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DAMAGE AND GRADIENT OF DAMAGE: THE UNILATERAL PHENOMENON

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ABSTRACT

We study the damage of concrete at the macroscopic level. The model involves the gradients of the damage quantities, one in tension and one in compression. The important unilateral phenomenon is taken into account and the model is mesh independent.

INTRODUCTION

Within the framework of continuum mechanics, we study the damage of concrete. This theory describes at the macroscopic level the effects of microfractures and microcavities which results in the decrease of the material stiffness. Let the scalar $\beta(x,t)$ be a damage quantity with value 1 when the material is undamaged and value 0 when completely damaged.

Within the solid, there exist microscopic movements which produce damage. We think that the power of these microscopic movements must be taken into account in the power of the internal forces. Thus we choose the power of the internal forces to depend, besides on the strain rates $\mathbf{D}(v)$ (v is the macroscopic velocity), also on $\dot{\beta}$ and $\text{grad}\dot{\beta}$. These latter quantities are clearly related to the microscopic movements. The gradient of damage is introduced to take into account the influence of damage at a material point on its neighbourhood.

The principle of virtual power gives a new equlibrium equation which describes the evolution of the damage quantity β. It is natural to assume that the free energy ψ is a function of ε the small deformation, β and $\text{grad}\beta$. For the sake of simplicity we assume that there is only dissipation with respect to $\dot{\beta}$.

PRINCIPLE OF VIRTUAL POWER, EQUATIONS OF THE MOVEMENT

The power of the internal forces which takes into account the microscopic movements in a domain Ω is chosen as :

$$P_i = -\int_\Omega \sigma:\mathbf{D}(v)\, d\Omega - \int_\Omega \left(\mathbf{B}\,\dot{\beta} + \mathbf{H}.\text{grad}\dot{\beta}\right)\, d\Omega, \qquad (1)$$

where σ is the stress tensor, $\mathbf{D}(v)$ the strain rates tensor with $D_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$, $i,j \in \{1,2,3\}$.

Two new non-classical quantities appear, $\mathbf{B}$, the internal work of damage, and $\mathbf{H}$, the internal work of damage flux vector.

One can check that $P_i = 0$ for any rigid body velocity ($\dot{\beta} = 0$ in such a movement because the relative distance of material points remains constant).

For quasi-static evolutions, the principle of virtual power gives two equlibrium equations :

$$\begin{cases} \mathrm{div}\,\sigma + \mathbf{f} = 0, & \text{in } \Omega, \\ \sigma.\mathbf{n} = \mathbf{T}, & \text{on } \partial\Omega, \end{cases} \tag{2}$$

$$\begin{cases} \mathrm{div}\,\mathbf{H} - \mathbf{B} = 0, & \text{in } \Omega, \\ \mathbf{H}.\mathbf{n} = 0, & \text{on } \partial\Omega, \end{cases} \tag{3}$$

where $\mathbf{n}$ is the outwards normal unit vector to Ω. Equation (3) is non-classical.

The constitutive laws are [1] :

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} \quad , \quad \mathbf{H} = \frac{\partial \psi}{\partial(\mathrm{grad}\beta)} \quad , \quad \mathbf{B} = \frac{\partial \psi}{\partial \beta} + \frac{\partial \phi}{\partial \dot{\beta}} \quad , \tag{4}$$

where $\psi(\varepsilon,\beta,\mathrm{grad}\beta)$ is the free energy and $\phi(\dot{\beta})$ the pseudo-potential of dissipation.

The equations describing the evolution of a piece of concrete are (2), (3) and (4) completed by initial and boundary conditions.

A MODEL WITH ONE DAMAGE QUANTITY β

It is known that damage is mainly produced by extensions during loading. Thus we are led to choose ψ and ϕ such as,

$$\psi = \frac{1}{2}\beta\left\{2\mu \mathrm{tr}[\varepsilon\varepsilon] + \lambda(\mathrm{tr}\varepsilon)^2\right\} + W(1-\beta) - M(\mathrm{Log}\,|\beta| - \beta + 1) + k\,\mathrm{grad}\beta^2 + I_{[0,1]}(\beta), \tag{5}$$

$$\phi = \frac{1}{2}c\dot{\beta}^2 - \frac{1}{2}\dot{\beta}\left\{2\mu \mathrm{tr}[\varepsilon^-\varepsilon^-] + \lambda(\langle \mathrm{tr}\varepsilon\rangle^-)^2\right\} + I_-(\dot{\beta}), \tag{6}$$

where λ and μ are the Lamé parameters, $\langle . \rangle^+$ and $\langle . \rangle^-$ are respectively the positive part and the negative part of the quantity $\langle . \rangle$: $\langle . \rangle^+ = \mathrm{Sup}\left\{0,\langle . \rangle\right\}$ and $\langle . \rangle^- = \mathrm{Sup}\left\{0,-\langle . \rangle\right\}$. The positive and negative parts of the strain tensor are obtained after diagonalisation [2]. The parameter W is the initial threshold of damage, M is the factor of displacement of this threshold, c is the viscosity parameter of the damage and k measures the influence of damage at a material point on its neighbourhood.

The functions $I_{[0,1]}$ and I_- are the indicator functions of the intervals $[0,1]$ and $]-\infty, 0]$ $\left(I_A(x)=0\right.$ if $x \in A$ and $I_A(x) = +\infty$ if $\left. x \notin A \right)$. The effect of these indicator functions is to make compulsory for β to be between 0 and 1 and for $\dot{\beta}$ to be negative.

The constitutive relations (4) give :

$$\sigma = \beta\{2\mu\varepsilon + \lambda(\mathrm{tr}\varepsilon)\mathbf{I_d}\} \quad , \quad \mathbf{H} = k\,\mathrm{grad}\beta \quad , \quad B = \frac{\partial\psi}{\partial\beta} + \frac{\partial\phi}{\partial\dot\beta} \quad , \tag{7}$$

where the generalized derivatives [3] of ψ and ϕ are,

$$\frac{\partial\psi}{\partial\beta} \in \frac{1}{2}\{2\mu\mathrm{tr}[\varepsilon\varepsilon]+\lambda(\mathrm{tr}\varepsilon)^2\} - W - M\left(\frac{1-\beta}{\beta}\right) + \partial I_{[0,1]}(\beta), \tag{8.a}$$

$$\frac{\partial\phi}{\partial\dot\beta} \in c\dot\beta - \frac{1}{2}\{2\mu\mathrm{tr}[\varepsilon^-\varepsilon^-]+\lambda((\mathrm{tr}\varepsilon)^-)^2\} + \partial I_-(\dot\beta)), \tag{8.b}$$

with $\partial I_{[0,1]}(x) = \{0\}$ if $0 < x < 1$, $\partial I_{[0,1]}(0) =]-\infty, 0]$ and $\partial I_{[0,1]}(1) = [0,+\infty[$, and $\partial I_-(x) = \{0\}$ if $x < 0$ and $\partial I_-(0) = [0,+\infty[$.

$\mathbf{I_d}$ is the identity matrix. The equations describing the evolution of concrete results from (2), (3), (7) and (8) :

$$\left|\begin{array}{ll} c\dot\beta - k\Delta\beta + \partial I_{[0,1]}(\beta) + \partial I_-(\dot\beta) \ni -\frac{1}{2}\{2\mu\mathrm{tr}[\varepsilon^+\varepsilon^+]+\lambda((\mathrm{tr}\varepsilon)^+)^2\}+W+ M\left(\frac{1-\beta}{\beta}\right), & \text{in } \Omega, \\ \mathrm{div}\left(\beta\{2\mu\varepsilon + \lambda(\mathrm{tr}\varepsilon)\mathbf{I_d}\}\right) + f = 0, & \text{in } \Omega. \end{array}\right. \tag{9}$$

The elements of $\partial I_{[0,1]}(\beta)$ are reactions which force β to remain between 0 and 1. For instance if $\beta = 1$ the elements of $\partial I_{[0,1]}(1)$ are positive numbers avoiding β to become larger than 1.

This model is sufficient to describe the whole damage phenomena due to tension. But the very important unilateral phenomenon of restoration of stiffness when the load goes from tension to compression [4] is not described.

A DAMAGE MODEL DESCRIBING THE UNILATERAL PHENOMENON

The unilateral phenomenon is taken into account by using two damage quantities : β_t for extension and β_c for contraction. We choose ψ and ϕ as in the previous section :

$$\begin{aligned}\psi(\varepsilon,\beta_t,\beta_c,\mathrm{grad}\beta_t,\mathrm{grad}\beta_c) = \frac{1}{2}\{\beta_t\{2\mu\mathrm{tr}[\varepsilon^+\varepsilon^+] + \lambda((\mathrm{tr}\varepsilon)^+)^2\} + \beta_c\{2\mu\mathrm{tr}[\varepsilon^-\varepsilon^-] + \lambda((\mathrm{tr}\varepsilon)^-)^2\}\} \\ + W_t(1-\beta_t) + W_c(1-\beta_c) - M_t(\mathrm{Log}\,|\beta_t| - \beta_t + 1) - M_c(\mathrm{Log}\,|\beta_c| - \beta_c + 1) \\ + k\,[\mathrm{grad}\beta_t^2 + \mathrm{grad}\beta_c^2] + I_C(\beta_t,\beta_c),\end{aligned} \tag{10}$$

$$\phi(\dot\beta_t,\dot\beta_c\,;\varepsilon) = \frac{1}{2}\left\{c_t\dot\beta_t^2 + c_c\dot\beta_c^2\right\} - \dot\beta_c\lambda((\mathrm{tr}\varepsilon)^-)^2 + I_-(\dot\beta_t,\dot\beta_c), \tag{11}$$

with here I_- and I_C are the indicator functions of the set $]-\infty, 0]\mathrm{x}]-\infty, 0]$ and of the triangle $\mathbf{C}$:

$$\mathbf{C} = \left\{(x,y) \mid x\in [0,1]\,,\, y\in [0,1]\,,\, x\le y\right\}. \tag{12}$$

Again the effect of the indicator function of the triangle $\mathbf{C}$ is to make compulsory for the vector $\beta = (\beta_t,\beta_c)$ to remain in the triangle. Therefore we have $\beta_t \le \beta_c$, which means that the damage in compression produces damage in tension and that the reverse is not true.

The constitutive law (4) give,

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} = \beta_t\{2\mu\varepsilon^+ + \lambda\langle tr\varepsilon\rangle^+ \mathbf{I_d}\} + \beta_c\{2\mu\varepsilon^- + \lambda\langle tr\varepsilon\rangle^- \mathbf{I_d}\}. \qquad (13)$$

Let us note that if the material is undamaged ($\beta_t = \beta_c = 1$) relation (13) is the classical elastic relation. The equations for the two damage quantities results from (3), (4) and (10), (11) :

$$A_1 + \partial I_-(\dot{\beta}_t) + c_t\dot{\beta}_t - k\Delta\beta_t \ni -\frac{1}{2}\{2\mu tr[\varepsilon^+\varepsilon^+] + \lambda(\langle tr\varepsilon\rangle^+)^2\} + W_t + M_t\left(\frac{1-\beta_t}{\beta_t}\right), \qquad (14)$$

$$A_2 + \partial I_-(\dot{\beta}_c) + c_c\dot{\beta}_c - k\Delta\beta_c \ni -\frac{1}{2}\{2\mu tr[\varepsilon^-\varepsilon^-]\} + W_c + M_c\left(\frac{1-\beta_c}{\beta_c}\right), \qquad (15)$$

where the vector $A = (A_1, A_2) \in \partial I_C(\beta)$ is a vector normal to the triangle **C** at the point β (A=0 if β is in the interior of the triangle) [3]. We can check that those equations give only one velocity $\dot{\beta}$ function of ε and β such that $\dot{\beta} \leq 0$ and β remains in the triangle.

Let us repeat that because β remains in **C**, damage in compression produces damage in tension.

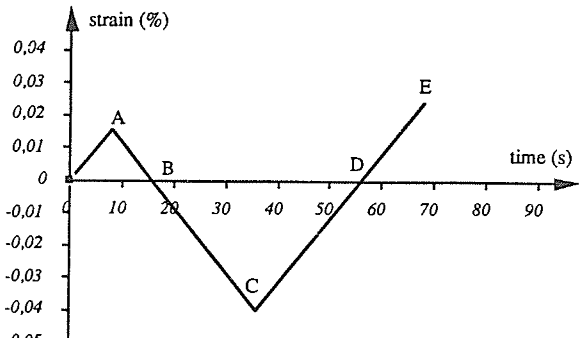

figure 1a. History of loading

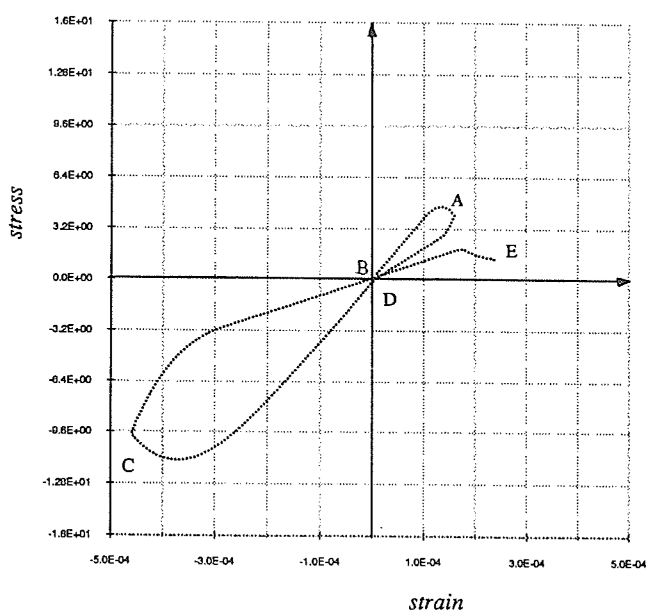

figure 1b. The stress versus the strain for the figure 1a loading.

Figure 1b shows the evolution of a sample submitted to the strain shown in figure 1a. We can see the unilateral phenomenon : from A to B the tension in the material decreases. The material is damaged ($\beta_t \leq 1$, $\beta_c = 1$) with modulus $\beta_t E$ (E is the undamaged Young's modulus): When going from B to C there is restoration of stiffness $\beta_c E = E$.

It is known that resistance is larger in compression than in tension. An actual example is treated in figures 2. The history of loading is given in figure 2a. The behaviour is in figure 2b where one can see again the restoration of stiffness.

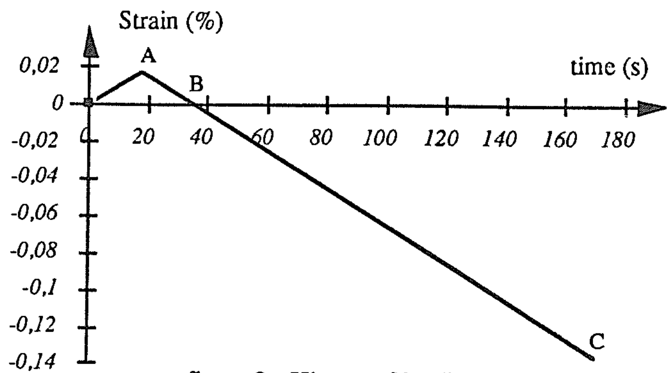

figure 2a. History of loading

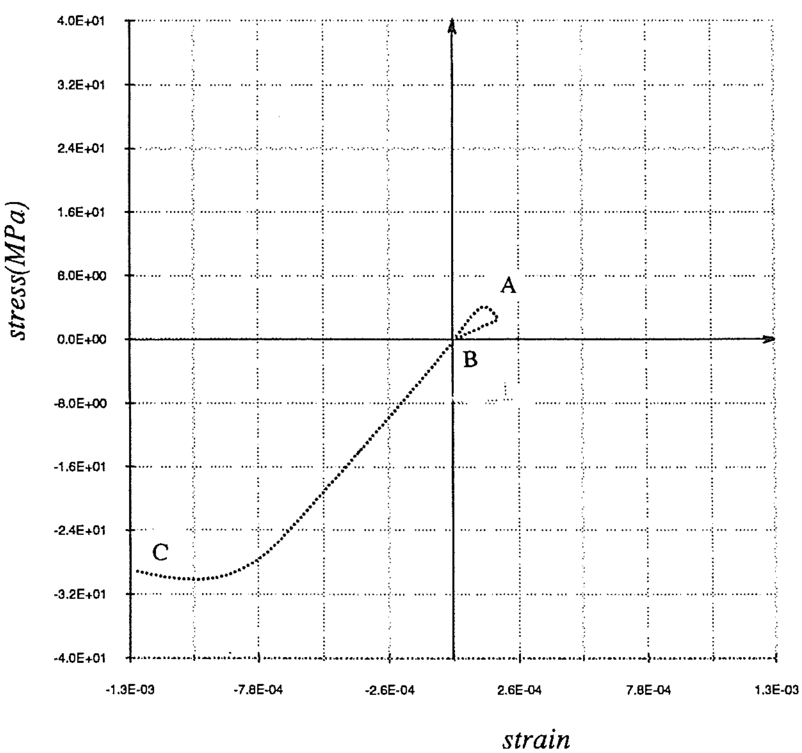

figure 2b. Behaviour of concrete . $E = 37000$ MPa, $\nu = 0.2$, $c_t = 0.002$, $c_c = 0.5$
$W_t = 1.15\ 10^{-4}$, $W_c = 0.7\ 10^{-2}$, $M_t = 0.25\ 10^{-3}$, $M_c = 0.3\ 10^{-1}$.

The actual data have been used for the splitting test (figure 3). The tension damaged zone appears in the middle part where the damage is due to tensions as it is found in experiments.

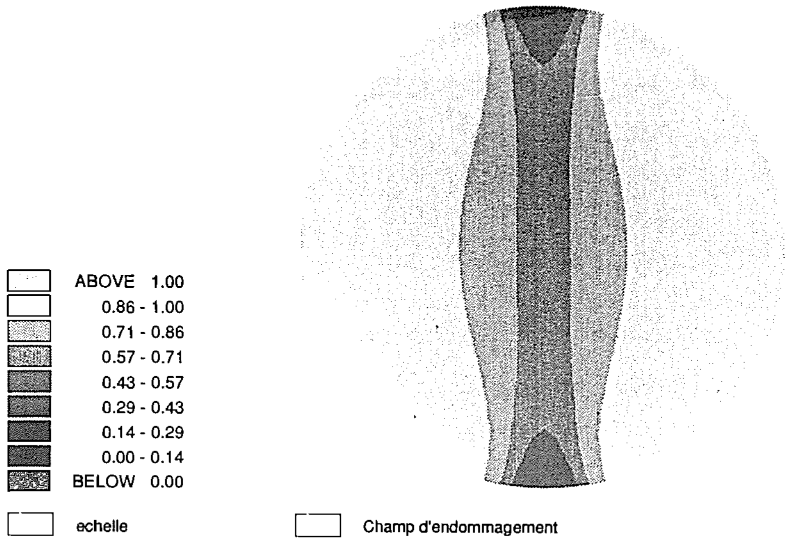

figure 3. Concrete splitting test.

CONCLUSION

This model shows qualitative agreement with experiments. Let us emphasize that it describes the unilateral phenomenon and allows different damage thresholds in tension and compression. Let us also say that it is a mesh independent model and that it shows no size effect.

REFERENCES

[1] Frémond M. "Sur l'inégalité de Clausius-Duhem", C. R. Acad. Sci. Paris t.311, serie II, pp. 757- 762,1990.

[2] Ladevèze P., "Sur une théorie de l'endommagement anisitrope", Rapport interne n° 34, LMT - ENS, CACHAN ,1983.

[3] Moreau J. J., "Fonctionnelle convexes", Séminaire sur les équations aux dérivées partielles, Collège de France, Paris, 1966.

[4] Chaboche J.L., "Une nouvelle condition pour décrire le comportement des matériaux avec dommage anisotrope", C. R. Acad. Sci. Paris t.314, serie II, pp. 1395- 1401,1992.