# On Thermomechanical Behaviour of Irradiated Fuel Sheath

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ABSTRACT

The present paper deals with the use of a thermoelastic-plastic finite element analysis for the prediction of the behaviour of fuel sheaths which exhibit "stain softening" in their material characteristic due to excessive neutron irradiation. It has been demonstrated numerically that the presence of the strain softening effect may cause up to 75% more additional deformation in the irradiated fuel sheath than that in the unirradiated state.

## 1. Introduction

In responding to the ever increasing demand for safe nuclear reactor operations, several sophisticated computer codes have been developed for the accurate prediction of the thermomechanical behaviour of key reactor core components during both normal operating and LOCA conditions. Computer codes such as ANSYS [1], SAFE-2D [2], TEPSA [3,4] and FULMOD/FAXMOD [5] were all developed on the basis of a multi-dimensional finite element principle and are capable of handling components of complex geometries and loading and boundary conditions. However, the accuracy of the code prediction relies strongly on the accurate inputs used for the computations. The most critical information required for the finite element analysis is the thermophysical and mechanical properties of the component materials. Much effort in material research has been made by various establishments for that purpose. Unfortunately, little is known about the behaviour of some components subjected to intense neutron irradiation. Virtually no attempt has been made by analysts on the thermomechanical behaviour of key reactor components, such as the fuel sheath, after subjecting to an extended period of radiation, probably because of the scarcity of material properties of irradiated Zircaloys.

It has been known for some time that excessive neutron irradiation in Zircaloys causes an increase in its yield strength and enhances anisotropy in the material [6]. Radical changes in the fracture and creep behaviour of the material also have been reported [7, 8]. However, one important phenomenon which may cause grave concern to analysts is the "strain softening" trend observed in the material constitutive relation as shown in Fig. 1, which was extracted from Ref. [6]. Coleman et al [9] showed that under certain conditions, the "strain softening" trend in the irradiated Zircaloy-4 fuel sheath at 300°C could be reversed to the "strain hardening" with additional load as illustrated in Fig. 2.

Implementation of the strain softening behaviour of the material into the elastic-plastic analysis induced the numerical instability and has been purposely neglected in the past. In the present analysis, the TEPSA code which adopts the incremental plasticity theory and the variable stiffness matrix concept originally proposed by Yamada et. al [10], can handle strain softening effect with little modification. Results from a case study on the deformation of a fuel canister comprised of a strain softening material subject to internal pressure indicated that the irradiated fuel sheath may deform a great deal more than the undamaged fuel material. The criticality of the radiation history of the fuel sheath in the assessments of LOCA and similar situations have been demonstrated by the outcome of this study.

## 2. Theoretical Formulations of Thermoelastic-plastic Analysis

The present analysis utilizes a generalized hardening rule and flow law to incorporate both temperature and strain rate dependent material properties. The yield surfaces which comply with von Mises' yield criterion may expand or contract as temperature and strain rate vary. As usual, the inertia effect is neglected and thermal and stress fields are assumed to be uncoupled, but both are made dependent upon the current deformed geometry and are solved quasi-simultaneously. The principle of incremental stationarity of potential energy is used with simplex elments giving in each

loading step:

$$\{\Delta Q_\ell\} = [K_\ell]\{\Delta u_\ell\} \tag{1}$$

where $\{\Delta Q_\ell\}$ = incremental load vectors,

$[K_\ell]$ = instantaneous stiffness matrix of the material,

$\{\Delta u_\ell\}$ = deformation vectors.

Thus by referring to the detailed derivations described in Ref. [3] the constitutive equations for the structure is expressed as:

$$\{d\sigma\} = [Cep]\{d\varepsilon\} - [Cep]\left(\{\alpha\}\Delta T + \frac{\partial [Ce]^{-1}}{\partial T}\{\sigma\}\Delta T + \frac{\partial [Ce]^{-1}}{\partial \dot{\bar{\varepsilon}}}\{\sigma\}\Delta\dot{\bar{\varepsilon}}\right.$$

$$\left. - [Ce]\left\{\frac{\partial F}{\partial \sigma}\right\}/\left[S\left(\frac{\partial F}{\partial T}\Delta T + \frac{\partial F}{\partial \dot{\bar{\varepsilon}}}\Delta\dot{\bar{\varepsilon}}\right)\right]\right) \tag{2}$$

where $\{d\sigma\}$, $\{d\varepsilon\}$ are the respective incremental stresses and strains; $\{\alpha\}$ is the thermal expansion coefficient matrix; $\Delta T$ and $\Delta\dot{\bar{\varepsilon}}$ are the respective temperature and effective strain rate.

The plastic potential function F is defined by the von Mises criterion as:

$$\left\{\frac{\partial F}{\partial \sigma'}\right\} = \{\sigma'\}$$

with $\{\sigma'\}$ to be the deviatoric stress components.

The elastic-plastic matrix [Cep] is:

$$[Cep] = [Ce] - [Cp]$$

where [ce] = elasticity matrix, or the coefficients of strain components in a generalized Hooke's law of stress and strain; the plasticity matrix, [Cp] can be expressed as:

$$[Cp] = \frac{[Ce]\{\sigma\}\{\sigma\}^T[Ce]}{S}$$

in which $S = 3G + H'$, with G = shear modulus of elasticity and H' is the slope of the effective stress $\bar{\sigma}$ vs. effective strain $\bar{\varepsilon}$ relation.

The stiffness matrix $[K_\ell]$ in Eq. (1) can be determined by the usual variational procedures identical to the elasticity analysis, or:

$$[K_\ell] = \int_v [B]^T[Cep][B]\, dv$$

where [B] is the matrix relating the element strain and displacement components.
v is the volume of the element.

The slope H' of the effective stress vs. effective plastic strain curve can be obtained by [11]:

$$H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}_p} = \frac{1}{1/\frac{d\bar{\sigma}}{d\bar{\varepsilon}} - 1/E} \tag{3}$$

which requires the evaluation of the derivative of the $\bar{\sigma}$ vs. $\bar{\varepsilon}$ curve of the material. For strain hardening materials, the experimentally determined uni-axial stress $\sigma$ vs. uni-axial strain $\varepsilon$ curve could be converted into $\bar{\sigma}$ vs. $\bar{\varepsilon}$ curves following the procedures specified in [12], and can be described by the following function [11]:

$$\bar{\sigma} = \frac{\bar{E}\bar{\varepsilon}}{\left\{1 + \left[\frac{\bar{E}\bar{\varepsilon}}{(1 - \bar{E}'/\bar{E})\sigma_k + \bar{E}'\bar{\varepsilon}}\right]^n\right\}^{1/n}} \tag{4}$$

in which $\bar{E} = 3E/2(1+\nu)$

$\bar{E}' = 3E'/[3 - (1-2\nu)E'/E]$

$E$, $\nu$ = respective modulus of elasticity and Poisson's ratio of the material

$E'$ = plastic modulus

$\sigma_k$ = auxiliary stress close to elastic to plastic transition

$n$ = parameter determining the abruptness of the transition.

For materials exhibiting the strain softening behaviour represented, for instance, by the $\sigma$ vs. $\varepsilon$ curves in Figs. 1 and 2, the function in Eq. (4) could be used with the following modification:

$$\bar{\sigma} = \sum_{i=1}^{M} C_i \bar{\sigma}_i \left(\bar{E}_i, \bar{E}_i', \sigma_{k,i}, n_i\right) \tag{5}$$

where $\bar{\sigma}_i$ take the form similar to Eq. (4). M is an odd number with $C_i$ being sign switches (e.g. $\pm 1$). The material parameters in Eq. (5) can be determined from the specific shape of the $\sigma$ vs. $\varepsilon$ curve.

## 3. Numerical Illustration

An elastic-plastic analysis was performed on a Zircaloy-4 tube which has a stress vs. strain curve as shown by the solid line in Fig. 2. Fig. 3 shows the dimensions and finite element idealization. The tube is subjected to internal pressure as illustrated in the same figure. The constitutive law which describes the curve shown in Fig. 2 can be expressed by Eqs. (4) and (5) with the following substitution of the material parameters:

$C_1 = C_2 = 1$ ; $C_3 = -1$ ; $E_1 = 37.85$ GPa ; $E_2 = E_3 = 5.17$ GPa ;
$E_1' = E_2' = E_3' = 1.20$ GPa ; $\sigma_{k1} = 351.64$ MPa ; $\sigma_{k2} = 31.03$ MPa ;
$\sigma_{k3} = 55.16$ MPa ; $n_1 = n_2 = n_3 = 100$.

This constitutive relation was included in the TEPSA code for the subsequent elasto-plastic analysis.

Fig. 4 shows the radial displacement at the half length of the fuel sheath at various internal pressures. The variation of the radial displacement along the length of the fuel sheath is depicted in Fig. 5. In all cases, the solutions obtained from the unirradiated condition (represented by the dotted line in Fig. 2) are presented for comparison.

## 4. Discussion

Result obtained from the above numerical illustration indicates that the irradiated Zircaloy-4 fuel sheath indeed deforms a great deal more than the unirradiated sheath. Fig. 4 shows a 75% increase in the maximum radial displacement due to the portion of strain softening in the $\sigma$ vs. $\varepsilon$ curve for the material shown in Fig. 2. The sequence of the region in the sheath that underwent strain softening during the loading process is illustrated in the upper portion of Fig. 5. From this figure it is observed that the strain hardening-strain softening-strain hardening cycle initiates at the mid-section of the canister and spreads with increasing pressure towards the ends. The inflection point, which shows the discrepancy of displacement slope due to the strain softening region, disappears gradually as the strain softening region approaches the end cap of the canister.

Although the TEPSA code can handle the strain softening behaviour of the material with a minimal amount of modification, it was found that small values of load increments had to be used in order to maintain gradual change of H' and the numerical stability. The mathematical expression in Eq. (2) indicates that numerical instability will take place when the slope of the strain softening portion designated by -H' approaches the value of 3G of the material. A new algorithm must be developed to handle this situation.

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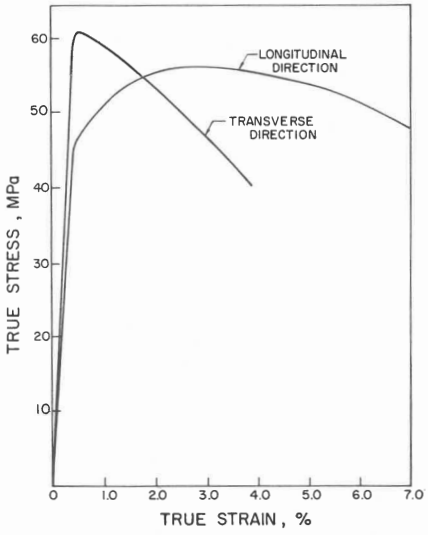


Fig. 1. Stress vs Strain Curves for an Irradiated Zircaloy - 2. (Translated from Ref. [6]).

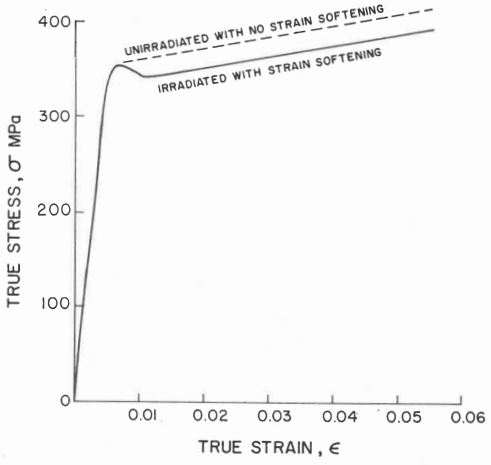


Fig. 2. Stress vs Strain Curve for Irradiated Zircaloy - 4 at 300°C.

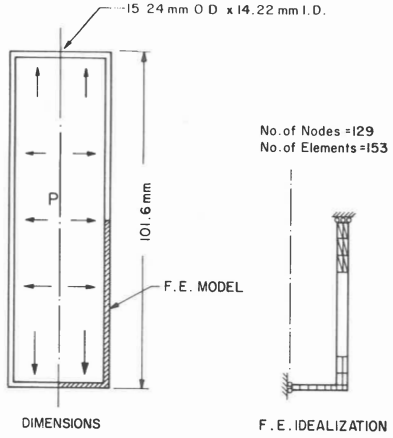


Fig. 3. Finite Element Idealization of an Irradiated Fuel Canister.

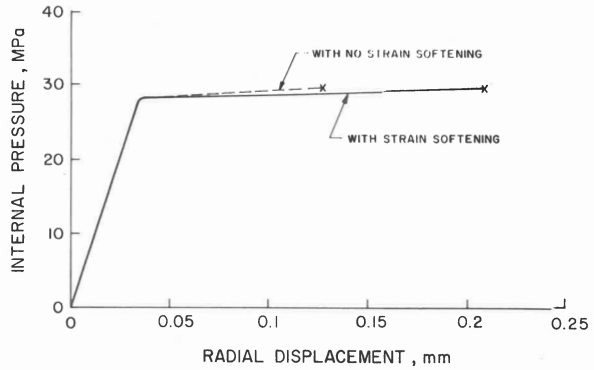


Fig. 4. Load-Displacement for Fuel Sheaths With and Without Strain Softening.

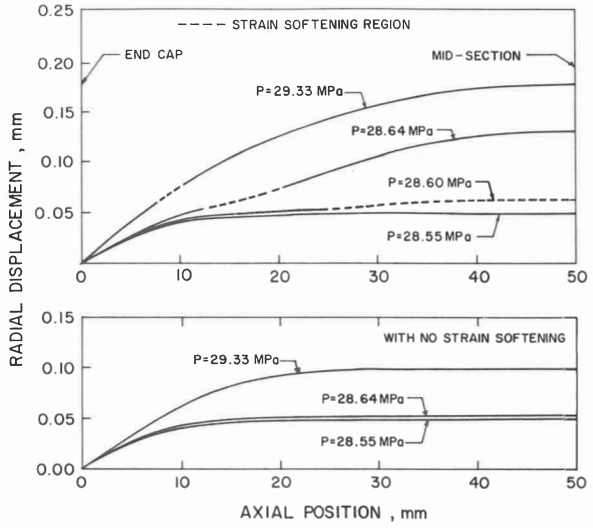


Fig. 5. Profiles of Deformed Fuel Sheaths With and Without Strain Softening.