Strain-Based Finite Elements for the Analysis of Cylinders with Holes and Normally Intersecting Cylinders

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Summary

A finite element solution to the problems of stress distribution for cylindrical shells with circular and elliptical holes and also for normally intersecting thin elastic cylindrical shells is given in the present paper. Quadrilateral and triangular curved finite elements are used in the analysis. The elements are of a new class, based on simple independent generalised strain functions insofar as this is allowed by the compatibility equations. The elements also satisfy exactly the requirements of strain free-rigid body displacements and uses only the external "geometrical" nodal degrees of freedom to avoid the difficulties associated with unnecessary internal degrees of freedom.

A rectangular curved element was first developed and applied to the analysis of the familiar pinched cylinder and barrel vault problems (Ashwell and Sabir |1|). The results converge rapidly for displacements as well as for stresses. Further tests were carried out to investigate the ability of this element in predicting the high stresses in the neighbourhood of applied concentrated loads, (Sabir and Ashwell |2|). The loads considered were either radial or axial forces as well as moments about tangents to the circular cross section. The results obtained were not only in agreement with those of Forsber and Flügge |3| but when plotted for the complex parameters defining proportions of the shell and flexibility as suggested by Calladine |4|, their general forms corresponded closely with theoretical predictions.

In the present paper we first develop strain based quadrilateral and triangular elements and apply them to the solution of the problem of stress concentrations in the neighbourhood of small and large circular and elliptical holes when the cylinders are subjected to a uniform axial tension. These results are compared with analytical solutions based on shallow shell approximations and show that the use of these strain based elements obviates the need for using an inordinately large number of elements.

Normally intersecting cylinders are common configurations in structural components for nuclear reactor systems and design information for such configurations are generally lacking. The opportunity is taken in the present paper to provide a finite element solution to this problem. A method of substructing will be introduced to enable a solution to the large number of none banded set of simultaneous equations encountered. The solutions show good agreement when compared with experimental results of Corum, Bolt, Greenstreet and Gwaltney |5|.

1. Introduction

Cylindrical shells containing holes and intersecting cylinders are commonly used in industrial applications such as pressure vessels, boilers, pipe connections and pressure hulls of submarines. In general, the presence of a hole in a structure causes perturbations in the nominal stress field with an associated stress concentration near the cut-out. Because of the importance of such a phenomena in design, the state of stress around a curvilinear cut-out in cylindrical shells has been the subject of numerous analytical, numerical and experimental investigations.

Most analytical solutions are based on shallow shell theories, and require that all stresses die out within shallow regions of the shell. This requirement restricts the applicability of the theory to cut-outs of radius less than a quarter of the radius of the shell. The dimensionless curvature parameter β, as defined in equation (1), is usually used in the various theoretical investigations

$$\beta = a|12(1 - \nu^2)|^{\frac{1}{4}}/2(Rh)^{\frac{1}{2}} \qquad (1)$$

where a is the radius of the circular hole or the semi-major axis if the hole is elliptical, ν is Poisson's ratio and R and h are the radius and the thickness of the shell.

A comprehensive survey of the published literature on cut-outs in shells is given by Guz |6| and Pattabiraman et al |7|. Lur'e |8| was the first to consider the stress state around a circular hole in a cylindrical shell loaded by either an axial tension or uniform internal pressure. He employed a perturbation method to obtain adequate solutions for cut-outs for β less than unity. Using a Fourier series expansion with the perturbation technique, Withum |9| extended the solution to twisted shells having β up to 2. In a series of studies, Eringen et al |10| covered a range of β up to 2.5 for several loading cases. Furthermore Lekkerkerker |11| modified Lur'e's work and extended it to a range of β of up to 4 by writing the solution in terms of Fourier Bessel series. He also conducted experiments and concluded that the analytical results are fully confirmed by the experiments. The progress was further continued by Van Dyke |12| and Yamamoto |13| who extended the range of β to 6.

The progress in the analytical solutions for cylindrical shells with elliptical holes followed the same process as that for circular holes. Among the first successful solutions is that of Savin and Guz |14|. The method did apply to small holes and required the contours of the holes to be suitable for mapping into circles. Murthy et al |15| improved the solution to overcome the difficulty of mapping. A new approach was later suggested by Christine and Harisen |16|. They carried out a formulation in terms of integral equations. This new approach was improved upon by Sánders |17| and Tsai and Sanders |18| to extend the range of application over a range of β of up to 8.

More recently, the finite element method has been applied to the problem of cylindrical shells with holes by Lindberg and Olson |19|, Key and Beisinger |20|, Prato |21| and Ando et al |22|. In all these applications the cut-out considered is restricted to small circles with β equal to less than unity.

The geometrical complexity of normally intersecting cylinders restricted the analytical solutions for the determination of the stress distribution in the vicinity of the junction to cases where the stresses die out within shallow regions. Attention was therefore turned to experimental testing using strain gauges |23-25| and photoelastic methods |25-26|.

The finite element method was later applied to this problem by Hermann and Campbell |27|, Prince and Rashid |28|, Ando et al |29| and Gwaltney et al |5| who used flat plate elements for the idealization of the entire structure. Krishnanurthy |30| and Rashid and Gilman |31| used three dimensional solid elements and Bakresbah et al |32| represented the immediate region of the intersection by three dimensional isoparametric elements and the rest of the structure by superisoparametric elements.

In the present paper we shall first develop new curved shell elements suitable for the analysis of cylinders with circular and elliptical holes and also for the problem of normally intersecting cylinders. These elements are based on deep shell theory formulation, (no restriction to shallowness is implied) and they have no additional unnecessary internal degrees of freedom. Solutions for cylinders with holes covering a range of β up to 8 is obtained and compared with existing solutions. The problem of the normally intersecting cylinders is solved, the geometry of the line of intersection of the two cylinders is first formulated to enable the computer program to automatically generate the co-ordinates for the common nodes and substructuring is used to ease the method of analysis.

2. Theoretical Considerations and Derivation of Shape Functions for the New Elements

Consider a cylindrical shell or radius R with curvilinear co-ordinate system x, y and z in the longitudinal circumferential and outward normal directions respectively. If the displacements in these directions are u, v and w, Sanders-Koiter strain displacement relationships become

$$\varepsilon_x=\partial u/\partial x\ ,\ \varepsilon_\phi=(\partial v/\partial\phi)/R+w/R\ ,\ \gamma_{x\phi}=(\partial u/\partial\phi)/R+\partial v/\partial x\ ,\ \chi_x=-\partial^2 w/\partial x^2 \tag{2}$$

$$\chi_\phi=(\partial v/\partial\phi)/R^2-(\partial^2 w/\partial\phi^2)/R^2\ ,\ \chi_{x\phi}=-(\partial^2 w/\partial x\partial\phi)/R+3(\partial v/\partial x)/4R-(\partial u/\partial\phi)/4R^2$$

where $\partial\phi=R\partial y$, ε_x, ε_ϕ are direct membrane strains, $\gamma_{x\phi}$ is the membrane shearing strain, χ_x, χ_ϕ are changes of principal curvatures and $\chi_{x\phi}$ is the twisting curvature.

Equations (2) show that the six strain components are not independent, for being derivable from three displacements u, v and w. They must therefore satisfy three compatibility relationships which can be obtained by eliminating u, v and w. Hence

$$\partial\chi_\phi/\partial x-(\partial\chi_{x\phi}/\partial\phi)/R-(\delta\gamma_{x\phi}/\partial\phi)/4R^2=0\ ,-\partial\chi_{x\phi}/\partial x+(\partial\chi_x/\partial\phi)/R+3(\partial\gamma_{x\phi}/\partial x)/4R-(\partial\varepsilon_x/\partial\phi)/R^2=0 \tag{3}$$

$$-\partial^2\varepsilon\phi/\partial_x{}^2+(\partial^2\gamma_{x\phi}/\partial x\partial\phi)/R-(\partial^2\varepsilon_x/\partial\phi^2)/R^2-\chi_x/R=0$$

Proceeding as with the new rectangular cylindrical element |1| we first integrate equations (2) with all the strains made equal to zero, to obtain :

$$\begin{aligned} u &= R\,\alpha_2\cos\phi + R\,\alpha_4\sin\phi + \alpha_5 \\ v &= (\alpha_1+\alpha_2 x)\sin\phi - (\alpha_3+\alpha_4 x)\cos\phi + \alpha_6 \\ w &= (\alpha_1+\alpha_2 x)\cos\phi - (\alpha_3+\alpha_4 x)\sin\phi \end{aligned} \tag{4}$$

These equations give the six components of rigid-body displacement, and require six of the displacement function constants α_1, α_2,....α_6. The number of the remaining constants available for expressing the deformation of the element will depend on the number of nods of the element and the number of degrees of freedom the element is allowed to have. Since the elements we seek have only external degrees of freedom, i.e. three linear displacements and two rotations, the quadrilateral element will have twenty degrees of freedom and the triangular element will have only fifteen degrees of freedom if the nodes are restricted to the corners only. Having used up six constants (α_1 to α_6) to satisfy the requirement of rigid body displacement, the deformed quadrilateral element should have fourteen more to be apportioned among the strains. This is done as follows :

$$\begin{aligned}
\varepsilon_x &= \alpha_7 + \alpha_8\,\phi \\
\varepsilon_\phi &= \alpha_9 + \alpha_{10}x - (\alpha_{12}\,x^2 + \alpha_{13}\,x^3/3 + \alpha_{14}\,x^2\phi + \alpha_{15}\,x^3\phi/3)\,2R \\
\gamma_{x\phi} &= \alpha_{11} + (4\,\alpha_8\,x/3R) \\
\chi_x &= \alpha_{12} + \alpha_{13}\,x + \alpha_{14}\,\phi + \alpha_{15}\,x\phi \\
\chi_\phi &= \alpha_{16} + \alpha_{17}\,x + \alpha_{18}\,\phi + \alpha_{19}\,x\phi \\
\chi_{x\phi} &= \alpha_{20} + (\alpha_{14}\,x/R + \alpha_{15}\,x^2/2R + \alpha_{17}\,R\phi + \alpha_{19}\,R\phi^2/2)
\end{aligned} \tag{5}$$

The unbracketted terms are first written down, the terms in brackets are then added to satisfy the compatibility equations (3).

Expressions (5) are equated to the corresponding expressions in terms of u, v and w from equations (2) and the resulting equations integrated to give :

$$\begin{aligned}
u &= \alpha_7\,x + (3\,\alpha_{11}\,R/4 + \alpha_{19}\,R^3 - \alpha_{20}\,R^2)\phi - \alpha_{17}\,R^3\phi^2/2 + \alpha_8\,x\phi - \alpha_{19}\,R^3\phi^3/6 \\
v &= (\alpha_{11}/4 - \alpha_{19}R^2 + \alpha_{20}R)x + \alpha_{16}R^2\phi + \alpha_8 x^2/6R + \alpha_{17}R^2x\phi + \alpha_{18}R^2\phi^2/2 + \alpha_{19}R^2x\phi^2/2 \\
w &= (\alpha_9 R - \alpha_{16}R^2) + (\alpha_{10}R - \alpha_{17}R^2)x - \alpha_{18}R^2\phi - \alpha_{12}x^2/2 - \alpha_{19}x\phi R^2 - \alpha_{13}x^3/6 - \alpha_{14}x^2\phi/2 - \alpha_{15}x^3\phi/6
\end{aligned} \tag{6}$$

The complete shape function is the sum of corresponding expressions from equations (4) and (6).

The shape function for the triangular element is derived by following the same above procedure, the strained element is allowed nine degrees of freedom apportioned as follows :

$$\begin{aligned}
\varepsilon_x &= \alpha_7 \\
\varepsilon_\phi &= \alpha_8 - (\alpha_{10}\,x^2/2R + \alpha_{11}\,x^3/6R) \\
\nu_{x\phi} &= \alpha_9 \\
\chi_x &= \alpha_{10} + \alpha_{11}\,x\,\phi \\
\chi_\phi &= \alpha_{12} + \alpha_{13}\,x\,\alpha_{14}\,\phi \\
\chi_{x\phi} &= \alpha_{15} + (\alpha_{11}\,x^2/2R + \alpha_{13}\,R\phi)
\end{aligned} \tag{7}$$

When these are integrated, we obtain for the displacements u, v and w :

$$\begin{aligned}
u &= \alpha_7\,x + (3\alpha_9\,R/4 - \alpha_{15}\,R^2)\phi - \alpha_{13}\,R^3\phi^2/2 \\
v &= (\alpha_9/4 + \alpha_{15}\,R)x + \alpha_{12}\,R^2\phi + \alpha_{13}\,x\phi R^2 + \alpha_{14}\,R^2\phi^2/2 \\
w &= (\alpha_8\,R - \alpha_{12}\,R^2) - \alpha_{14}\,R^2\phi - \alpha_{10}\,x^2/2 - \alpha_{11}\,x^3\phi/6 - \alpha_{13.}\,R^2\,x
\end{aligned} \tag{8}$$

The complete shape function is the sum of corresponding expressions for equations (4) and (8).

Both the quadrilateral and the triangular elements are non-conforming and have five external degrees of freedom per node, these are u, v, w and the two rotations $\partial w/\partial x$ and $\partial w/\partial y - v/R$. The origin of the co-ordinates are taken at the centroids and the stiffness matrices are calculated using numerical integration techniques.

3. Finite Elements Idealization for Cylindrical Shells with Circular or Elliptical Holes

Consider a cylindrical shell with either a circular or an elliptical hole if it is subjected to a rotationally symmetric load applied at the ends, a quarter of the shell need only be analysed. To obtain an insight into the finite element model and the associated mesh required to produce converged results, numerous preliminary tests were carried out. The quarter of the shell is first thought of as made up of two zones. A zone in the neighbourhood of the hole where the stresses vary rapidly and a second zone away from the hole. These tests showed that reasonable converged results are obtained if the first zone is idealised by eight quadrilateral elements in the tangential direction to the hole and that the number of elements in the radial direction dependend on the relative dimensions of the hole and cylinder. These tests also showed that if the first zone extended to a radial length of about 3 times the radius of the hole, the rest of the shell where the stresses are nearly uniform need only be idealised by one longitudinal quadrilateral element.

Experience with finite elements based on strain assumptions indicated that these elements provide acceptable stresses if they are calculated at the centroids of the elements and that generally in regions of rapid variation in stresses further improvement of results are obtained by refinement of the mesh. However we observed that if we calculate the stresses at selected points along the radial centre lines of the elements, the discontinuities of stresses at the elements interfaces which are inherent to the finite element method can be smoothed out using the NAG library routine EO 2ADF. This procedure was used to interpolate the stresses at the edge of the hole.

We also observed that the quadrilateral element predicts stresses more accurately than the triangular element and hence the triangular element is used only in regions away from the hole and only when needed to avoid the use of needle-like quadrilateral elements.

3.1 Finite Elements Results for Cylinders with Circular Holes

Cylindrical shells with circular holes subjected to a uniform axial tension were analysed using the NODAL solution routine |33|. Since the curvature parameter β is dependent on the radius and thickness of the shell and the size of the hole the results are grouped so that the effect of varying each of these geometric properties is obtained. We first considered the effect of varying the radius of the hole. A group of four cylinders having β = 2,4,6 and 8 and a range of a/R between 0.5 and 2.0 were analysed and the variation of the circumferential membrane and bending stresses around the holes are obtained. The results are expressed in terms of a non-dimensional quantity referred to as the stress concentration factor. This factor is the ratio of the calculated stresses in the presence of the hole to the uniform stress in the absence of the hole. for β=2 agreement with the analytical solution given by Tsai and Sanders |18| which is based on shallow shell approximations is satisfactory. For the larger holes β=4, 6 and 8 the results progressively differed, the finite element method gave higher maximum circumferential membrane stress concentration factors. The differences were about 10%, 65% and 180% for β=4, 6 and 8 respectively.

In a second group of cylinders the parameter β is varied by changing the thickness of the shell while the radii of both hole and cylinder are kept constant. The ratio of radius of hole to cylinder considered was about 0.1. The geometrical properties of these cylinders are within the range of applicability of the shallow shell theories as suggested by Van Dyke |12|. The results obtained by the present finite element as expected, show good agreement with Tsai and Sanders solutions for β=2, 4, 6 and 8.

To find the range of applicability of Tsai and Sanders solutions a third group were considered. The radius of the hole is kept the same as in the previous group, the radius of the shell was reduced to one fifth of that of the previous group and the thickness is again varied to give β=2, 4, 6 and 8. The geometrical properties of this group can be shown to be outside the range of shallowness but the finite element results which gave good agreement with the analytical solutions suggests that Tsai and Sanders solutions are acceptable for cylinders with medium size holes where a/R is as large as 0.5.

3.2 Cylinders with Elliptical Holes

Elliptical holes with the minor axis in the direction of the longitudinal axis of the shell were considered. The cylinders were subjected to a uniform axial tension and as in the case of circular holes the finite element results show that the size of the elliptical hole has the greatest effect on the stress concentration factors. The solutions again agree with the analytical results for small and medium size holes of up to a ratio of semi major axis of hole to radius of the cylinder of 0.5 and for a range of values of β between 2 and 8. A group of cylinders with the same dimensions as the third group of the previous section with elliptical holes having a ratio of minor to major axis of 0.8 were analysed. The results show that the maximum circumferential membrane stresses are about 20% higher than those with circular holes having a radius equal to the major radius of the elliptical holes.

In all the above results attention was given to the stresses around the holes. To give a general picture of the distribution of stresses a cylinder having β=6 and a ratio of semi-minor axis to radius of cylinder of 0.2 was considered. Contour lines for various values of the stress concentration factors were plotted for the entire developed surface of the cylinder. The results show that outside an area of length of four times the semi-major axis of the elliptical hole the stress concentration factor is less than 1.25.

4. Finite Elements Results for Perpendicularly Intersecting Cylinders

The dimensions and the geometrical arrangements of the intersecting cylinders are as for model 1 of reference |5|. The main cylinder is rigidly built in at one end and the free ends of both cylinders are closed by end plates. Two loading cases were analysed. A uniform axial tension was applied at each of the free ends in turn.

Convergence tests were carried out by considering several finite element idalizations. We found that acceptable converged results are obtained when the idealization consists of 169 nodes in the cylinder and 187 nodes in the nozzle. If the overall structural matrix including the end plates is assembled in the usual way it will result into a non banded matrix of approximately 2000 number of degrees of freedom. To reduce the storage and to eliminate a large amount of computational effort a method of substructing for structures with several interconnected branches was suggested and detailed in reference |34|. This method of substructing can be applied to the problem as follows.

1 - The elements within the nozzle only are assembled first and Gaussian forward elimination corresponding to all the nodes except those on the junction are carried out. In this way the response of the nozzle to loads applied at the nodes of the line of intersection is obtained. The associated stiffness matrix (stitching matrix) and the condensed load vector are then transformed to the main cylinder's co-ordinate system.

2 - The stiffness matrix for the main cylinder is then assembled, the transformed stitching matrix is added to it and a solution for the cylinder is obtained.

3 - The nodal displacements of the line of intersection are then transformed back to the nozzle's co-ordinate system and multiplied by the stitching matrix to obtain the nodal forces applied to the nozzle. These forces are then used to obtain a solution for the nozzle.

The computer program uses the established equation for the line of intersection to calculate the co-ordinates of the nodes on this line and proceeds to calculate the stiffness matrices numerically. The substructing procedure is then incorporated in the NODAL solution routine |33| to obtain the stresses along the elements centre lines. The stresses at the junction are then interpolated using NAG EO 2ADF.

The stresses obtained in this way show satisfactory agreement when compared with the experimental results given in reference |5|. The solution using the strain based curved elements and the substructing procedure requires about 40 seconds CPU time on the CDC-7600 computer to solve for a system of 2000 unknowns. A more refined mesh with 3000 unknowns required only 100 seconds. This compares favourably with the solution of reference |5| which required more than 3 minutes of CPU time on an IBM 360/91 computer for 3000 equations.

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