Modelling the Response of Reinforced Concrete Structures to Thermal Loads

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ABSTRACT

An analytical procedure is presented for modelling the response of reinforced concrete structures to thermal load. It is based on an iterative, nonlinear, "secant stiffness" method of frame analysis, and incorporates realistic constitutive relations for concrete and reinforcement. The details and preliminary results of an experimental program are also described. Test results are compared to the predictions of the proposed method as well as to other existing methods.

1. INTRODUCTION

The magnitude and pattern of forces exerted on a reinforced concrete containment structure, due to thermal loading, depend greatly on the effective stiffnesses of the structural members. The stiffnesses, in turn, depend on the degree of cracking and yielding sustained and thus on the magnitudes of the forces acting. The result is a complex problem in structural analysis, where conventional linear elastic procedures will not give realistic results.

The analytical procedures (eg 1,2,3) initially advanced resorted to major assumptions and simplifications. They typically did not adequately account for such factors as concrete tensile strength, tension stiffening, mechanical load interactions, nonlinear thermal gradients, or nonuniformly cracked members. In many cases, these various procedures yielded radically differing results. The more recently proposed "effective stiffness" procedures (eg 4,5) overcome many of the weaknesses, but still rely on semi-empirical relationships for effective stiffness which may not adequately represent, in all cases, the critical effects of concrete tensile strength and tension stiffening. More accurate analyses can be obtained by utilizing a computer-based nonlinear frame analysis procedure employing fundamentally sound and rational constitutive relations for concrete and reinforcement.

2. PROPOSED ANALYTICAL PROCEDURE

Standard linear frame analysis methods are initially used to determine force resultants acting on the frame members. Each member is then analyzed for nonlinear axial force-elongation and moment-curvature response, and secant stiffness factors are determined. The "new" stiffness factors are used in a re-analysis of the structure, and revised force resultants are determined. The process is repeated until all stiffness factors converge to constant values, at which point final calculations are made.

Response of each individual reinforced or prestressed concrete member is determined using "layered" section analyses. In this method, the member's section is discretized into a number of concrete layers, rebar elements, and prestressing steel elements, and each is analyzed separately. The temperature and strain at the mid-depth of each element are determined as functions of the section's top and bottom fibre strains, subject to the condition that plane sections remain plane. Stresses in these components are then calculated using appropriate constitutive relations, and a numerical integration of the resulting moments and forces is performed. Estimates of the top fibre strain and the bottom fibre strain are adjusted until equilibrium conditions are satisfied.

The constitutive relation used for concrete in compression is the simple parabolic expression proposed by Hognestad, as follows:

$$fc = -fc' \cdot \left[2 \left(\frac{\varepsilon_c}{\varepsilon_o} \right) - \left(\frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right] \qquad (1)$$

For concrete in tension, where the concrete strain does not exceed the tensile cracking strain, the concrete tensile stress is computed from the following relations:

$$f_c = E_c \cdot \varepsilon_c; \qquad o < \varepsilon_c < \varepsilon_{cr} \qquad (2)$$

$$\varepsilon_{cr} = \frac{f'_t}{E_c} \qquad (3)$$

$$\varepsilon_c = \frac{2f'c}{E_o} \qquad (4)$$

If the concrete strain exceeds the cracking strain, then concrete tension stiffening effects should be considered. The relationship suggested is

$$f_c = f'_t \cdot \left[\frac{1}{1 + \sqrt{200\ \varepsilon_c}} \right] \qquad (5)$$

subject to the condition that combined concrete and steel tensile forces do not exceed the steel yield force. Post-cracking tensile stresses are assumed to exist only in that concrete which is defined as being the effective embedment area for a rebar. This area should not exceed the area of a square whose sides are 7.5 times the rebar diameter, and reductions due to overlapping embedments and proximity to edges of the section must be taken into account.

For reinforcing steel, a bilinear stress-strain relationship is used. Perfect bond between the reinforcement and the concrete is assumed, and strain hardening effects are ignored. Thus

$$f_s = E_s \cdot \varepsilon_s; \qquad |f_s| < f_y \qquad (6)$$

$$\varepsilon_s = \varepsilon_c \qquad (7)$$

Secant stiffness factors are evaluated for a given member under a specific load condition by examining that member's cross-sectional strains both with and without the loads acting. Assume that under the action of thermal load ΔT, axial load P, and end-moment M, the section's center fibre strain is ε_c and the curvature is ϕ. Further, under thermal load alone, the center fibre strain and curvature are ε_c' and ϕ, respectively. Given these values, effective stiffness factors are computed as follows:

$$A_{eff} = \frac{P}{E_c \cdot (\varepsilon_c - \varepsilon_{c'})} \tag{8}$$

and

$$I_{eff} = \frac{M}{E_c \cdot (\phi - \phi')} \tag{9}$$

It is necessary to re-evaluate thermal load fixed-end forces during each iteration of the solution process. Having calculated effective stiffness factors for a member, at a particular load level, the thermal load fixed-end forces are checked by examining initial strain and free thermal strain conditions. If the member's initial center fibre strain and curvature are $\varepsilon_c°$ and ϕ', and if under free thermal load the strain conditions are $\varepsilon_{c'}$ and ϕ', then the fixed-end forces are determined as follows:

$$P_F = \frac{(\varepsilon_{c'} - \varepsilon_c^{\circ})}{E_c \cdot A_{eff}} \tag{10}$$

$$M_F = \frac{(\phi' - \phi^{\circ})}{E_c \cdot I_{eff}} \tag{11}$$

3. EXPERIMENTAL PROGRAM

An extensive three-year experimental research program was initiated in 1984 to study thermal gradient effects in reinforced concrete containment structures. Three large-scale models will be tested with the objective of obtaining data necessary to verify and refine the analytical procedure described.

The first model, currently being tested, is schematically shown in Figure 1. It is essentially a reinforced concrete portal frame consisting of two columns (each 300x800x2350 mm) and one beam (300x800x2200 mm). The model sits in an inverted position with the ends of the columns connected by two 25 mm diameter tie-rods, and two reinforced concrete side panels (each 125x1500x2200 mm) span the interior of the frame to form a tank-like structure. Flexible silicone waterstops bridge between the panels and the frame, allowing the frame to be structurally independent of the panels and thus unaffected in its stiffness or response. Water is placed in the tank to a depth of 1500 mm and a 120 kW heater, immersed in the water, is used to apply thermal loads to the structure. The test specimen is heavily instrumented with load cells, thermocouples, strain gauges and displacement transducers to monitor its response.

The first series of tests performed was with the columns unrestrained, and with thermal gradients of up to 70°C applied gradually. The objective was to monitor thermal deformations, the development of nonlinear temperature profiles, and the magnitude of "primary" stresses induced through the wall thickness as a result of the nonlinear gradients. A second series of tests was conducted with the columns restrained from deflecting outward, the magnitude of restraint force induced being of prime interest. One test involved the gradual application of thermal load over a 4-day period, incrementing by 5°C to a maximum of 80°C. This was followed by eight "shock" load tests ranging from 10°C to 80°C. Future tests will involve the simultaneous application of thermal and mechanical loads.

4. COMPARISON OF RESULTS

Shown in Figure 2 are the bending moments induced at the midspan of the portal frame beam as a result of thermal gradients applied to the restrained test model. A moment of 16.2 kN•m was acting initially due to the weight of the structure, the weight of the water contained, and a small pre-load in the tie-rods. The experimental line is an envelope of the peak values attained during the shock load tests. Also shown are the values predicted by four different methods.

The "fully cracked sections" method of analysis suggested by Gurfinkel (1), Pajuhesh (2) and others assumes that the entire tensile region of a member is fully cracked in all loading cases. For frame analyses, all members assume stiffnesses equal to their cracked moment of inertia, I_{cr}. As can be seen in Figure 2, this procedure greatly underestimates the forces measured in the test structure.

Conversely, the method suggested by ACI Committee 349 (3) recognizes that uncracked lengths might exist in a member near the regions of contraflexure. The cracked and uncracked regions are determined on the basis of mechanical moments alone, and thermal moment distribution is carried out considering this variation in stiffness along the member. For the test specimen, which was uncracked under mechanical loads alone, the method implies that the uncracked or gross moments of inertia, I_g, be used. This results in greatly overestimated restraint forces.

The "effective stiffness" methods proposed by Mentes et al (4) and Thurston (5) assume that the effect of cracked and uncracked lengths in a member is equivalent to an effective moment of inertia given by Branson's formula, ie

$$I_{eff} = \left(\frac{M_a}{M_{cr}}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_a}{M_{cr}}\right)^3\right] \cdot I_{cr} \qquad (12)$$

With this relationship, standard methods of frame analysis can be used in an iterative procedure to solve for the forces acting. As can be seen from Figure 2, the method predicts well the experimental results, although the tendency is to underestimate slightly at higher load levels.

Also shown is the predicted response obtained using the analytical procedure described in this paper. It too predicts well the experimental results, marginally better than does the empirical effective stiffness method. The over-estimation of force in the 20 to 30°C range is likely related to the fact that the structure was pre-cracked slightly due to previous testing. The chief advantage of the proposed procedure, however, is that it is based on a rational method of section analysis, taking into account compatibility requirements, equilibrium conditions, and realistic constitutive relations. Thus, unlike the empirical effective stiffness method, it is an adaptable procedure which will permit the input of more advanced material behaviour models, as well as models for creep, shrinkage, load history, time history, changes in material properties at elevated temperatures, and other factors which may influence response.

5. CONCLUSIONS

A nonlinear, iterative, secant stiffness method of frame analysis, where the stiffness factors are determined on the basis of section analyses, can be used to accurately model the response of reinforced concrete structures to thermal load. Predictions of the method compare well to experimental results, although additional work is needed in its treatment of tension stiffening effects. The method is completely adaptable to the input of improved material behaviour models as they become available.

The effective stiffness procedures based on Branson's empirical formulation for I_{eff} also give reasonably good predictions of response. They tend, however, to underestimate tension stiffening effects and are not readily adaptable to refinements in material response models. The methods based on either fully cracked members or uncracked members tend to give very poor predictions of response.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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8. NOTATION

A_{eff} - effective cross-sectional area of member
ε_c - strain in the concrete
ε_{cr} - concrete cracking strain
ε_o - concrete strain at maximum compression stress, f_c'
ε_s - strain in the reinforcing steel
E_c - modulus of elasticity of the concrete
E_s - modulus of elasticity of the reinforcing steel
f_c - stress in the concrete
f_c' - concrete compressive strength
f_s - stress in the reinforcing steel
f_t' - concrete tensile strength
f_y - yield stress of the reinforcing steel
I_{cr} - cracked moment of inertia
I_{eff} - effective moment of inertia
I_g - gross moment of inertia
M_a - maximum moment acting on member
M_{cr} - cracking moment
M_F - fixed-end moment due to thermal load
P_F - fixed-end axial force due to thermal load

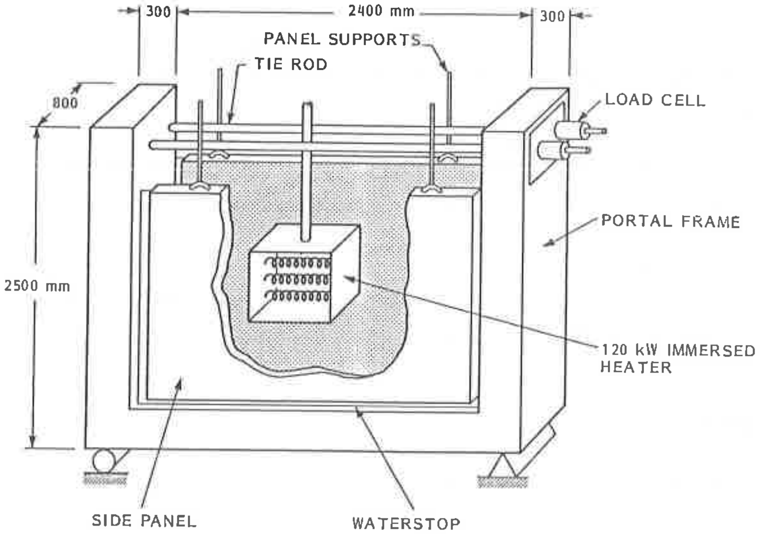

FIGURE 1
THERMAL GRADIENTS TEST SPECIMEN NO 1

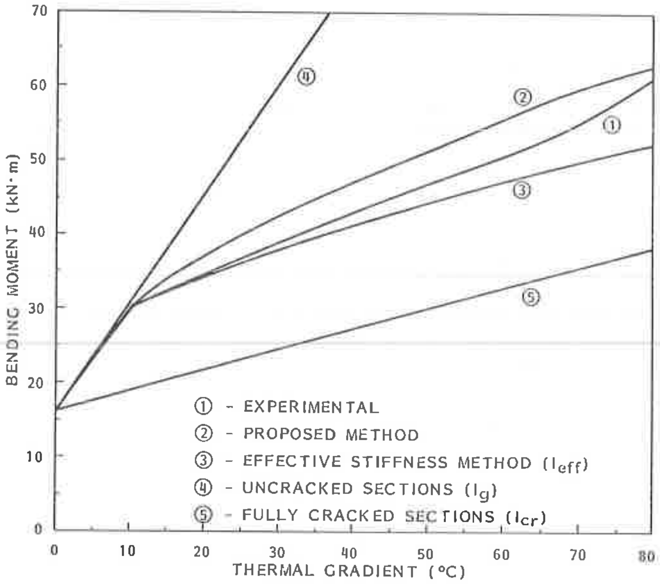

FIGURE 2
COMPARISON OF EXPERIMENTAL AND PREDICTED RESULTS