ESTIMATION OF FAILURE PROBABILITY OF EXPANSION BELLOW AT RCB CONTAINMENT PENETRATION OF PFBR USING IMPROVED HIGHER ORDER STOCHASTIC RESPONSE SURFACE METHOD

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ABSTRACT

Expansion bellows are an integral part of the pressure boundary in the nuclear power plants, when they are used in the containment penetrations as soft connection to mitigate the effects of differential movements between the penetrating pipe and containment shell. The expansion bellow at the Reactor Containment Building penetration of secondary sodium line of Prototype Fast Breeder Reactor experience high axial and lateral duty at normal operating condition and have lesser design margin then other mechanical components. These conditions make them a potential source of leakage of contaminations to atmosphere in the event of failure of their structural integrity. This report presents a methodology that estimates the probability of failure of expansion bellow based on higher order response surface method called as improved Higher Order Stochastic Response Surface Method (iHOSRSM), which is well validated with wide range problems given in the literature. The procedure first accurately estimates the order of the Chebyshev polynomial and subsequently estimates the significant mixed order terms required for limit state approximation. Investigation of probable failure modes of expansion bellow is carried out by deterministic finite element analysis of bellow. The fatigue failure mode of bellow is used for development of limit state, which will account the variations in material and geometric properties as well variations in loading condition. Sensitivity analysis and stochastic response analysis infers that failure probability of bellows has high dependence on material properties and loading conditions.

Keywords: expansion bellow, failure probability, response surface, Chebyshev polynomial, fatigue

INTRODUCTION

Expansion bellows are an integral part of the pressure boundary in the nuclear power plants. They are used in the containment penetrations as soft connection to mitigate the effects of differential movements between the penetrating pipe and containment shell [1]. Expansion bellows are also used as inline components in the pipeline or in the components like heat exchangers to provide flexibility for thermal expansion and to function as pressure containing elements. As piping penetration bellows, it is the last engineered barriers to prevent the release of the radioactive materials to the atmosphere from containment. Therefore they are a potential source of leakage of contaminations to atmosphere in the event of failure of their structural integrity. This paper presents a methodology to estimate the probability of failure of expansion bellows at the Reactor Containment Building (RCB) penetration of secondary sodium line of Prototype Fast Breeder Reactor (PFBR).

PFBR is a 500 MWe sodium cooled pool type fast reactor. During normal operation, the heat from the core is removed by primary sodium flowing through the core and is transported to the Intermediate Heat Exchangers (IHX) where it transfers heat to secondary sodium. The secondary sodium in turn transfers the heat to water in steam generator [2]. This secondary sodium line is at the temperature of 525^0C which passes through RCB penetration between IHX and surge tank. High temperature of the line cause large thermal expansion of this pipe segment. To accommodate this expansion four expansion bellows in series are provided at the RCB penetration. Isolation of RCB will be jeopardize if there will be any breach of structural integrity of these bellows.

American Society of Mechanical Engineers Boiler & Pressure Vessel (ASME B&PV) code, section VIII, Div-2 [3] has given the criteria for checking the stability and fatigue failure of the expansion joints for the static or quasi-static load. These criteria need evaluation of equivalent axial duty from the axial, lateral and bending duty of the bellows, calculated from relative displacement of ends of these bellows. Numbers of cycles allowed to prevent fatigue damage are determined from the equations given in this code. The allowable number of cycles calculated

from these codes have design margin of 3 on cycles and 1.25 on stress [3]. However, the allowable number of cycles calculated from ASME B&PV, section III, Div-1, subsection-NB for pipelines and components have design margin of 20 on cycles and 2.0 on stress. Since bellows have lower design margins, its probability of failure for actual condition need to be evaluated to achieve reliability target of containment isolation.

Thus, in view of safety importance of these expansion bellows, probability of failure of these bellows due to fatigue failure is carried out. Probability of failure of these bellows are estimated with a procedure based on response surface methods called as improved Higher Order Stochastic Response Surface Method (iHOSRSM). Deterministic analysis of expansion bellow during normal operating condition (NOC) is performed to identify the failure mode and formulate the limit state functions, which will be used for the estimation of probability of failure of bellow.

FINITE ELEMENT ANALYSIS OF EXPANSION BELLOW

Description of Bellow

Expansion bellows are part of the line segments which penetrated the RCB between IHX and surge tank. At normal operating conditions pipeline as at 525^0C, whereas in the shutdown condition it is at 150^0C. In case of draining of sodium from pipeline, it will reach the temperature of 40^0C. Expansion bellows will approximately 50^0C lower than the pipeline temperature. Two expansion bellows are provided in both side of the containment wall to accommodate duty on the bellows at NOC. All these bellow is attached with the sleeve in series. One end of this sleeve is welded with guard pipe inside RCB Containment and other end with the main pipe outside RCB containment. Middle portion of the sleeve is welded with the embedment penetration (EP) coming from the containment opening. Expansion bellows configuration near containment penetration is shown in the figure-1. The expansion bellow is made up of austenitic steel SS304L. The geometric details of these bellows are as given in Table-1 with their nomenclatures in figure-2.

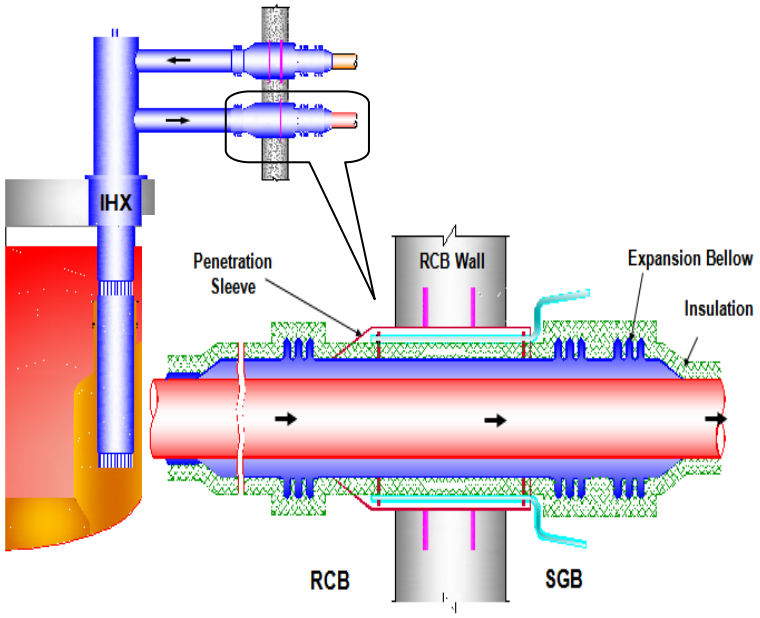

Fig.1: Secondary Sodium piping expansion bellows configuration near RCB penetration

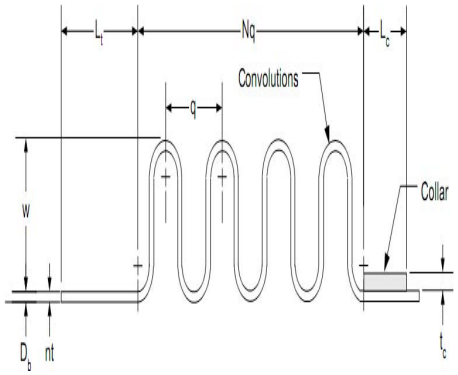

Fig.2: The sketch of the corrugation of expansion bellow

Finite Element Analysis

Finite Element (FE) model is developed by meshing the Area of solid model using 8-noded SHELL93 element. SHELL93 is particularly well suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. The deformation shapes are quadratic in both in-plane directions [4]. Integrated FE model of piping systems, which has piping elements modeled with pipe elements, PIPE16 and PIPE18, an expansion joints modeled with 8-noded shell

element 'shell93'. The connectivity between pipe and shell element is provided by a spider of beam element shown in figure 3. The material properties of bellows are taken RCC-MR [5] and ASME [3]. An elastic analysis of bellow is performed to evaluate the equivalent local membrane and bending stress for fatigue evaluation.

Table 1: Geometric properties of the expansion joints

S.N	Geometric Properties	
1	Length of expansion joints (L)	: 440 mm
2	Inside Diameter of expansion joints convolution (Db)	: 840 mm
3	Convolution height (w)	: 29.5 mm
4	No. of corrugation(N)	: 10
5	Convolution pitch (q)	: 26 mm
6	End tangent length (Lt)	: 90 mm
7	Crest Convolution radius (ric)	: 7.5 mm
8	Root Convolution radius (rir)	: 7.5 mm
9	Design number of Cycles (N_{DBE})	: 1000

Pressure load on the expansion bellows is very low and the stresses induced due to pressure are insignificant. Only significant load at the normal operating condition is thermal expansion of pipe segment between IHX and surge tank. The Von Mises stress in the below at the inner surface is 708.34 MPa as shown in figure 4. This is higher than the twice of material yield strength and will not lead to shakedown of the bellow. Therefore fatigue-ratcheting due to thermal load became significant failure modes for bellow and considered as criteria of limit state formulation.

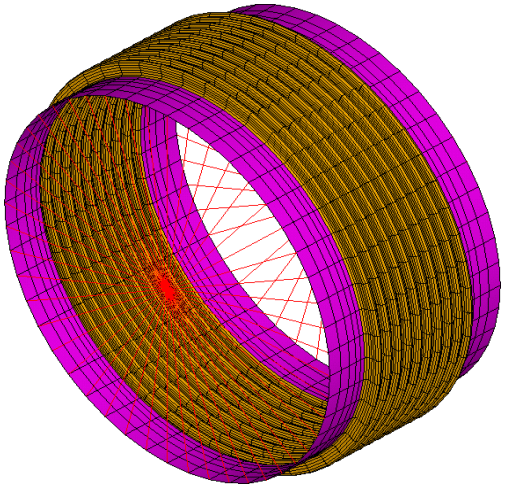

Fig.3: Finite element model of expansion bellow at the RCB penetration of PFBR

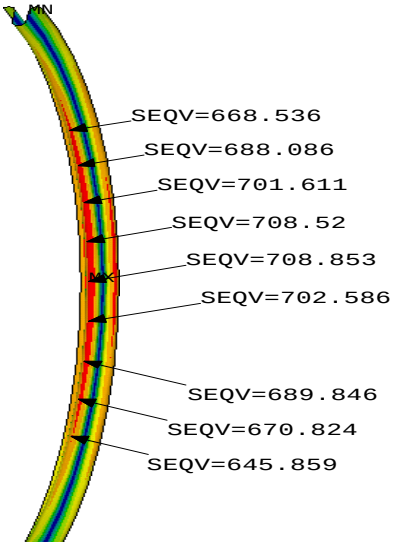

Fig. 4: Von Mises stress range in the expansion bellow during NOC

PROCEDURE OF FAILURE PROBABILITY EVALUATION

Evaluation of failure probability of components needs to estimate the limit state function $g(X) = 0$ under various loading states defined by loads, material strengths and dimensions as defined in eq-1:

$$p_f = P[g(X) \le 0] = \int_{g(X)\le 0} f_X(x)dx \qquad (1)$$

Where, $f_X(x)$ is the joint probability distribution function for the vector X of basic random variables. In structural reliability analysis, Monte Carlo Simulation (MCS) is a well-known technique to accurately estimate the

failure probability, p_f, regardless of complexity of the system or the limit state [6, 7, 8]. But MCS becomes computationally intensive for structural reliability analysis of complex systems with low failure probabilities and MCS can be infeasible when the analysis requires a large number of computationally intensive simulations.

Other methods like First Order Reliability Method (FORM) or Second Order Reliability Method (SORM) are also developed as the computation of failure probabilities using full distribution approach is close to impossible. Both these methods require explicit or implicit limit state functions defined a priori, but in actual problems like expansion bellow, where limit state is evaluated using finite element analysis true limit states cannot be expressed in analytic form [6]. The Response Surface Method (RSM) is a recently developed technique which can provide an efficient and reasonable estimation of structural reliability regardless of the complexity of the failure process of the actual system. In this method, successive approximation of limit state around design point is performed using FORM or SORM to estimate the reliability index and probability of failure [9]. A particular disturbing feature in this method is that for some type of problems there can be instability in the probability estimate as the location of the points keep oscillating or diverging [10]. Thus to remove the shortcoming of this methods a procedure called improved Higher Order Stochastic Response Surface Method (iHOSRSM) is developed for bellow failure probability estimation. A brief description of this procedure is given below:

iHOSRSM

iHOSRSM is an improved procedure developed by author based on the Higher Order Stochastic Response Surface Method (HOSRSM) given in the ref. [7]. Although, it is found that in many cases HOSRSM give reasonable approximation of the limit state functions, yet there are cases where it gives erroneous results. Especially for the problem where order estimation of the variable is insufficient or the higher order terms are present as mix-term in limit state. For example consider the simple equation with cosine terms as given in equation-2:

$$g(X) = X_2 + \cos(2X_1) + \frac{X_1}{5} + 3 = 0 \qquad (2)$$

The approximation of limit state function using the algorithm proposed by Boucher and Bourgund [10] with second order polynomial is shown in the figure 5(a). It can be clearly seen that second order polynomial will not be able to align the true limit state curve as it is of higher order. Figure 5(b) represents the approximation using the HOSRSM with of two different random number sets. This method fails to estimate the good approximation of limit state because it incorrectly estimated the order of Chebyshev polynomial. HOSRSM is improved by authors for better order estimation as well as for mixed term estimation which is explained in next sections.

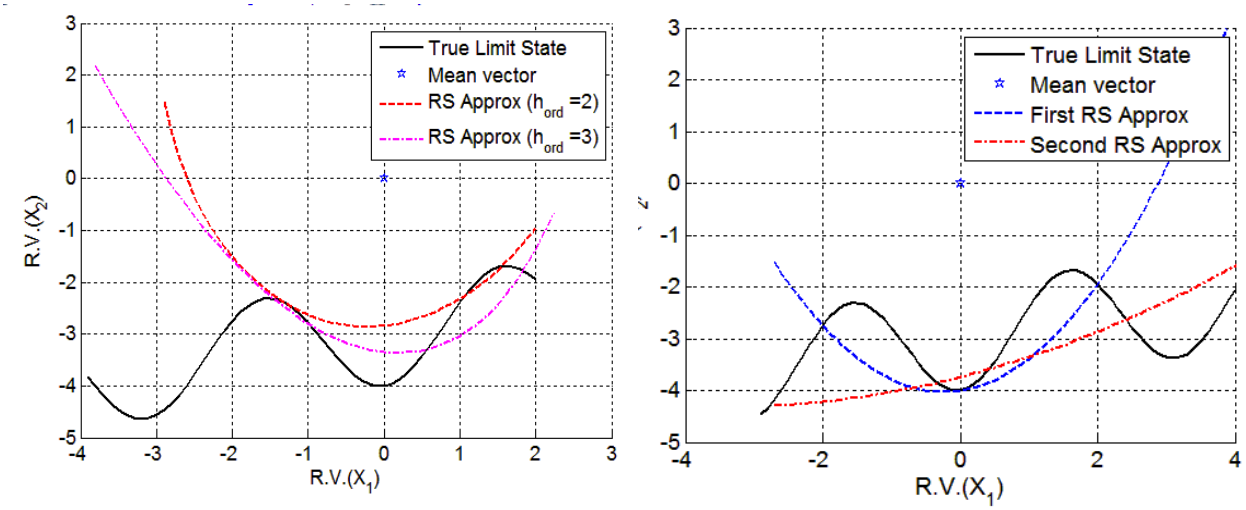

Fig. 5: Response surface approximation of equation 3.1 (a) using FORM for two successive approximation (b) using HOSRSM with two different sets of random variables

Order estimation

Algorithm of the order estimation is improved to better estimate the order of the polynomial at lower computational cost. Utilizing the discrete orthogonality conditions of Chebyshev polynomial over the discrete set of

Chebyshev nodes, the limit state is evaluate at nine nodes along each random variable axes. Since Chebyshev polynomial is orthogonal and bounded between interval [-1, 1], the coefficient matrix is compared among themselves to check their significance. Order estimation starts with first order and continues till the sixth order. If the coefficients of higher order polynomial are 0.01% of the maximum of all its lower coefficients, it is considered significant. These nodes are sufficient to test the order of the approximating limit state functions up to six orders, which can be considered as sufficient for a wide range of problem. In case a typical problem has sixth order term higher than 1% of maximum of all its lower order coefficients, then order is estimated till eighth order. Beyond that it is assumed that no problem will have significant coefficients.

Mixed term estimations:

In problems with many random variables, mix-terms (i.e. $T_{p1}(x_1)T_{p2}(x_2)..... T_{pn}(x_n)$ where p_1, p_2 and p_n are order of the polynomial $T(x_1)$, $T(x_2)$ and $T(x_3)$ respectively) increase rapidly with increasing order of polynomials. Thus a suitable procedure has to be developed to filter out the insignificant mix-terms from all possible mixed terms. To keep the number of deterministic evaluation of LS to minimum, stepwise forward selection procedure is employed for model building. The model is presumed to have all the terms, including mixed term if any, with sum of orders less than equal to minimum order, i.e. $\sum_i p_i \le \min(k_i)$. Subsequently, one term at a time is added in the model and following two criteria is employed to check the significance of added terms:

(i) The polynomial coefficients of the added term are not less than (0.1%) of maximum coefficient.

(ii) Adding of the new terms improve the Adjusted R^2 of the regression model.

It is worthwhile to mention here that Chebyshev polynomials with mix-terms are also bounded to within the range of [-1, 1], but they are not orthogonal to each other. This bound property of mix-term is used for checking their numerical significance.

Response surface approximation

Once all the terms involved for the estimation of response surface is identified, the coefficients are estimated using least squares method. Regression analysis is performed with Chebyshev polynomial as basis function instead of regular polynomial. This significantly reduces the total number of deterministic analysis involved in the procedure. In the last stage, a full scale MCS on the approximated limit state is carried out to determine the p_f. The procedure is validated with various numerical examples given in the literatures [11, 12]. The probability of failure of bellow using this method in explained in next section against limit state based on fatigue failure.

PROBABILITY OF FAILURE OF BELLOW

Limit States

Secondary nature of thermal stress will lead to the gradual accumulation of plastic strain in the bellow and will cause failure of bellow due to low cycle fatigue. ASME section-VIII division-2 has given the fatigue curve of bellow which is used for estimation of allowable number of cycles for bellow. Thus limit state of the bellow is defined as equation-3

$$g(x) = 1.0 - \frac{N_{DBE}}{N_{Allowable}} \tag{3}$$

where, N_{DBE} is number of cycles given in the design basis events of PFBR, whereas $N_{Allowable}$ is number of cycles allowed to avoid fatigue failure. $N_{Allowable}$ is estimated considering the variation in geometric and material properties of bellow as well as variation in the load or bellow. The uncertainties in these variables are explained in next section.

Uncertainties in the Variables

The bellow is made up of material SA-240, Grade TP304L alloy steel. The minimum Young modulus (E) for this material at the temperature of 475^0 C specified in the RCC-MR is 157GPa. The Coefficient Of Variation (COV) in young modulus is estimated as 0.031based on the 90% confidence bound given in the ref [11] for material 316L(N). Assuming that minimum yield stress value is given at 95 percentile, mean of young modulus is estimated as 168.4 MPa.

The RCC-MR code specified minimum and mean yield strength (S_y) at the temperature of 475^0 C is 109.9MPa and 140.6MPa respectively. Assuming that yield stress follows lognormal distribution and minimum value is given at 95 percentile, COV is estimated at 0.145. Also minimum and mean tensile strength (S_u) specified in the RCC-MR is 373.7 MPa and 415.57 MPa. Again assuming that tensile strength follows lognormal distribution and minimum value is given at 95 percentile, COV is estimated at 0.063.

Fatigue analysis of the bellow is carried out using the design S-N Curve given in ref [3]. The allowable number of cycles from this curve has a design margin of 3 of cycles and 1.25 on the stress. A mean curve for fatigue is generated from design curve by removing the design margin. The coefficient of variation in the allowable number of cycles is taken as 0.225, as there is large dispersion in fatigue data reported in the literature. Design and mean fatigue curve is given in the figure 6.

Diameter of the bellow is 840 mm and thickness of the bellow is 0.63 mm. Thickness and diameter is considered as random variables with coefficient of variation 0.03. The COV in temperature range is considered as 10%. Pressure load are very low on the bellow and treated as deterministic load in the analysis.

Table 3.1 Statistical properties of the random variables of expansion bellow

Random variables	Young Modulus (E) (GPa)	Tensile Strength (S_u) (MPa)	Fatigue Curve- F_C	Diameter- (OD) (mm)	Thickness (t) (mm)	Temperature Range (ΔT) (^{0}C)
Distribution	Lognormal	Lognormal	Lognormal	Lognormal	Lognormal	Lognormal
Mean (μ)	168.4	415.6	As per Fig. 3.3	840	0.63	325
COV (Ω)	0.031	0.145	0.063	0.03	0.03	0.10

Order of Random Variables

Orders of random variables are determined by the as described in the procedure iHOSRSM. Thirty-three limit state evaluations are performed for the estimation of the order. There is strong dependence of limit state with material properties, temperature and outer diameter, but relatively smaller dependence for thickness. Order of random variables associated with material properties, diameter, thickness and temperature estimate by procedure is 3, 6, 6, and 6 respectively. Figure 7 and Table- 2 shows the dependence of limit state function with random variables. Temperature has strong linear and quadratic dependence, whereas diameter and thickness has very weak linear dependence. Other higher order terms are less significant. They contribute not much in term of sensitivity estimation but they make good contribution in estimating the failure probability of bellow.

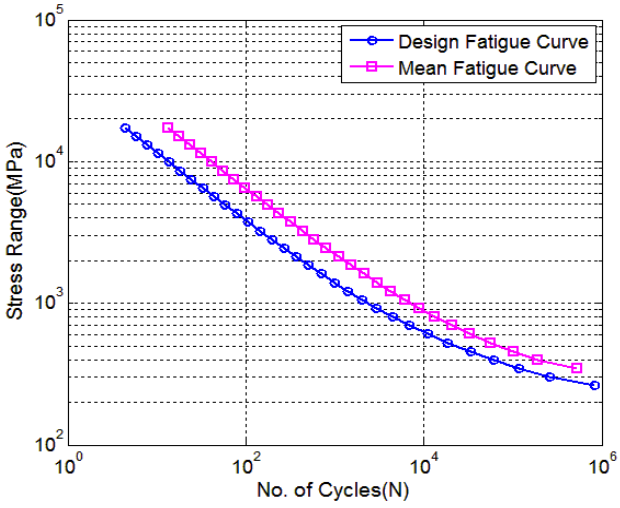

Fig. 6: Fatigue Curve for bellow material SA-240, Grade TP304L alloy steel

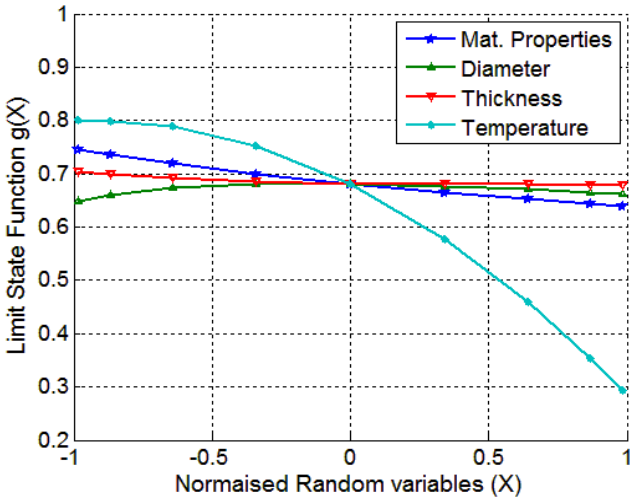

Fig. 7: Sensitivity of limit state function g(X) with random variables

Coefficient Estimation of Limit state Function

Once the polynomial order gets estimated, next step is estimation of significant mixed order terms from hundred of mix-terms. This is carried out as explained in the previous section. The limit state defined by the polynomial is shown in the Table- 3. Polynomial order has four digits, where each digit symbolizing the order of random variables. The sequence of digits on the Polynomial order is material properties, diameter, thickness, and temperature respectively.

Table 2: Coefficient of polynomial order defining sensitivity of random variables w.r.t limit state function

S.N	Polynomial Order	Mat. Properties	Diameter	Thickness	Temperature
1	T0	6.87E-01	6.69E-01	6.86E-01	6.12E-01
2	T1	5.30E-02	3.56E-03	1.14E-02	2.57E-01
3	T2	5.17E-03	1.39E-02	4.94E-03	6.97E-02
4	T3	7.31E-04	4.02E-03	2.12E-03	3.85E-05
5	T4	7.31E-06	1.01E-03	5.97E-05	1.67E-04
6	T5	2.92E-05	2.38E-04	3.04E-04	1.09E-04
7	T6	1.03E-17	7.78E-05	1.33E-04	2.44E-04

Probability of failure

Probability of failure of expansion bellow is estimated using fatigue failure criteria. The evaluation of limit state defined by equation 3 gives probability of failure as 1.53E-06. The number of limit state evaluation i.e. number of deterministic finite element analysis required for failure probability estimation is 103. This is the average value of five different stochastic set of variables given in Table-4. The nearness of the failure probability for various set of variables shows the robustness of the procedure. Also the procedure is highly computationally efficient as Monte Carlo simulation will take minimum of 10^8 number of simulation for same level of approximation of failure probability. The failure probability is of the order of 10^{-6}, which indicated that the expansion bellow will maintain the structural integrity with high reliability at the normal operating condition. In the future work, module will be added to estimate the reliability of component by accounting operating condition as well as accidental loads.

Table-3: Significant coefficients of Chebyshev polynomial for limit state approximation

S.N	Poly. Order	Coeffic ients		S.N	Poly. Order	Coeffic ients
1	0000	0.5640		**7**	0101	0.0266
2	1001	0.2037		**8**	3001	0.0250
3	1000	0.1472		**9**	3000	0.0165
4	0100	0.0551		**10**	1013	0.0156
5	1110	0.0485		**11**	1130	0.0151
6	2101	0.0340		**12**	3110	0.0140

Table 4: Probability of failure of expansion joint by fatigue under NOC

S.N	**iHOSRSM**	
	Probability of Failure(P_s)	*No. of Iterations*
1	1.81E-6	97
2	1.03E-6	116
3	1.75E-06	88
4	1.497E-6	102
5	1.55E-06	111
Average	**1.53e-06**	**103**

CONCLUSION

Expansion bellows at the containment penetrations are an integral part of the pressure boundary in PFBR, where they are used as soft connection to mitigate the effects of differential movements between the penetrating pipe and containment shell. The expansion bellow at RCB penetration of secondary sodium line of PFBR experience high axial and lateral duty at normal operating condition. Deterministic analysis of bellow shows that most probable failure mode is low cycle fatigue. The probability of failure of bellows is estimated using the method based on higher order response surface method.

The proposed procedure, iHOSRSM, first accurately estimates the order of the Chebyshev polynomial required for limit state approximation. Subsequently significant mixed order terms are selected based on their numerical significance and its contribution to adjusted R^2 of the regression model. Check for significant mixed order

terms reduce the total number of terms in the approximating polynomial, thus save the unnecessary computation work. The procedure is integrated with a FEA code ANSYS for failure probability estimation.

Failure probability is estimated against low cycle fatigue failure criteria, which has strong dependence on material properties and temperature variations, but less significant dependence on geometric properties like thickness and diameter. Probability of failure of expansion bellow estimated by iHOSRSM, which required only103 number of deterministic evaluation, is 1.53e-6. The failure probability is of the order of 10^{-6}, which indicated that the expansion bellow will maintain the structural integrity with high reliability at the normal operating condition.

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