of the strong motion section as defined by Trifunac and Brady is the time at which 5% of Arias intensity is reached and the end the time that yields 95% of the total Arias intensity. At fig.4 is showed the recorded earthquake motion and the strong motion part as resulted by the above given definition.

Moreover, the concept of principal axes is considered in the present research. Principal axes are defined as the axes along which the components of earthquake ground motions have maximum, intermediate and minimum values and covariances equal to zero. Examination of real accelerograms reveals that the major principal axis points in the general direction of the epicenter and the minor principal axis is nearly vertical. Watabe et al. presented that in order to synthesize the three dimensional earthquake ground motions appropriate to the design response spectrum, it is useful to apply the phase differences of recorded 3-dimensional earthquake ground motions transformed to the principal axes. Based upon this consideration, the under research earthquake motions were transformed to the principal axes and the phase differences characteristics along these axes, were studied.

3 PROBABILITY DENSITY FUNCTION OF PHASE DIFFERENCES

As it can be observed in fig.2 the distribution of phase differences is composed of four parts over the interval $(0,-2\pi)$; built-up, high relative frequency, decaying and near the value (-2π) another increase of relative frequency ranging from low to moderately high. Moreover, it is observed that the relative frequencies for values of phase differences belonging to 0 and -2π areas are approximately the same. Due to the presence of the fourth part, the value of phase difference in which the maximum relative frequency occurs does not coincide to the sample mean. In order to make to sample mean to be obtained when the maximum occurs the distribution must be changed to symmetrical one. For this reason the

fourth part of the distribution has been cut-off and transferred to the positive phase differences axis in advance of the other three parts as showed in fig.5. The criterion used in this transformation was the minimazation of the sample standard deviation. By the above-described technique the sample mean and the value at which the maximum relative frequency occurs, coincide.

Regression analysis of the obtained mean value (μ) and standard deviation (σ) of phase differences was conducted. Linear functions of independent variables such as magnitude (M) and epicentral distance (R) were used in order to accomplish a first order approximation of μ and σ. The results of the regression analysis showed that relationships between μ and σ and the independent variables M and R can not be established due to the small number of used accelerograms. Although, there is not sufficient correlation, the following useful observations have been made. The mean value depends slightly on the magnitude but increases significantly with the distance. The standard deviation is not depended on both magnitude and epicentral distance. Mean and Standard deviation for both major and intermediate axes are approximately the same but in the case of minor axis an insignificant increase was observed. The calculated mean values of mean and standard deviation of phase differences are summarized in Table I.

In order to obtain the most appropriate probability distribution function the third and fourth order moments, skewness and Kurtosis, have been obtained. These coefficients let us know that the demanded distribution function must be symmetric with high relative frequencies near the maximum. The normal distribution function corresponds to the above-mentioned demands and represents the best choise on the basis of simplicity and a first order approximation of the derived distribution of phase differences. The distribution derived by the use of phase differences of a real accelerogram with estimated mean (μ) and standard deviation (σ) and the normal distribution with the same parameters are plotted simultaneously in fig.6. It can be observed that the fitting of the normal distribution is satisfactory, fact that confirmed conducting chi-square test.

4 SIMULATION OF EARTHQUAKE MOTION

Taking into account the parameters described in Table 1, the distribution of phase differences for major, intermediate and minor axis can be obtained. Assuming that the initial value of phase angle is equal to 0, by the use of relationship Eq. (2), the phase angles over the interval $(0, 2\pi)$ can be generated. After, the target response spectrum proposed by Ohsaki for earthquake magnitude 8.0 R. and epicentral distance 100 km has been set. Following, inverse Fourier transformation represented by Eq. (1) non-stationary acceleration time histories along the principal axes were composed, as showed in fig.7.

5 CONCLUSIONS

The probability distribution of phase differences, appear to be normal with mean value and standard deviation approximately the same for both major and intermediate axes and slightly greater values for the minor principal axis. The mean value depends slightly on the earthquake magnitude but increases considerably with epicentral distance. The

standard deviation is approximately constant for a wide range of magnitudes and epicentral distances of engineering interest. The simulated wave shape and the used distribution of phase differences bear a close similarity.

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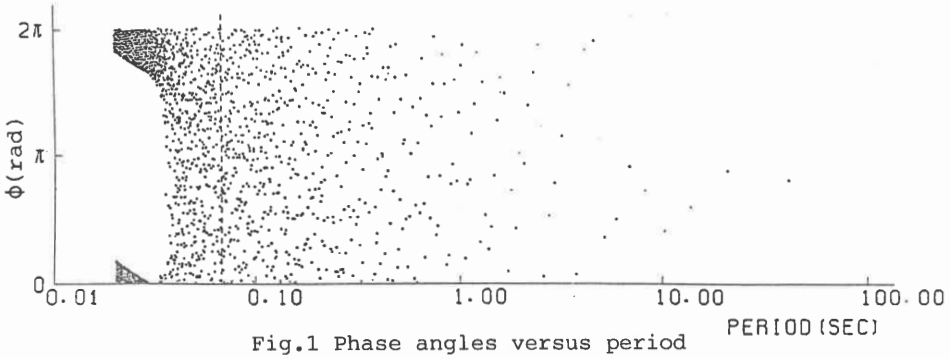

Fig.1 Phase angles versus period

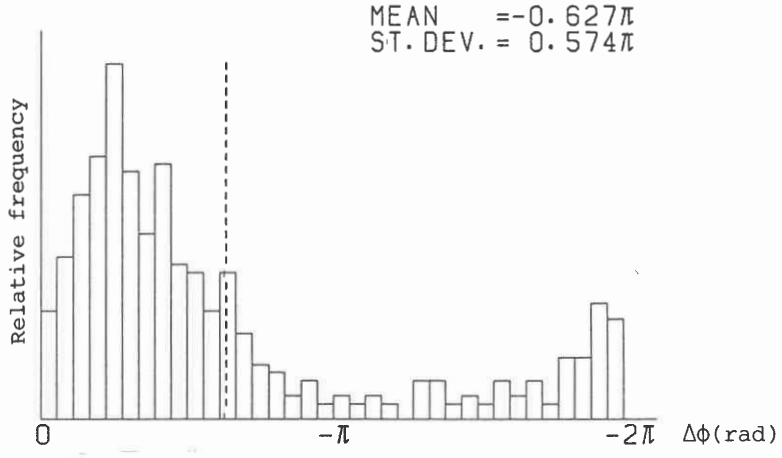

Fig.2 Distribution of phase differences

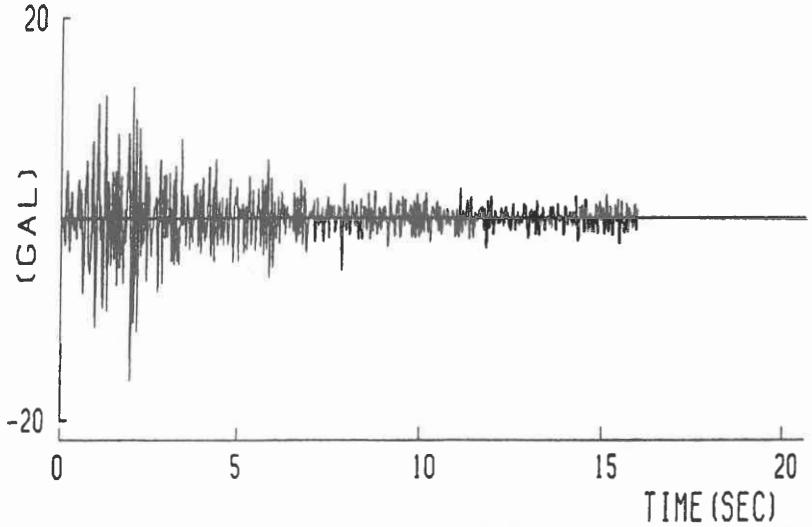

Fig.3 Strong motion part of recorded earthquake motion used in the present research

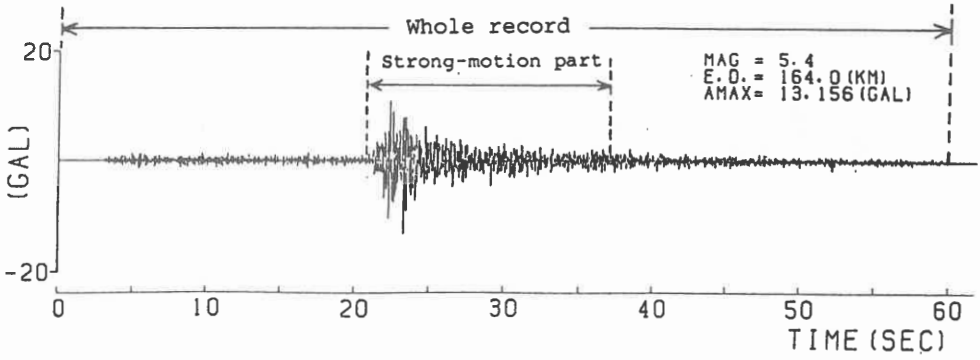

Fig.4 Recorded earthquake and strong-motion part

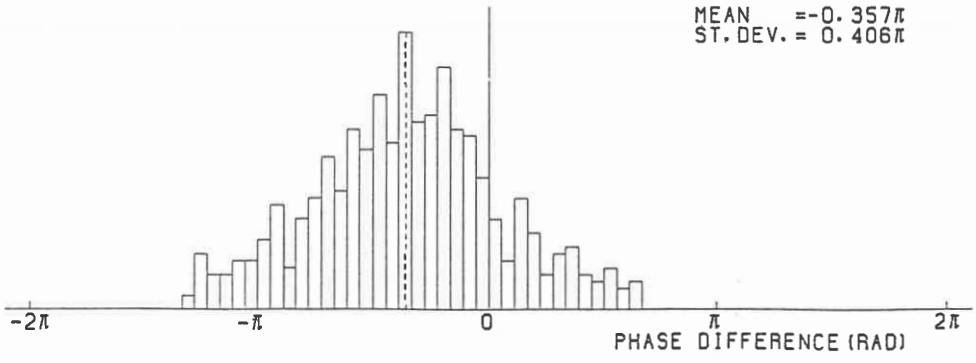

Fig.5 Transformed distribution of phase differences

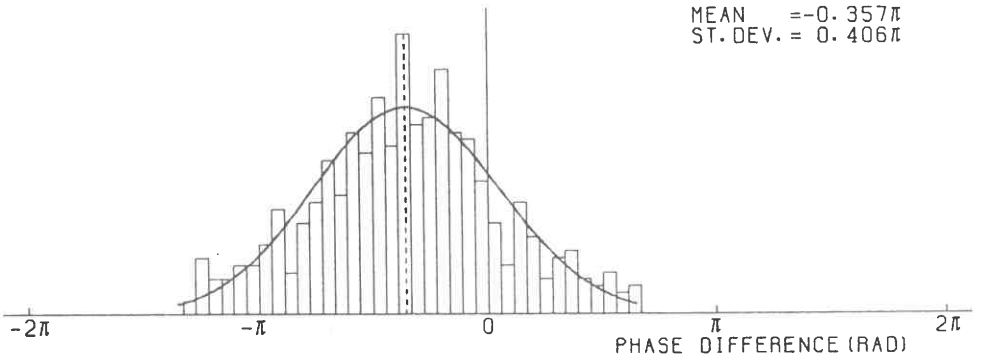

Fig.6 Normal distribution fitted to the transformed distribution of phased differences

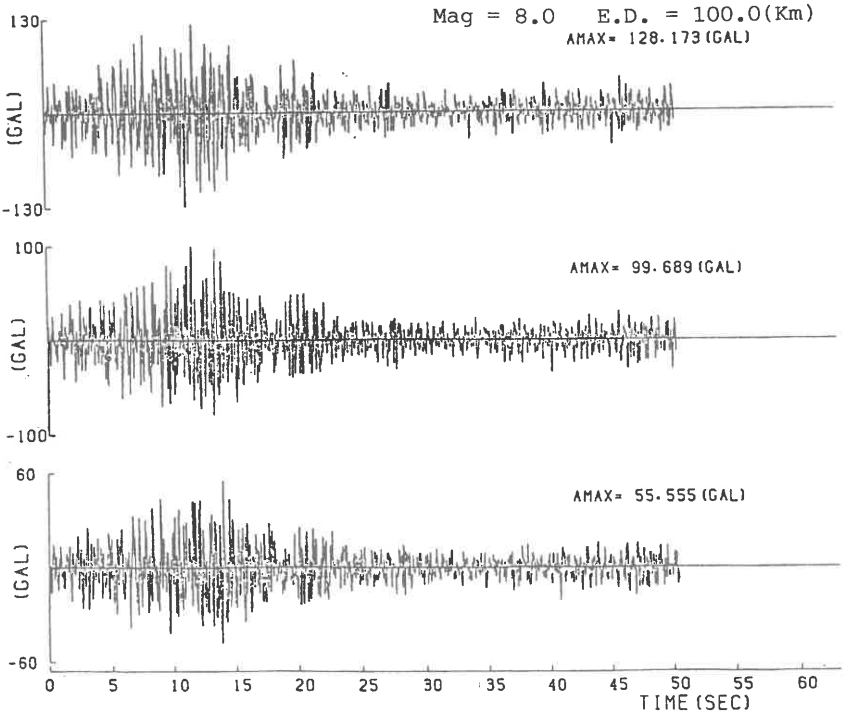

Fig.7 Simulated earthquake motion along the principal axes

TABLE I

	Major	Intermediate	Minor
Mean value(μ)	-0.2319π	-0.2482π	-0.2687π
Standard deviation(σ)	0.3262π	0.3260π	0.3378π