

Measurement Errors by Gages Integration Effect

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ABSTRACT

The paper studies the error magnitudes, due to integration effect, for a strain gage installed tangent to the hole of a thin disc in uniformly rotation as steam discs, electric ventilator, electric motor, etc.

The authors proposed a new calculus method based on the integration on the all filaments length which allows a comparative study between the different strain gages types.

INTRODUCTION

In practical experimental stress analysis, a common objective is to determine the maximum stresses existing in the structures or machine elements. In many cases the stresses will be maximum at points of stress concentration. But in these area the peak strain is highly localized.

It is known that every strain gage has a finite grid area, and produces an output corresponding approximately to the average strain under the grid. The error is one of under-measuring the peak strain and can easily be in the range of 20-30 percent for commonly encountered strain measurement tasks (Perry, 1982). That is the integration effect (or tendency) of the strain gage. In order to calculate the errors introduced by the integrating tendency, two methods have been suggested (Soete, Van Crombrugge, 1950) and (Boiten, Ten Cate, 1952). Although both methods have been used in the case of determining the residual stresses by the hole drilling method, they can not be limited to this case. The W. Soete and R. Van Crombrugge method presumes that the strain of all the filaments is identical. This is true for a uniaxial state of stress and that is why it does not give good results in the case of a biaxial state of stresses. The suggested formula can be written as follows:

$$\bar{\epsilon}_L = \frac{1}{R_2 - R_1} \int_{R_1}^{R_2} \epsilon_L \cdot dl \quad (1)$$

where

ϵ_L is the longitudinal strain, parallel to grid lines of strain

$\bar{\epsilon}_L$ gage;

$\bar{\epsilon}_L$ is the output of the strain gage;

$R_2 - R_1 = L$, is active grid length.

The R.G.Boiten and W.Ten Cate method presumes that the whole surface of the grid is sensitive, not only the surface occupied by the filaments. The suggested formula can be written:

$$\bar{\epsilon}_L = \frac{1}{S} \int_S \epsilon_L \cdot dS$$

where

S is the active grid surface.

This method gives quite good results for a biaxial stresses.

However, it presents following disadvantages:

- it does not take into account the real geometry of the grid, which is supposed to be sensitive in any point of the surface;
- the integration of the strain function on the surface is difficult;

-it does not allow a clear comparison between the strain gages with different geometries of the grid. One can compare strain gages with same electric resistance, but the surface of the grid does not give any clear indication in this respect.

That is why the authors of the present paper have suggested several methods of calculus in order to remove these disadvantages (Gafitanu, Barsanescu, Poterasu, 1988). It has been demonstrated that the signal given by strain gage is the average of the signals given by the (n) filaments:

$$\bar{\epsilon}_L = \frac{1}{n} \sum_{i=1}^n \epsilon_{Li} \quad (2)$$

After that, the method of integration on the length of all the filaments is presented, method which will applied in the studied case.

The method consist in calculating, using the W.Soete and R.Van Crombrugge method, the output given by each of the filaments, and then the output given the strain gage, according to the formula (2). In the case of a strain gage whose grid is tangent to the hole in a thin plate which is subjected to uniformly distributed uniaxial stress, it has been shown that this method gives more precise results than that suggested by R.G.Boiten and W.Ten Cate (Gafitanu, Barsanescu, Poterasu, 1988).

THIN DISC IN UNIFORMLY ROTATION

Assume that we wish to measure the maximum strain at the edge of a hole in a thin disc in uniformly rotation.

It is known that a surface element dA taken from a rotating disc is subjected to stress in two directions by the stresses σ_r , σ_t and centrifugal force dF_c (see Fig. 1).

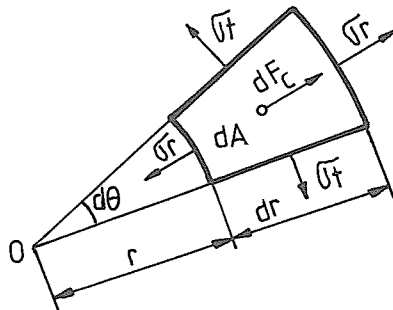


Fig. 1. A surface element taken from a disc.

For a disc with a radius R_2 having a hole with radius R_1 , made in a material with specific weight γ and the Poisson coefficient ν , stresses σ_r and σ_t can be written as follows:

$$\sigma_r = \frac{(3+\nu)\gamma R_2^2 \omega^2}{8g} \cdot \left(\frac{R_1^2}{R_2^2} + 1 - \frac{R_1^2}{r^2} - \frac{r^2}{R_2^2} \right) \quad (3)$$

$$\sigma_t = \frac{(3+\nu)\gamma R_1^2 \omega^2}{8g} \cdot \left(\frac{R_1^2}{R_2^2} + 1 - \frac{R_1^2}{r^2} - \frac{1+3\nu}{3+\nu} \cdot \frac{r^2}{R_2^2} \right)$$

where ω is the angular speed of the disc. The highest value of σ_t lies on the inside of the hole ($r=R_1$), and it is:

$$\sigma_{t \max} = \frac{\gamma \omega^2}{4g} \cdot [(3+\nu)R_2^2 + (1-\nu)R_1^2] \quad (4)$$

In Fig.2, the variation of these stresses is shown. It is known that the dangerous state of stresses lies on the inside of the hole ($\sigma_t = \sigma_{t \max}$, $\sigma_r = 0$). This is demonstrated by the fact that the rotating discs break due to a crack starting from the inside of the hole and spreading radially (the crack is therefore the finding of $\sigma_{t \max}$ - see Fig.2). It is therefore the finding of $\sigma_{t \max}$ that counts in practical experimental stress analysis.

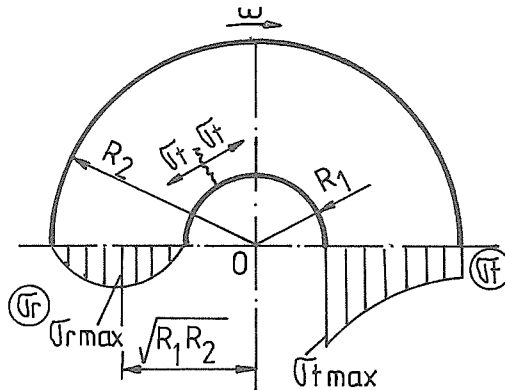


Fig.2. The variation of stresses for a half disc.

Because the strain gage responds to strain rather than stress, we will need to know the strain distribution in the area near the hole in order to assess the measurement error.

If (u) is the radial displacement we can write:

$$\epsilon_r = \frac{du}{dr} \quad (5)$$

$$\epsilon_t = \frac{u}{r}$$

with

$$u = Ar + \frac{B}{r} - (1-\nu^2)Cr^3$$

and

$$\frac{du}{dr} = A - \frac{B}{r^2} - (1-\nu^2) \cdot 3Cr^2$$

where

$$A = (1-\nu) \cdot (3+\nu) \cdot (R_1^2 + R_2^2)C$$

$$B = (1+\nu) \cdot (3+\nu) \cdot R_1^2 \cdot R_2^2 \cdot C$$

$$C = \frac{\gamma \omega^2}{8gE}$$

However, the primary sensing direction of the strain gage is parallel to the grid lines, and this direction is, in general, neither radial nor tangential (Perry, 1982). The strain expressions is needed to describe the strain parallel to the grid lines. For this purpose, we apply the transformation relationships for the strain distribution at a point:

$$\epsilon_L = \frac{\epsilon_t + \epsilon_r}{2} + \frac{\epsilon_t - \epsilon_r}{2} \cdot \cos 2\theta + \frac{\gamma_{rt}}{2} \sin 2\theta$$

When passing in the Cartesian co-ordinates, for the filament (i) situated at the distance x_i from the centre of the disc, we get (see Fig.3):

$$\epsilon_{Li}(y) = C(3+\nu) \cdot \left[(1-\nu) \cdot (R_1^2 + R_2^2) + (1+\nu) R_1^2 R_2^2 \cdot \frac{x_i^2 - y^2}{(x_i^2 + y^2)^2} - \frac{(1-\nu^2)}{3+\nu} \cdot (x_i^2 + 3y^2) \right] \quad (6)$$

where

$$x_i = R_1 + \delta + (i-1)p$$

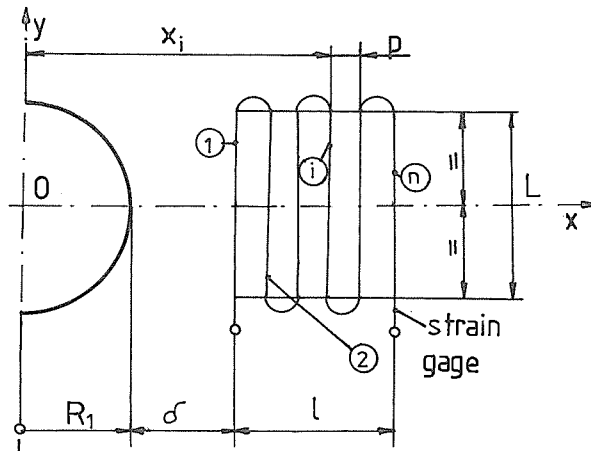


Fig.3. A strain gage installed near the hole of the disc.

The output given by the filament (i) is :

$$\bar{\epsilon}_{Li} = \frac{1}{L} \cdot \int_0^L \epsilon_{Li}(y) dy$$

from where there results:

$$\bar{\epsilon}_{Li} = C(3 + \nu) \cdot \left[(1 - \nu)(R_1^2 + R_2^2) + (1 + \nu)R_1^2 R_2^2 \frac{L}{x_1^2 + L^2} - \frac{1 - \nu^2}{3 + \nu} (x_1^2 + L^2) \right] \quad (7)$$

By using the relation (4) and the Hooke's law one can write:

$$\epsilon_{t \max} = 2C(3 + \nu) \cdot \left[\frac{R_1^2(1 - \nu)}{3 + \nu} + R_2^2 \right] \quad (8)$$

The output of the strain gage will be calculated with (2).
The expression of the standard error is:

$$e = \frac{\bar{\epsilon}_L - \epsilon_{t \max}}{\epsilon_{t \max}} \cdot 100 \% \quad (9)$$

One can notice that the error does not depend on C. In other words it does not depend on ν, ω, g , and E, anylonger, but in a small extent on the material of the disc (through ν).

From (2), (7), (8), (9) we get the relation (10):

$$e = \frac{(1 - \nu)(R_1^2 + R_2^2) + (1 + \nu)R_1^2 R_2^2 \cdot \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{x_i^2 + L^2} \right) - \frac{2R_1^2(1 - \nu)}{3 + \nu} - 2R_2^2}{2 \left[\frac{R_1^2(1 - \nu)}{3 + \nu} + R_2^2 \right]} \cdot 100\% \quad (10)$$

By neglecting the terms with a smaller influence one can get the a simplified formula, valid for $R_2 \gg R_1$:

$$e \approx 50(1 + \nu) \cdot \left[\frac{R_1^2}{n} \cdot \sum_{i=1}^n \left(\frac{1}{x_i^2 + L^2} \right) - 1 \right] \% \quad (11)$$

This formula is much simpler than (10), but still gives a good approximation.

Example

As an example, we took of a steel disc ($\nu = 0.29$) with $R_1 = 15\text{mm}$ and $R_2 = 150\text{mm}$. For any grid the error is minimal if the edge of the grid is tangent to the hole ($\delta = 0$). Taking this case, the error has been calculated for a 10H type strain gage, made in Romania, with the following characteristics:

-the gage length $L=10$ mm;
-the gage width $l=3.5$ mm;
-the number of lines of the grid $n=8$;
-the pitch of the network $p=0.5$ mm.
For this case, an error $e = -25.914\%$ with (10) and $e = -25.75\%$ with (11) was found.
Of course, the error would have been smaller for a strain gage with the grid having more reduced dimensions.

CONCLUSIONS

The formulae (10), (11) can be used to calculate the error magnitudes, due to integration effect, for a strain gage installed near the hole of a thin disc in uniformly rotation. In practice, such measurements can be performed for the seat plates in the rotor of an electro-motor or of an electric generator, fan, compressor or turbine disc. In the latter case one will have take into consideration the effect of temperature.

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