The Determination of Dymanic Stress Intensity Factors of Multiple Craks in a Layered Medium

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ABSTRACT

An analysis of scattering of elastic waves in a layered medium with multiple non–coplanar interface Griffith cracks is presented in this paper. Making use of Fourier integral transform, the problem is reduced to a set of dual integral equations which can be reduced to a set of singular integral equations and then solved by numerical methods. As an example, the case of two layered half–space with two cracks has been inverstigated in more detail. The features of the scattered waves at the crack–tips are discussed. Finally, the numerical values of the dynamic stress intensity factors are plotted versus the frequency of the incident waves.

1. INTRODUCTION

The problem of elastic wave scattering in a media with multi–cracks is difficult to deal with mathematically, but duo to its practical significance, more and more anthors pay attention to this subject now. Itou 1980 has first considered the diffraction of an antiplane shear wave in an infinite elastic medium with two coplanar Griffith cracks. Zhang et al. 1988 using the Green function and integral equation methods, have analysed the scattering of elastic waves due to multi–cracks and its feature at the far–fields. For the case of multiple interface cracks, Karim and Kundu 1988 have considered the transient surface response of SH wave in a layered half space with two interface cracks and the dynamic interaction of SH waves with two interface cracks in a three layered plate.

This paper is concerned with the scattering of SH waves in a layered medium with multiple non–coplanar interface cracks. Using Fourier transform, the problem is reduced to a set of dual integral equations. Then take the case of two layered half–space with two cracks as an example, the feature of the scattered waves at the crack tips has been investigated in more detail. And finally, the numerical results and discussions are presented.

2. DUAL INTEGRAL EQUATIONS

Suppose n layeres and a half–space are bonded together perfectly, except in the regions $|X| < 1$, $y = d_{j(i)} (i = 1,2 \cdots K)$, where the interface Griffith cracks exist as shown in Fig.1. λ_j, μ_j, $\rho_j (j = 0,1,2 \cdots n)$ are the constants of materials. The interface between the j th and j–1 th layeres is represented by j.

When the incident waves are harmonic SH waves, Let ω be its frequency. The governing equations in displacements is

$$\nabla^2 U_{zj} + K_{Tj}^2 U_{zj} = 0 \quad j = 0,1,2 \cdots n \quad (2-1)$$

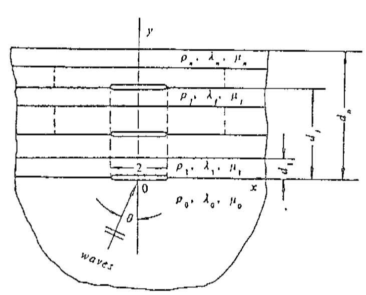

Fig.1. The mechanic model

where $K_{Tj}^2 = \omega^2 \rho_j / \mu_j$, $\nabla^2 = \partial^2/\partial X^2 + \partial^2/\partial y^2$.

The relations between stresses and displacements are

$$\sigma_{xzj} = \mu_j \frac{\partial u_{zj}}{\partial x}, \quad \sigma_{yzj} = \mu_j \frac{\partial u_{zj}}{\partial y} \qquad j = 0,1,2\cdots n \tag{2-2}$$

The elastic fields can be expressed in the sum of the incident field and scattered field as follows

$$\{U^{(t)}, \sigma^{(t)}\} = \{U^{(i)}, \sigma^{(i)}\} + \{U^{(s)}, \sigma^{(s)}\}$$

where $\{U^{(i)}, \sigma^{(i)}\}$ represents the incident field with no cracks, $\{U^{(s)}, \sigma^{(s)}\}$ the scattered field and $\{U^{(t)}, \sigma^{(t)}\}$ the total field.

Defining the openning displacement functious of the crack surfaces as:

$$\triangle U_{zj(i)} = \begin{cases} U_{zj(i)} - U_{zj(i)-1} & |X| < 1 \\ 0 & |X| > 1 \end{cases} \qquad y = d_{j(i)-1}, \quad i = 1,2\cdots K \tag{2-3}$$

The boundary and coutinuity conditions of the scattered field are expressed as

$$\begin{aligned} &\sigma_{yzn}^{(s)} = 0 && y = d_n, \ |X| < \infty \\ &\sigma_{yzj}^{(s)} = \sigma_{yzj-1}^{(s)} = -\sigma_{yzj-1}^{(i)} = -\sigma_{yzj}^{(i)} && y = d_j, j \in j(i), \quad |X| < 1 \\ &\sigma_{yzj}^{(s)} - \sigma_{yzj-1}^{(s)} = 0, \quad U_{zj}^{(s)} - U_{zj-1}^{(s)} = \triangle U_{zj(i)} \delta_{j(i),j} && y = d_j, \quad j = 1,2\cdots n, |X| < \infty \end{aligned} \tag{2-4}$$

In addition, the scatered field must satisfy the following radiation condition

$$\lim_{r\to\infty} r^{\frac{1}{2}} \left\{ \frac{\partial U_o^{(s)}}{\partial r} - i|K| U_o^{(s)} \right\} = 0 \tag{2-5}$$

where $r = (x^2 + y^2)^{1/2}$, $|K|$ is the wave number vector modulus.

For convenience, we omit the superscripts of the scattered field in what follows.

Applying Fourier integral transfonm to the equation (2–1)and taking account of the radiation condition (2–5), we have

$$U_{zj} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[A_j(\xi) e^{i\gamma_{Tj} y} + B_j(\xi) e^{-i\gamma_{Tj} y} \right] e^{-i\xi x} d\xi \qquad j = 0,1,2\cdots \tag{2-6}$$

where $\gamma_{Tj} = \left(K_{Tj}^2 - \xi^2 \right)^{1/2}$, $A_j(\xi).B_j(\xi)$ are unknown functions. $A_o(\xi) \equiv 0$.

Substituting (2–6) into the boundary and continuity condition, one obtains the following results

$$[A_j] = \sum_{J=j(i)}^{j(k)} [L_{jl} + K_{jl} H(i - j - 1)][S_j] \tag{2-7}$$

where $[L_{jl}]$、 $[K_{jl}]$ are known matrixes. H(i–j–1) is a Heavside function,

$$[A_j] = [A_j(\xi), B_j(\xi)]^T, [S_j] = [0, S_j]^T, S_j = \int_{-\infty}^{\infty} \triangle U_{zj(i)} e^{i\xi x} dx \delta_{j(i)j}.$$

Then substituting (2–7) into (2–6), from the surface condition of the cracks and the definition of the openning displacement functions, we have

$$\left. \begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{j=j(i)}^{j(k)} [F_{jl}][S_j] e^{-i\xi x} d\xi = -\sigma_{yz}^{(i)} && |x| < 1 \\ &\frac{1}{2\pi} \int_{-\infty}^{\infty} [S_j] e^{-i\xi x} d\xi = 0 && |x| > 1 \end{aligned} \right\} \qquad j = j(i)\cdots j(k) \tag{2-8}$$

where $[F_{jj}]$ is a known matrix.

The equations (2–8) are a set of dual integral equations for the scattering of SH waves in a layered medium with multiple non–coplanar Griffith cracks.

3. THE SINGULAR INTEGRAL EQUATIONS AND ITS SOLUTION

As an example, the case of two layered half–space with double Griffith cracks has been investigated here in more detail.

Let n = 2, j(1) = 1, j(2) = 2, then the equation (2–8) can be simplified as

$$\left.\begin{aligned} &\frac{i\mu_1}{2\pi}\int_{-\infty}^{\infty}\{P_1(\xi)S_1(\xi)+P_2(\xi)S_2(\xi)\}\frac{e^{-i\xi x}}{\triangle(\xi)}d\xi = -\sigma_{yz1}^{(i)}(x,d_1)\\ &\frac{i\mu_1}{2\pi}\int_{-\infty}^{\infty}\{P_3(\xi)S_1(\xi)+P_4(\xi)S_2(\xi)\}\frac{e^{-i\xi x}}{\triangle(\xi)}d\xi = -\sigma_{yz}^{(i)}(x,0)\end{aligned}\right\} \quad |x|<1$$

$$\left.\begin{aligned}&\frac{1}{2\pi}\int_{-\infty}^{\infty}S_1(\xi)e^{-i\xi x}d\xi=0\\ &\frac{1}{2\pi}\int_{-\infty}^{\infty}S_2(\xi)e^{-i\xi x}d\xi=0\end{aligned}\right\} \quad |X|>1 \qquad (3\text{–}1)$$

Where $P_1(\xi), P_2(\xi), P_3(\xi), P_4(\xi)$ and $\triangle(\xi)$ are known functions of ξ. $\sigma_{yz1}^{(i)}(x,d_1), \sigma_{yz1}^{(i)}(x,0)$ are the incident stresses on the faces of the cracks.

Using $R_1(x)$, $R_2(x)$, instead of $\triangle U_{z1}$ and $\triangle U_{z2}$, it can be seen that

$$S_1(\xi)=\int_{-1}^{1}R_1(u)e^{i\xi u}du, \qquad S_2(\xi)=\int_{-1}^{1}R_2(u)e^{i\xi u}du \qquad (3-2)$$

substituting (3–2)into (3–1). it can be reduced to a set of singular integral equations of first kind with Canchy kernel.

$$\left.\begin{aligned}&\int_{-1}^{1}\frac{f_1(u)}{u-x}du-\frac{1}{i\beta}\int_{-1}^{1}[N(u,x)f_1(u)+T(u,x)f_2(u)]du=\frac{\pi}{\mu_1\beta}\sigma_{yz1}^{(i)}(x,0)\\ &\int_{-1}^{1}\frac{F_2(u)}{u-x}du+\frac{1}{i\alpha}\int_{-1}^{1}[L(u,x)f_1(u)+M(u,x)f_2(u)]du=-\frac{\pi}{\mu_1\alpha}\sigma_{yz1}^{(i)}(x,d_1)\end{aligned}\right\} \quad |x|<1 \qquad (3\text{–}3)$$

where $f_1(u)=R_1'(u)$, $f_2(u)=R_2'(u)$, α, β are know constants, N(u,x), T(u,x), L(u,x), M(u,x) are Fredholm kernal.

Then expanding $f_1(u)$, $f_2(u)$ into a series of Chebyshev polynomials in the form:

$$f_1(u)=(1-u^2)^{-1/2}\sum_{j=0}^{\infty}A_{1j}T_j(u) \qquad f_2(u)=(1-u_2)^{-1/2}\sum_{j=0}^{\infty}A_{2j}T_u(u) \qquad (3-4)$$

and using the orthogonality of Chebyshev polynomials. the singular integral equation can be reduced to a set of algebraic equations:

$$\begin{aligned}&\frac{\pi}{2}A_{1k+1}+\sum_{j=1}^{\infty}[a_{jk}A_{1j}+b_{jk}A_{2j}]=q_{1k}\\ &\frac{\pi}{2}A_{2k+1}+\sum_{j=1}^{\infty}[C_{jk}A_{1j}+d_{jk}A_{2j}]=q_{2k}\end{aligned} \qquad k=0,1,2\cdots \qquad (3\text{–}5)$$

where:

$$\begin{aligned}&q_{1k}=\frac{1}{\mu_1\beta}\int_{-1}^{1}\sigma_{yz1}^{(i)}(x,0)(1-x^2)^{1/2}U_k(x)dx\\ &q_{2n}=-\frac{1}{\mu_1\alpha}\int_{-1}^{1}\sigma_{yz}^{(i)}(x,d_1)(1-x^2)^{1/2}U_k(x)dx\\ &a_{jk}=\frac{i}{\pi\beta}\int_{-1}^{1}\int_{-1}^{1}(1-u^2)^{-1/2}(1-x^2)^{1/2}N(u,x)T_j(u)U_k(x)dudx\\ &b_{jk}=\frac{i}{\pi\beta}\int_{-1}^{1}\int_{-1}^{1}(u-u^2)^{-1/2}(1-x^2)^{1/2}T(u,x)T_j(u)U_k(x)dudx\\ &C_{jk}=\frac{1}{i\pi\alpha}\int_{-1}^{1}\int_{-1}^{1}(1-u^2)^{-1/2}(1-x^2)^{1/2}L(u,x)T_j(u)U_k(x)dudx\end{aligned} \qquad (3-6)$$

$$d_k = \frac{1}{i\pi\alpha}\int_{-1}^{1}\int_{-1}^{1}(1-u^2)^{-1/2}(1-x^2)^{1/2}M(u,x)T_j(u)H_k(x)dudx$$

$T_j(u)$and $U_k(x)$are the first and second kind of Chebyshev polynomials respectively. From (3–5) the unknown constants A_{1j}、A_{2j}($j=1,2\cdots$)can be determined.

4. THE STRESS INTENSITY FACTORS AN ITS NUMERICAL RESULTS

Now we derive the expnessions of the stress intensity factors which is of particular interest in fracture mechanics.

Defining the dynamic stress intersity factor as

$$K_{31} = \lim_{x\to 1^+}\left|(x^2-1)^{1/2}\sigma_{yz}(x,0)\right|, \qquad K_{32} = \lim_{x\to 1^+}\left|(x^2-1)^{1/2}\sigma_{yz}(x,d_1)\right| \quad |x|>1 \qquad (4-1)$$

From (3–3), it is known that the dominat pants of the scattered stress near the crack tip at the interfaces are

$$\sigma_{yz}(x,0) = -\frac{\mu_1\beta}{\pi}\int_{-1}^{1}\frac{f_1(u)}{u-x}du, \qquad \sigma_{yz}(x,d_1) = \frac{\mu_1\alpha}{\pi}\int_{-1}^{1}\frac{f_2(u)}{u-x}du \qquad (4-2)$$

substitutinng (4–2) into (4–1) and with (3–4) one can obtuin

$$K_{31} = \left|-\frac{\mu_0\mu_1}{\mu_0+\mu_1}\sum_{j=1}^{\infty}A_{1j}\right|, \qquad K_{32} = \left|\frac{\mu_1^2\mu_2}{(\mu_0+\mu_1)(\mu_1+\mu_2)}\sum_{j=1}^{\infty}A_{2j}\right| \qquad (4-3)$$

The formula (4–3) indicates the dynamic feature of the scattered field at the tips of the cracks. For the case of bonded material with the half–space, middle layer and top layer being iron、aluminium and rickel respectively, its variation with the frequency of the incident wave are shown in Fig.2 and Fig.3 where $\Omega = \omega d_1 / C_{To}$

The Fig.2 shows the stress intensity factors versus the frequency of the incident wave with the incident angle $\theta=0$. Obviously, there are several resonance peaks in some range of frequencies, which increase as the thickness of the top layer increase, this is because that the interference of two scattered fields of the two cracks will intensified as the relative distance decreases. In addition, the magnitude of K_{32} is much larger then that of K_{31}. From Fig.3 it can be seen that, with the increament of the incident angle, the magnitudes of K_{31} and K_{32} become small.

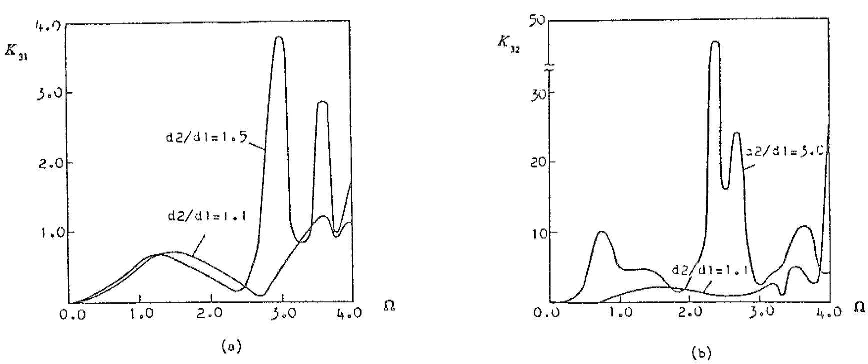

Fig.2 Variation of the SIF versus the frequency of incident waves

with incident angle $\theta=0$, (a) K_{31}, (b) K_{32}.

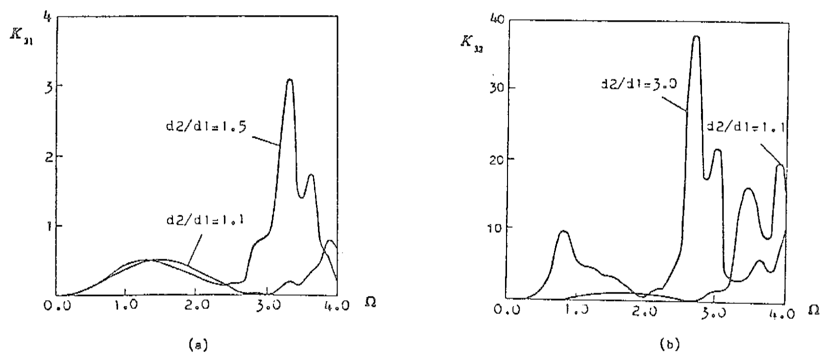

Fig.3 Variation of the SIF versus the frequency of the incident waves

with incident angle $\theta = 30°$, (a) K_{31}, (b) K_{32}。

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