ACOUSTIC PRESSURE PULSATIONS IN PRESSURIZED WATER REACTORS

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This paper is an examination of pressure pulsations within a PWR resulting from circulating pump excitations. Evaluation of these pulsations are necessary in determining the structural integrity of the reactor internals.

Previously published solutions involving the coolant annulus were formulated by the use of an equivalent body force to circumvent the complexities of a time-dependent mixed-value boundary condition problem. However, due to divergent physical arguments concerning the correct form of the body force, different solutions were obtained. In addition these publications are based upon assumptions regarding the value of the inlet pressure (i.e. pressure at the inlet pipe/coolant annulus interface).

This analysis addresses the following two problems:

1. Derivation of a theoretical solution that resolves discrepancies between varying physical arguments.

2. Analytical determination of the inlet pressure.

The solution involves examination of two separate regions:

1. The inlet pipe which includes the pump, inlet pipe and inlet pipe/ coolant annulus interface is described by a one-dimensional acoustic wave equation with time-dependent non-homogeneous boundary conditions.

2. The coolant annulus is described by a three-dimensional acoustic wave equation in cylindrical coordinates with a non-homogeneous boundary condition at the inlet pipe/coolant annulus interface. The remaining boundary conditions are assumed to be homogeneous.

A transformation technique is utilized to reduce each region to a non-homogeneous differential equations with homogeneous boundary conditions. Standard normal mode supernosition methods are employed to formulate the transformed forced vibration response of both regions. The inverse transformation solutions are then derived and the complete description of the system is obtained by mathematically requiring pressure continuity at the inlet pipe/coolant annulus interface.

By considering the inlet pipe and introducing the mathematical transformation a theoretical solution is developed that resolves the discrepancies in previous literature. The concept of an equivalent body force is shown to be analogous to the mathematical transformation and that certain restrictions on the use of the body force had been previously neglected. Furthermore it is possible to analytically determine the inlet pressure. Predicted inlet pressures, utilizing the present methodology, are shown to be in agreement with reactor data. Numerical results for typical reactor steady state pressure pulsations are presented for both the inlet pipe and coolant annulus.

1. INTRODUCTION

When a compressible liquid confined in a finite enclosure is excited, pressure waves are produced throughout the medium. Such pressure waves exist in the coolant medium of a PWR as a result of circulating pump excitations. A description of these pressure waves is necessary in formulating the vibration levels of PWR internals.

Pressure pulsations in a PWR were first examined by Penzes, reference [1], and later by Bowers and Horvay [2]. Both Penzes and Bowers treated the problem of pulsations in the coolant annulus by introducing a body force representative of the time dependent boundary condition at inlet pipe/coolant annulus interface. However the correct form of the body force was questioned by Bowers and Horvay and resulted in an alternate solution.

Fisher et al. [3] demonstrated an improved technique in solving one dimensional wave propagation problems with time dependent boundary conditions. This technique introduces a transformation equation in terms of a new variable and auxiliary functions. The mathematical description is recast in the form of a non-homogeneous differential equation. The boundary conditions become homogeneous by restricting the auxiliary functions at the boundaries. Knowledge of the auxiliary functions on the interior of the medium is not required as demonstrated in reference [3]. This method is utilized in the examination of the three dimensional wave propagation in the coolant annulus and the one dimensional wave propagation of the inlet piping.

The body force concept is demonstrated to be analogous to the transformation utilized here. Both Penzes and Bowers neglected to recognize restrictions on the body force necessary to make the boundary conditions homogeneous. In addition, by coupling the solutions of the inlet pipe and coolant annulus, the inlet pressure is calculated, a parameter assumed by Penzes and Bowers.

2. MATHEMATICAL FORMULATION

The geometric representation of the inlet pipe and coolant annulus (henceforth designated as region I and region II, respectively) is shown in figure 1. It has been assumed that the inlet pipe of length ℓ_I can be represented as being straight, rigid and having wave propagation in the x-direction with speed of sound $C_{o,I}$. The coolant annulus with length ℓ_{II} and speed of sound $C_{o,II}$ is bounded by the core support barrel and reactor vessel. The core support barrel, having a radius a_1, is assumed rigid. The reactor vessel, having a radius a_2, is assumed rigid everywhere except at the region I / region II interface. The axial end conditions of the coolant annulus may be open (zero pressure) or closed (maximum pressure).

2.1 GOVERNING EQUATIONS

The equations governing both region I and region II are homegeneous acoustical wave equations for pressure with non-homogeneous time dependent boundary conditions. The mathematical description of region I is a one dimensional acoustic wave equation

$$L_I\ [P_I\ (X,t)] = 0 \qquad (1\text{-a})$$

where the operator $L_I = \frac{\partial^2}{\partial x^2} - \frac{1}{C_{o,I}^2}\frac{\partial^2}{\partial t^2}$ and with boundary conditions

$$P_I(o,t) = P_o \mathrm{COS}\omega_p t \quad (1\text{-b}) \qquad \text{and} \qquad \left.\frac{\partial P_I}{\partial x}\right|_{x = \ell_I} = A\ \mathrm{COS}\omega_p t \qquad (1\text{-c})$$

The pressure exiting the pump, P_o can be obtained through analytical means, [4], or experimental means. The pump driving frequency, ω_p, represents either the pump rotor frequency or its resulting harmonics, pump blade passing frequency or its harmonics. The pressure gradient at the region I / region II interface is assumed to have the time dependency of the pump with unknown amplitude A.

Region II can be described by a three dimensional wave equation in cylindrical coordinates given by:

$$L_{II}[P_{II}(\vec{r},\ t)] = 0 \qquad (2\text{-a})$$

where $L_{II} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{C_{o,II}^2}\frac{\partial^2}{\partial t^2}$

and with radial boundary conditions

$$\left.\frac{\partial P_{II}(\vec{r},t)}{\partial r}\right|_{r=a_1} = 0 \quad (2\text{-b}) \qquad \text{and} \qquad \left.\frac{\partial P_{II}(\vec{r},t)}{\partial r}\right|_{r=a_2} = h\ (z)\ h\ (\theta)\ A\ \mathrm{COS}\omega_p t \qquad (2\text{-c})$$

and axial boundary conditions

$$\text{open end} \quad \left. P_{II}(\vec{r},t)\right|_{z=0,\ z=\ell_{II}} = 0 \quad (2\text{-d}) \qquad \text{closed end} \quad \left.\frac{\partial P_{II}(\vec{r},t)}{\partial z}\right|_{z=0,\ z=\ell_{II}} = 0 \quad (2\text{-e})$$

The radial boundary conditions at the reactor vessel has been described by the use of Heaviside step functions, H( ), by defining $h(z) = H(z-z_1) - H(z-z_2)$ and $h(\theta) = H(\theta - \theta_1) - H(\theta - \theta_2)$. The choice of equation (1-c) is dictated by equation (2-c) where an unconventional boundary condition is introduced to avoid complexities of a mixed valued boundary condition. Equations (1-c) and (2-c) couple the two regions by requiring continuity of the pressure gradient at the interface of region I and region II.

2.2 TRANSFORMATION TECHNIQUE

The solution of both equations (1-a→1-c) and equations (2-a→2-e) is difficult in their present form. This difficulty is circumvented by choosing the following transformations:

$$P_I\ (X,t) = q_I\ (X,t) + f_1(x)\ P_o\ \mathrm{COS}\omega_p t + f_2(X)\ A\ \mathrm{COS}\omega_p t \qquad (3)$$

$$P_{II}(\vec{r},t) = q_{II}\ (\vec{r},t) + h(z)\ h\ (\theta)\ g\ (r)\ A\ \mathrm{COS}\omega_p t \qquad (4)$$

Upon substitution of equation (3) into equations (1-a→1-c) yields:

$$L_I\ [q\ (x,t)] = -\ L_I\ [P_o\ \mathrm{COS}\omega_p t\ \ f_1(x) + A\ \mathrm{COS}\omega_p t\ f_2(x)] \qquad (5\text{-a})$$

$$q_I\ (o,t) = P_o\ \mathrm{COS}\omega_p t - P_o\ \mathrm{COS}\omega_p t\ f_1(0) - A\ \mathrm{COS}\omega_p t\ f_2(0) \qquad (5\text{-b})$$

$$\left.\frac{\partial q_I(X,t)}{\partial X}\right|_{X=\ell_I} = A\ \mathrm{COS}\omega_p t - P_o \mathrm{COS}\omega_p t \left.\frac{df_1(X)}{dX}\right|_{X=\ell_I} - A\ \mathrm{COS}\omega_p t \left.\frac{df_2(X)}{dx}\right|_{X=\ell_I} \qquad (5\text{-c})$$

To make equations (5-b) and (5-c) homogeneous, the following restrictions must be imposed on the auxiliary functions $f_1(X)$ and $f_2(X)$:

$$f_1(0) = 1 \quad (6\text{-a}) \qquad \left.\frac{df_1(X)}{dx}\right|_{X=\ell_I} = 0 \quad (6\text{-b})$$

$$f_2(0) = 0 \quad (7\text{-a}) \qquad \left.\frac{df_2(X)}{dx}\right|_{X=\ell_{II}} = 1 \quad (7\text{-b})$$

Similarly, substitution of equation (4) into equations (2-a→2-e) yields:

$$L_{II}[q_{II}(\vec{r},t)] = - L_{II}[h(Z)h(\theta)g(r)\ A\ \cos\omega_p t] \qquad (8\text{-}a)$$

$$\left.\frac{\partial q_{II}(\vec{r},t)}{\partial r}\right|_{r=a_1} = - h(Z)h(\theta)A\ \cos\omega_p t \left.\frac{dg(r)}{dr}\right|_{r=a_1} \qquad (8\text{-}b)$$

$$\left.\frac{\partial q_{II}(\vec{r},t)}{\partial r}\right|_{r=a_2} = h(Z)h(\theta)A\ \cos\omega_p t - h(z)A\ \cos\omega_p t \left.\frac{dg(r)}{dr}\right|_{r=a_2} \qquad (8\text{-}c)$$

open end $\left. q_{II}(\vec{r},t)\right|_{Z=0,\ Z=\ell_{II}} = 0$ (8-d) closed end $\left.\frac{\partial q_{II}(\vec{r},t)}{\partial Z}\right|_{Z=0,\ Z=\ell_{II}} = 0$ (8-e)

To render equation (8-b) and (8-c) homogeneous, the following restrictions must be imposed on the auxiliary function g(r)

$$\left.\frac{dg(r)}{dr}\right|_{r=a_1} = 0 \quad (9\text{-}a) \quad \text{and} \quad \left.\frac{dg(r)}{dr}\right|_{r=a_2} = 1 \qquad (9\text{-}b)$$

The description of both region I and region II now consists of a non-homogeneous differential equation with boundary conditions becoming homogeneous by requiring restrictions (6), (7) and (9) on the auxiliary $f_1(X)$, $f_2(X)$ and g(r), respectively. Normally these auxiliary functions are chosen and the solution is obtained by standard modal analysis. However, due to the many choices of auxiliary functions that fit these restrictions at the boundaries, there can be confusion as to the proper solution. Reference [3] demonstrates that choosing an auxiliary function is not necessary for a one dimensional wave equation. Employing this procedure results in the solution of region I, given by:

$$P_I(X,t) = - C_{o,I} \sum_{i=1,3,5}^{\infty} \frac{P_o\ \omega_i + A\ C_{o,I}\ \sin\frac{i\pi}{2}}{(\omega_p^2 - \omega_i^2)\int_0^{L_I} Q_i^2(X)dx}\ Q_i(X)\ \cos\omega_p t \qquad (10)$$

where the frequencies $\omega_i = \frac{i\pi C_{o,I}}{2\ell_I}$ and mode shapes $Q_i(X) = \sin\frac{i\pi X}{2\ell_I}$

Extending this procedure to the three dimensional geometry of the coolant annulus, the pressure of region II becomes:

$$P_{II}(\vec{r},t) = \sum_{n=j}^{\infty}\sum_{m=0}^{\infty}\sum_{s=1}^{\infty} A\ C_{nms}Q_n(Z)Q_m(\theta)Q_s(r)\cos\omega_p t \qquad (11)$$

where:

$$C_{nms} = \frac{C_{o,II}^2\ a_2\ Q_s\ (a_2)\ I_1}{(\omega_{nms}^2 - \omega_p^2)\quad I_2} \qquad (12\text{-}a)$$

$$I_1 = \int_{\theta_1}^{\theta_2} Q_m(\theta)d\theta \int_{z_1}^{z_2} Q_n(z)\ dz \qquad (12\text{-}b)$$

$$I_2 = \int_0^{\ell_{II}} \int_0^{2\pi} \int_{a_1}^{a_2} Q_n^2 (z)\, Q_m^2 (z)\, Q_s^2 (r)\, r dr d\theta dz \qquad (12\text{-c})$$

$$Q_n(Z) = B_1 \,\mathrm{Sin}\, \alpha_n Z + B_2 \,\mathrm{COS}\, \alpha_n Z \qquad (13\text{-a})$$

$$Q_m(\theta) = \mathrm{COS} m\theta \qquad (13\text{-b})$$

$$Q_s(r) = J_m (\tau_{nms} r) + \eta_{nms} Y_m (\tau_{nms} r) \qquad (13\text{-c})$$

J_m and Y_m are Bessel functions of the first and second kind. B_1, B_2, j and α_n depend on the choice of axial boundary conditions and are found in Table I of [1]. η_{nms} are given by:

$$\eta_{nms} = \left. \frac{-\frac{d}{dr} Jm (\tau_{nms} r)}{\frac{d}{dr} Ym (\tau_{nms} r)} \right|_{r=a_1} \qquad (14\text{-a})$$

where $\tau_{nms}^{\;2} = \frac{\omega_{nms}^{\;2}}{C_{o,II}^2} - \alpha_n^{\;2}$

The frequencies, ω_{nms}, are obtained from the transcendental equations:

$$\left. \frac{d\, J_m (\tau_{nms} r)}{dr} \right|_{r=a_2} = \eta_{nms} \left. \frac{d\, Ym (\tau_{nms} r)}{dr} \right|_{r=a_2} \qquad (14\text{-b})$$

The unknown constant, A, can now be obtained by requiring pressure continuity at the region I/region II interface., i.e.:

$$P_I (\ell_{I,t}) = P_{II} \left(r = a_2, \theta = \frac{\theta_1 + \theta_2}{2}, Z = \frac{Z_1 + Z_2}{2}, t\right) \qquad (15)$$

Substitution of equations (10) and (11) into (15) results in:

$$A = \frac{K_1 P_o}{K_3 - K_2} \qquad (16)$$

where:

$$K_1 = - \sum_{i=1}^{\infty} \frac{C_{o,I}\, \omega_i \,\mathrm{Sin}\, \frac{i\pi}{2}}{(\omega_p^2 - \omega_i^2) \int_o^{\ell_I} Q_i^2 (X) dx} \qquad (17\text{-a})$$

$$K_2 = - \sum_{i=1}^{\infty} \frac{C_{o,I}^{\;2}}{(\omega_p^{\;2} - \omega_i^{\;2}) \int_o^{\ell_I} Q_i^2 (X) dx} \qquad (17\text{-b})$$

$$K_3 = \sum_{n=j}^{\infty} \sum_{m=o}^{\infty} \sum_{s=1}^{\infty} C_{nms}\, Q_n\left(\frac{z_1 + z_2}{2}\right) Q_m\left(\frac{\theta_1 + \theta_2}{2}\right) Q_s (a_2) \qquad (17\text{-c})$$

The solution of region I and region II is now complete and is given by equations (10) and (11) with the unknown constant A obtained by equation (16).

3. EQUIVALENT BODY FORCE

The mathematical transformation, equation (4) for the coolant annulus, can be obtained using physical arguments as suggested by Penzes. It is necessary to find an equivalent system that includes a body force in lieu of a time dependent boundary condition. Compare the linearized equations of motion with and without a body force,

$$\rho_o \frac{\partial \vec{V}_1}{\partial t} + \nabla P = 0 \tag{18}$$

$$\rho_o \frac{\partial \vec{V}_2}{\partial t} + \nabla q = - \nabla \phi \tag{19}$$

where the body force $\vec{B}$ has been written as the gradient of a scalar ϕ. For equation (18) and equation (19) to be equivalent and to have identical velocity distributions, i.e., $\vec{V}_1 = \vec{V}_2$, the following relationship must be recognized:

$$P = q + \phi \tag{20}$$

The linearized continuity equation reads:

$$\frac{\partial \rho}{\partial t} + \rho_o \nabla \cdot \vec{V} = 0 \tag{21}$$

Care must be taken, however, in deriving the relationship for sonic velocity. Without a body force the sonic velocity relationship reads:

$$P = C_o^2 \rho \tag{22}$$

Introducing equation (20) into equation (22) results in:

$$q = C_o^2 \rho - \phi \tag{23}$$

An alternate derivation of equation (23) can be obtained by standard acoustic pertubation techniques in conjunction with a Taylor series expansion of pressure in terms of the two variables ρ and ϕ. The body force acoustic wave equation can now be obtained by taking the divergence of equation (19), substituting for $\nabla \cdot \vec{V}$ from equation (21) and substituting for ρ from equation (23) yielding:

$$L_{II}\, q = - L_{II}\, \phi \tag{24}$$

Note the equivalence of the transformation equations (20) and (4) and also the wave equations (24) and (8-a). Substitution of equation (20) into (2-c) yields:

$$\left.\frac{\partial\, q\,(\vec{r},t)}{\partial r}\right|_{r=a_2} = A\ \mathrm{COS}\omega_p t\ H(\theta)H(Z) - \left.\frac{\partial \phi}{\partial r}\right|_{r=a_2} \tag{25}$$

For (25) to be homogeneous

$$\left.\frac{\partial \phi}{\partial r}\right|_{r=a_2} = \left|\, \vec{B}_r \right|_{r=a_2} = A\ \mathrm{COS}\omega_p t\ H(\theta)H(Z) \tag{26}$$

Substitution of (20) into (2-b) yields

$$\left.\frac{\partial\, q\,(\vec{r},t)}{\partial r}\right|_{r=a_1} = \left.\frac{\partial\phi}{\partial r}\right|_{r=a_1} \tag{27}$$

For equation (27) to be homogeneous

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=a_1} = \left|\vec{B}_r\right|_{r=a_1} = 0 \tag{28}$$

A body force that fulfills requirements (26) and (27) is:

$$\frac{\partial\phi}{\partial r} = \left|\vec{B}_r\right| = H\,(r-\varepsilon)\; A\; COS\omega_p t\; H(\theta)H(Z) \tag{29}$$

where $a_1<\varepsilon\leq a_2$. Upon solving equation (24) with the body force, defined by (24), and taking the limit as $\varepsilon\to a_2$ one obtains identical results as found in equation (11).

4. NUMERICAL RESULTS

The coolant annulus parameters considered are identical to those found in references [1] and [2]:

ℓ_{II} = 328 in; a_1 = 76 in; a_2 = 86 in; Z_1= 54 in; Z_2 = 94 in; ω_p = 2π (100) rad/sec; θ_2 - θ_1 = 26.6°; $C_{o,II}$ = 3850 ft/sec. A pump exit pressure of 1.0 psi is assumed. The acoustic medium within the inlet pipe having a speed of sound $C_{o,I}$ = 3800 ft/sec is considered using three inlet pipe lengths (ℓ_I, $\ell_I/2$ and $\ell_I/4$) with ℓ_I = 217 in. The axial pressure distribution of the coolant annulus is shown in figure 2. This distribution is normalized to the inlet pressures. The pressure distributions in the inlet pipe are shown in figure 3. The largest pressure at the inlet, for the three inlet pipe lengths considered, occurs for an inlet pipe length of $\ell_I/2$ as a result of one of the liquid natural frequencies being close to the pump forcing frequency.

To illustrate a case where the inlet pipe lengths are approximately the same, test data from two of C-E's Precritical Vibration Monitoring Programs (PVMP) will be compared with predictions based on the present methodology.

The Maine Yankee PVMP [5] and Omaha PVMP [6] note an inlet pressure of 0.354 psi and 0.096 psi, respectively. The inlet pipe lengths of both Maine Yankee and Omaha are approximately equal. For the two reactors, based on [4], the fluctuating pressures at the pump discharge are identical. However, the size of the annulus, particularly the annulus length, differ for both reactors.

Based on the present theory the resulting predicted inlet pressure for Maine Yankee is 0.298 psi and Omaha is 0.1 psi. From the two numerical examples considered it is illustrated that the coupling of the inlet pipe and coolant annulus is essential in predicting acoustic pressure pulsation in PWR's.

5. CONCLUSIONS

1. An alternate derivation of the coolant annulus resolves discrepancies between varying physical arguments found in previously published solutions.

2. The present solution permits determination of the inlet pressure.

REFERENCES

1. Penzes, L. E. "Theory of Pump Induced Pulsating Coolant Pressure In Pressurized Water Reactors", 2nd Intl. Conf. on Structural Mechanics in Reactor Technology, Berlin, Germany, September 10-14, 1973, Paper E 5/1*.

2. Bowers, G. and Horvay, G., "Forces Vibrations of a Shell Inside of a Narrow Water Annulus", 3rd Intl. Conf. on Structural Mechanics in Reactor Technology, London, United Kingdom, September 1-5, 1975, Paper F 2/8.

3. Fisher, H. D., Cepkauskas, M. M. and Chandra, S., "Transient Response of a Non-Homogeneous Bar by the Boundary Operator Method", Accepted for publication, International Journal of Solids and Structures.

4. Simpson, H. C., Clark, T. A., "Noise Generation in Centrifugal Pumps", ASME 70-FE-37.

5. Maine Yankee Precritical Vibration Monitoring Program, Combustion Engineering, Inc., Report CENPD-93.

6. Omaha Precritical Vibration Monitoring Programs, Combustion Engineering, Inc., Report CEN-7(0).

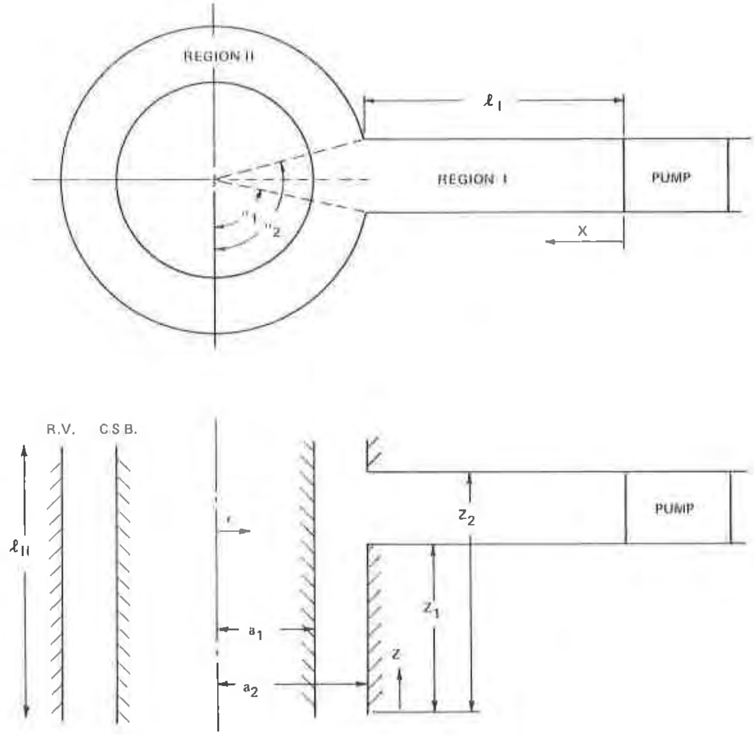

Figure 1

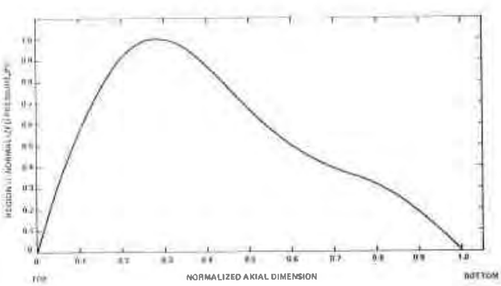

Figure 2

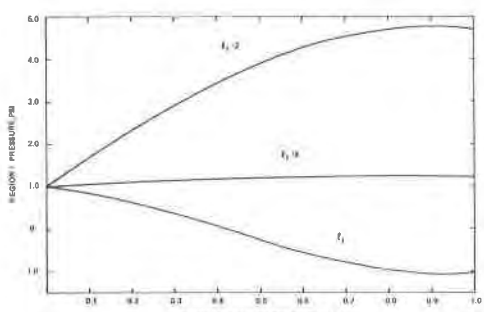

Figure 3