A Simple Scheme of Time History Response Analysis with Dynamic Stiffness Depending Strongly on Frequencies

Y. SHIMOMURA[1] and Y. IKEDA[2]

1)Junior College of Nihon University, Funabashi City, JAPAN
2)Taisei Corporation, Tokyo, JAPAN

ABSTRACT

The paper first presents a simple time history response analysis scheme of the system that involves dynamic stiffness functions depending strongly on frequencies. Second, the numerical analyses of dynamic soil-structure interaction problems are conducted to confirm the validity of the above scheme. Interaction forces in the time domain can be regard as being equal to interaction forces concerning dynamic problems in the frequency domain by taking into account causality and relation in terms of the Fourier transform of displacements, velocities and accelerations. In order to obtain the interaction forces in the time domain, one has to perform the convolution integral calculus involved qualities of the past responses. Therefore it is quite difficult to conduct the time history response analysis in consideration of the accurate interaction forces. In fact, the interaction forces in the time domain are often calculated for response analyses by using the discretized model, that is, k-c or m-k-c model. In the paper, it is assumed that we can divide the interested frequency range referring to the characteristics of the impedance function as well as the corresponding input motion into some segments. It means that the impedance function is approximated as a sequential single function in each segment. Some appropriate discretized models are applied to each segment. Whole response of the system has been obtained by aggregating the response of model in each divided segment. It turned out that the accuracy of the result by the proposed multi discretized model is fairly better than that by a single discretized model. Finally, the proposed scheme is also employed to the nonlinear response analysis of the structure with bi-linear type hysteresis.

1 INTRODUCTION

It is necessary to take account of the dynamic soil-structure interaction so as to investigate earthquake response of the deeply embedded structure. In general, the soil underlying the embedded structure is relatively harder than the side one. Because the dynamic soil spring estimated by the above soil depends strongly on the frequency, we have to perform the convolution integral calculus involved qualities of the past responses for obtaining the interaction forces in the time domain. A lot of researches such as Wolf [1], Hayashi [2,3]), Motosaka [4] and etc. presented the numerical calculation methods concerning the convolution integral. However, once the frequency dependency of the dynamic soil spring becomes strong, the practicability of their methods would be insufficient for estimating the interaction forces rigorously. Therefore it is quite difficult to conduct the time history response analysis in consideration of the accurate interaction forces. In fact, the interaction forces in the time domain are often calculated for response analyses by using the discretized model, that is, k-c or m-k-c model. The stiffness and damping coefficients of the model are estimated by the dynamic soil impedance functions at zero frequency or the predominant frequency of the system. It is foreseen that the result of the time history response analysis utilizing the discretized model will be different from the rigorous one in the high frequency range. It is indicated that the dynamic impedance function should avoid being approximated by a sequential single function in entire frequency band range.

In the paper, it is assumed that we can divide the interested frequency range referring to the characteristics of the impedance function as well as the corresponding input motion into some segments. It means that the impedance function is approximated as a sequential single function in each segment. Some appropriate discretized models are applied to each segment. Consequently, values of soil stiffness and damping in each segment are constant and these provably become discontinuous at the boundary of adjacent segments. Whole response of the system has been obtained by aggregating the response of model in each divided segment. It turned out that the accuracy of the result by the proposed multi discretized model is fairly better than that of a single discretized model.

Finally, the proposed scheme is also applied to the nonlinear response analysis of the structure with bi-linear type hysteresis. The result of a single discretized model based on zero frequency or the predominant frequency of the system differs greatly from that of this proposed scheme.

2. DYNAMIC SOIL STIFFNESS

The dynamic soil stiffness in frequency domain is expressed in the following form:

$$K(i\omega) = K_R(i\omega) + iK_I(i\omega) \tag{1}$$

The above equation is substituted by Taylor expand in consideration of the even function of the real part and odd function of the imaginary part.

$$K(i\omega) = K_0 + K_1(i\omega)^2 + \cdots + \{C_0 + C_1(i\omega)^2 + \cdots\}(i\omega) \tag{2}$$

Also, the above equation can be written as

$$K(i\omega) = K_0 + i\omega C_0 + M_r(i\omega) \tag{3}$$

where,

$$M_r(i\omega) = \frac{Re.[K(i\omega)] - K_0}{-\omega^2} + \frac{Im.[K(i\omega)]/\omega - C_0}{i\omega} \tag{4}$$

Here, the interaction force is given by

$$R(i\omega) = K_0 U(i\omega) + i\omega C_0 U(i\omega) + (i\omega)^2 M_r(i\omega) U(i\omega) \tag{5}$$

Taking into account the causality and relationship of Fourier transform of displacements, velocities and accelerations ($u(t) \Leftrightarrow U(i\omega)$, $\dot{u}(t) \Leftrightarrow i\omega U(i\omega)$, $\ddot{u}(t) \Leftrightarrow -\omega^2 U(i\omega)$), the interaction force in the time domain can be obtained by

$$R(t) = K_0 u(t) + C_0 \dot{u}(t) + \int_0^t M_r(\tau)\ddot{u}(t-\tau)d\tau = K_0 u(n\Delta t) + C_0 \dot{u}(n\Delta t) + M_0 \ddot{u}(n\Delta t) + \Delta t \sum_{k=1}^{n} M_r(k\Delta t)\ddot{u}((n-k)\Delta t) \tag{6}$$

In order to obtain the interaction forces in the time domain, one has to perform the convolution integral calculus involved qualities of the past responses. Furthermore, it is complicated to compute the numerical inverse transform of $M_r(i\omega)$. Therefore it is quite difficult to conduct the time history response analysis in consideration of the accurate interaction forces. On the other hand, when it is assumed that $M_r(i\omega)$ is approximately equal to $(i\omega)^2 M_a$, the interaction force in the time domain is simplified like the next expression.

$$R(t) = M_a \ddot{u}(t) + C_0 \dot{u}(t) + K_0 u(t) \tag{7}$$

3. PROPOSAL OF SIMPLE SCHEME OF TIME HISTORY RESPONSE

3.1 Interaction Force for Practical Use

In practical calculations, Eq. (6) has been often evaluated as the following simplified forms:

(a) k-c model I (based on the static soil stiffness)

$$R(t) = K_0 u(t) + C_0 \dot{u}(t) \tag{8}$$

(b) k-c model II (based on the soil stiffness at the natural frequency ω_0)

$$R(t) = Re.[K(i\omega_0)]u(t) + \frac{Im.[K(i\omega_0)]}{\omega_0}\dot{u}(t) \tag{9}$$

(c) m-k-c

$$R(t) = K_0 u(t) + \frac{Im.[K(i\omega_0)]}{\omega_0}\dot{u}(t) + \frac{Re.[K(i\omega_0)] - K_0}{\omega_0^2}\ddot{u}(t) \tag{10}$$

3.2 Improvement of m-k-c Model

In this paper, it is assumed that we can divide the interested frequency range referring to the characteristics of the impedance function as well as the corresponding input motion into some segments as follows

$$K(i\omega) \approx \tilde{K}(i\omega) = \sum_{\Gamma=1}^{n} \tilde{K}^{(\Gamma)}(i\omega) \tag{11}$$

where,

$$\widetilde{K}^{(\Gamma)}(i\,\omega) = K^{(\Gamma)} - \omega^2 M^{(\Gamma)} + i\,\omega C^{(\Gamma)} \tag{12}$$

It means that the impedance function is approximated as a sequential single function in each segment. Some appropriate discretized models are applied to each segment. As an example, consider the rigid body of the mass m supported by this dynamic interaction spring. Then, the response of the approximate model in the segment Γ can be given by using the next equation

$$(m + M^{(\Gamma)})\ddot{u}_f^{(\Gamma)} + C^{(\Gamma)}\dot{u}_f^{(\Gamma)} + K^{(\Gamma)}u_f^{(\Gamma)} = -m\ddot{u}_g^{(\Gamma)} \tag{13}$$

Whole response of the system has been obtained by aggregating the response of model in each divided segment.

3.3 Superstructures

It is assumed that the relationship between the restoring force and displacemnet can be represented by a bi-linear type hysteresis. Its characteresitics are defined by Eq. (14) and Fig. 1.

$$Q = kd + Q_r \tag{14}$$

where, d denotes the relative story displacement. Indices 1, 2, and 3 in Fig. 1 represent the state of condition. In addition, the secant moduls of spring constant k and reference restoring force Q_r have the values corresponding in each state in Table 1.

Table 1 Spring Constant and Reference Restoring Force in Each State

state	k	Q_r
1	k_e	$-k_e(1-\phi)d_r$
2	ϕk_e	$k_e(1-\phi)d_e$
3	ϕk_e	$-k_e(1-\phi)d_e$

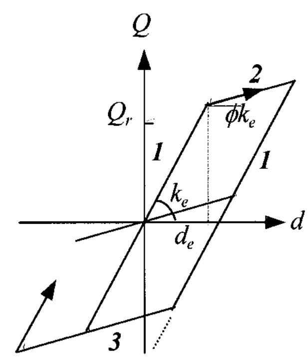

Fig. 1 Hysteresis Loop of Superstructure

where, k_e : inclination of elasticity spring constant
ϕk_e : second inclination in plasticity region
d_e : limit displacement at yield point
d_a : the maximum displacement caused at state 2
d_b : the maximum displacement caused at state 3
d_r : reference displacement depended on the state, $d_r = d_a - d_e$ at state 2, $d_r = d_b + d_e$ at state 3.

The restoring force model is divided into n bi-linear model segments corresponding to each frequency segment of the soil stiffness. Then,

$$Q = \sum_{\Gamma=1}^{n} Q^{(\Gamma)}, \qquad d = \sum_{\Gamma=1}^{n} d^{(\Gamma)} \tag{15}$$

where,

$$Q^{(\Gamma)} = k^{(\Gamma)}d^{(\Gamma)} + Q_r^{(\Gamma)}, \quad k^{(1)} = k^{(2)} = \cdots = k, \quad Q_r = \sum_{\Gamma=1}^{n} Q_r^{(\Gamma)}, \quad Q_r^{(\Gamma)} = Q_r / n \tag{16}$$

4 NUMERICAL MODEL

The superstructure with a single lumped mass and shearing type stiffness is employed for a numerical model. The stiffness has the bi-linear type hysteresis and the structure is supported on rigid foundation embedded deeply in the soil. Using this model the earthquake response analysis is carried out. In the analysis, it is assumed that the deposit under the foundation is a rigid bedrock and the translation motion of the rigid body can be neglected, *i.e.* the rigid body has only rotation degree of freedom. Thus, the equation of motion of the numerical model can be given as follows,

$$[M]\{\ddot{u}\} + \{R\} = -[M_E]\{I\}\ddot{u}_g + \{p\} \tag{17}$$

where,

$$[M]=\begin{bmatrix} m_b & m_b H_b \\ m_b H_b & I_f + m_b H_b^{\,2} + m_f H_G^{\,2} \end{bmatrix}$$
$$\{u\}=(u_b \quad \theta_f)^T,\ \{R\}=diag.(Q_b \quad R_f),\ [M_E]=diag.(m_b \quad m_b H_b + m_f H_G) \tag{18}$$

Q_b :restoring force of superstructure including damping force, $R_f = K_R \theta_f$: resultant resistance moment with side soil around the rotational axis at the bottom of rigid body, and $\{p\}=(0, p)^T$: driving force vector. For the simplification, surface soil layer is modeled in one degree of freedom system and driving force is derived as $p = K_R u_s / H_E$, where u_s is the displacement of the surface soil layer.

Fig. 3 shows the dynamic rotational soil stiffness utilized for the numerical analysis. Here, it is assumed that the rigid body lies on a bedrock and is surrounded by a soft surface layer. While the dynamic soil stiffness becomes idealized simply, its characteristics drastically change nearby the predominant frequency of the surface soil layer, that is, the cut-off frequency [5]. Here, all frequency band regions are divided into four segments, and four 2-degree of freedom models, that is, bi-linear type superstructure and m-k-c soil model, are adopted. Then, the earthquake input motion and the driving force are also divided into four segments corresponding to the structure models. Finally, the following vibration equation of each segment model is obtained,

$$[M]\{\ddot{u}^{(\Gamma)}\}+\{R^{(\Gamma)}\}=-[M_E]\{1\}\ddot{u}_g^{(\Gamma)}+\{p^{(\Gamma)}\} \tag{19}$$

where,

$$R_f^{(\Gamma)} = K^{(\Gamma)}\theta_f^{(\Gamma)} + C^{(\Gamma)}\dot{\theta}_f^{(\Gamma)} + M^{(\Gamma)}\ddot{\theta}_f^{(\Gamma)},\quad p^{(\Gamma)} = (K^{(\Gamma)}u_s^{(\Gamma)} + C^{(\Gamma)}\dot{u}_s^{(\Gamma)} + M^{(\Gamma)}\ddot{u}_s^{(\Gamma)})/H_E \tag{20}$$

The real response is evaluated as the sum of the response value of each segment model. The dynamic rotational soil stiffness of each segment utilized for the numeric calculation is shown in Fig.3. In Table 2, ω represents the circular frequency of the segment boundary, ω_g denotes the cut-off frequency, and K_0 is the value of the dynamic soil stiffness at zero frequency, respectively. Fig. 4 shows the concept chart of the proposed numerical calculation scheme.

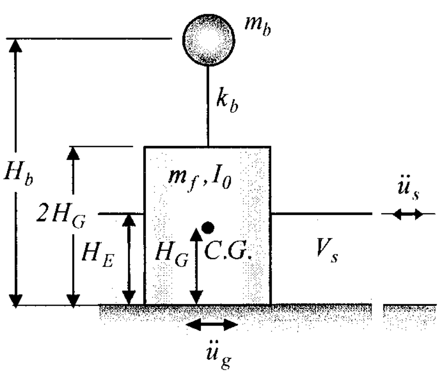

Fig. 2 Numerical Model for Seismic Analysis

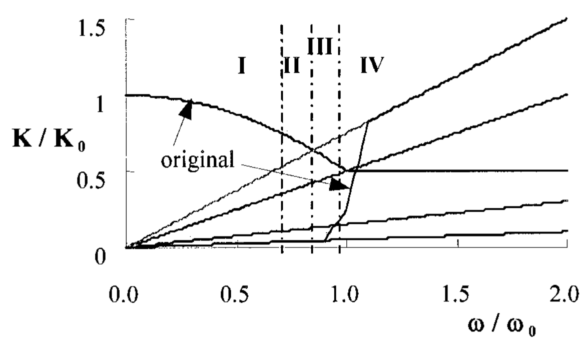

Fig. 3 Dynamic Soil Stiffness divided into Four Segments

Table 2 Static Spring, Mass and Damping Coefficient in Each Segment

Segment	I	II	III	IV
Region	$0 \le \omega < 0.9\omega_g$	$0.9\omega_g \le \omega < \omega_g$	$\omega_g \le \omega < 1.1\omega_g$	$1.1\,\omega_g \le \omega$
Static spring (K) / K_0	1	1	$1/2$	$1/2$
Add. Mass (M) / K_0	$1/2\omega_g^{\,2}$	$1/2\omega_g^{\,2}$	0	0
Damping Coef. (C) / K_0	$0.05/\omega_g$	$0.15/\omega_g$	$0.5/\omega_g$	$0.75/\omega_g$

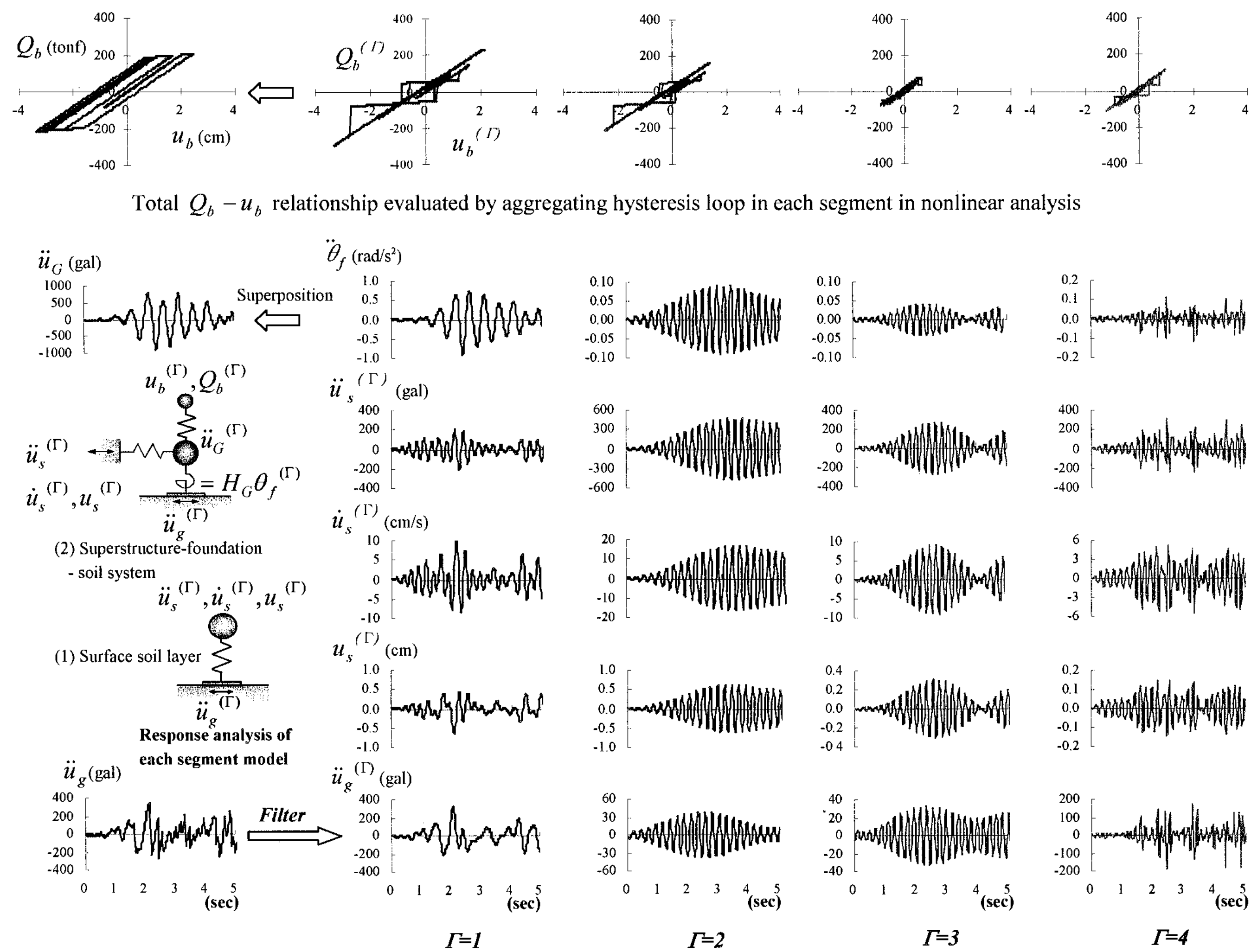

Fig. 4 Concept Chart of Proposed Analysis Scheme for Obtaining Time History Response

5 VERIFICATION OF PROPOSED SIMPLE SCHEME

The accuracy of the numerical analysis method, which uses the proposed multi m-k-c model, is examined through the numerical analysis of the model represented in the previous chapter. The physical constants of the superstructure, the soil properties, and the natural frequencies of the soil-structure system are shown in Table 3, 4, and 5, respectively. Here the behavior of the superstructure is assumed to be linear. EL CENTRO NS (1940) is adopted as an earthquake input motion. The maximum acceleration of the input motion at the bedrock is normalized to 100gal. In this chapter, the response by inverse Fourier transform of the result in the frequency domain analysis (fft) is determined as a rigorous solution to verify the results of the proposed simple scheme (pro). Furthermore, the practical method (a) and (b), which are based on the static stiffness (sta) and the soil stiffness at the predominant frequency (eig), are employed so as to compare with the proposed scheme.

Fig. 5 shows the example of time history of the response obtained by the above four numerical schemes. The horizontal response acceleration at the gravity center of the embedded rigid foundation, $\ddot{u}_G$, and the horizontal response acceleration at the mass of the superstructure, $\ddot{u}_T$, are shown in Fig. 5(A) and (B), respectively. It is clear that the obtained maximum response acceleration value and the time dependency of the time history by the rigorous method (fft) and the proposed scheme (pro) have a good agreement. The practical methods (sta) and (eig) overestimates the response value of $\ddot{u}_G$. On the other hand, both (sta) and (eig) methods underestimate the response value of $\ddot{u}_T$. Fig. 6 shows the transfer functions of $\ddot{u}_G$ and $\ddot{u}_T$ to the input ground motion, $\ddot{u}_g$. The second natural frequency of the soil-structure system which is calculated by the method (sta) and (eig) is higher than that of the exact one.

Further, the both methods (sta) and (eig) cannot take into account the increase of the damping coefficient of the soil stiffness in the high frequency range. Therefore, the results of the both methods overestimate the response amplification nearby the second natural frequency. The hysteresis loops of the resultant reaction moment, R_f, and rotational angle, θ_f are shown in Fig. 7. Consequently, the hysteresis loop of the soil stiffness by the proposed scheme has a good agreement with that of the rigorous method.

6 EXAMPLE OF NONLINEAR RESPONSE ANALYSIS

It is assumed that the model in this chapter employed for a nonlinear response calculation is the same one as in the previous chapter. The elasticity limit displacement of the superstructure is assumed to be 2.5cm (rotation angle:1/200). As for the maximum input level, we determine two levels, 100gal and 200gal at the bedrock. The three schemes are applied to the numerical analysis except for the method (fft).

Fig. 8 shows the relative displacement response of the superstructure for both input levels. In comparison among the method (sta), (eig) and the proposed scheme, drift phenomenon cannot be seen in the method (sta) and (eig) at 100gal input level. The hysteresis loops of the restoring force and displacement are shown in Fig. 9. The agreement between the results of (eig) and the proposed scheme is better in case of 200 gal input level. While the method (sta) and (eig) are often used as practical calculation methods, it is suggested that the effect of the higher-oreder mode be taken account to implement the nonlinear analysis of the system depending strongly on frequencies.

7 CONCLUSION

The paper proposed a simple time history response analysis scheme of the system whose dynamic stiffness functions depend strongly on frequencies. The numerical analyses were also conducted for dynamic soil-structure interaction problems to confirm the validity of the above scheme. It turned out that the accuracy of the result by the proposed multi discretized model is fairly better than that of a single discretized model. Finally, the proposed scheme is applied to the nonlinear response analysis of the structure with bi-linear type hysteresis. While the single discretized model like (sta) or (eig), which adopt the soil spring based on zero frequency and the predominant frequency, is often used as practical calculation methods, it is suggested that the effect of the higher-order mode be taken into account to implement the nonlinear analysis of the system depending strongly on frequencies.

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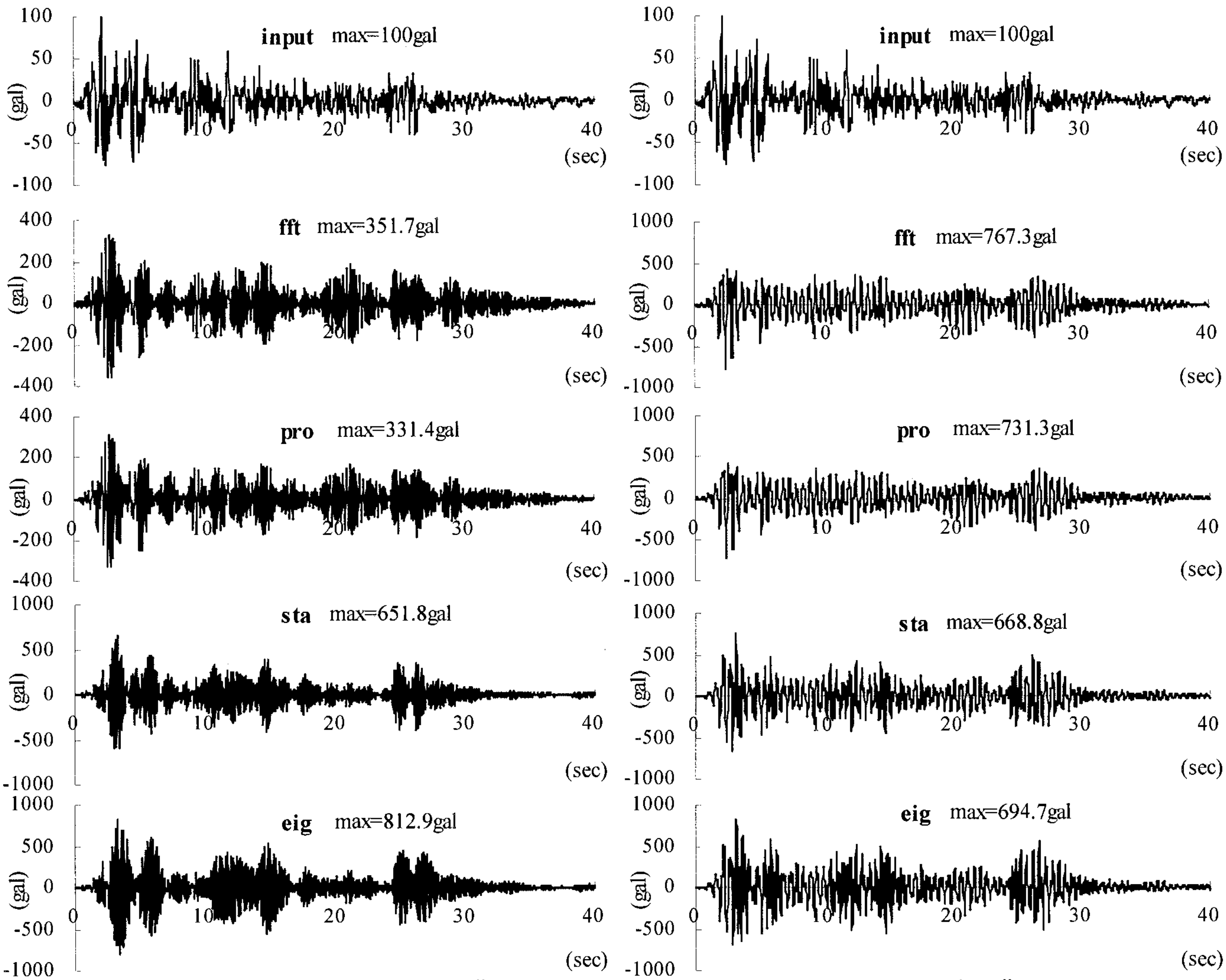


(A) Response Acceleration $\ddot{u}_G$ (B) Response Acceleration $\ddot{u}_T$

Fig. 5 Example of Time History by Four Numerical Calculation Schemes

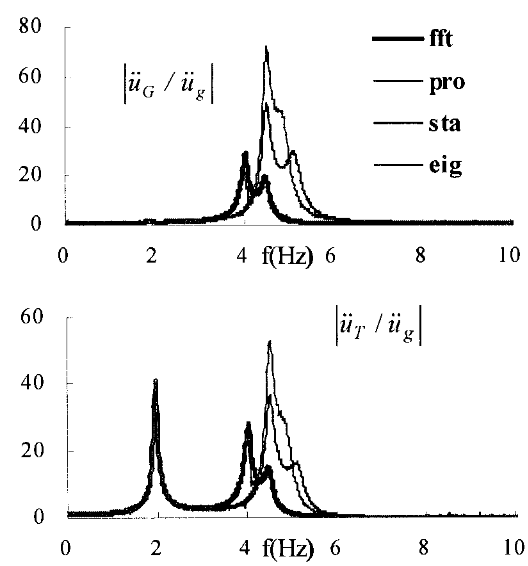


Fig. 6 Frequency Transfer Functions

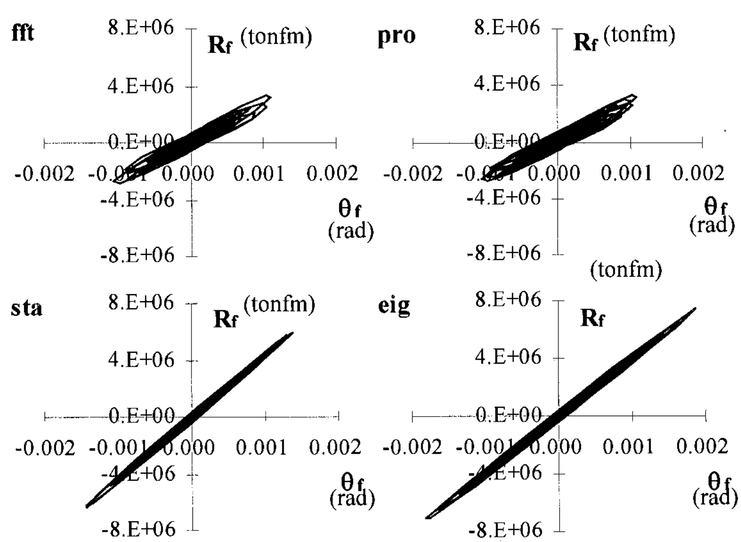


Fig. 7 Loops of Resultant Reaction Moment and Rotational Angle of Rigid Foundation

Table 3 Superstructure

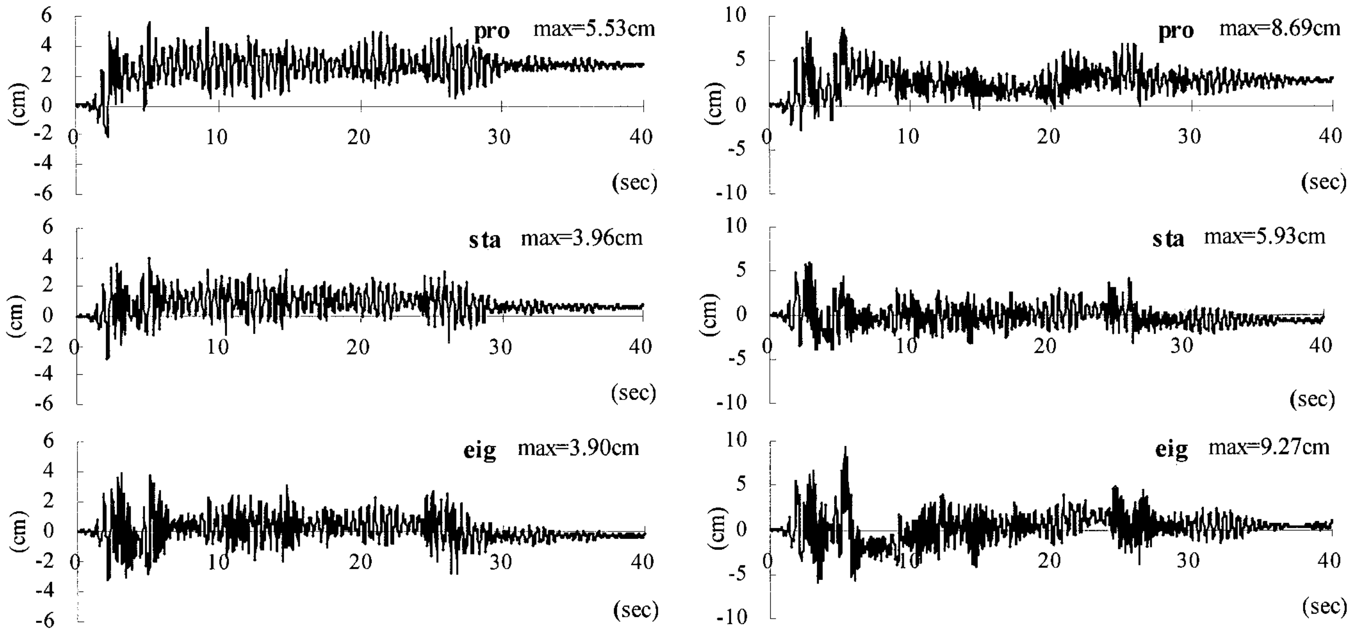
m_b	0.3tonfcm^2/sec
k_b	48.31tonf/cm
H_b	15m
f_b	2Hz

Table 4 Soil Stiffness

V_s	200m/sec
H_E	($2H_G$) 10m
f_g	4.5Hz

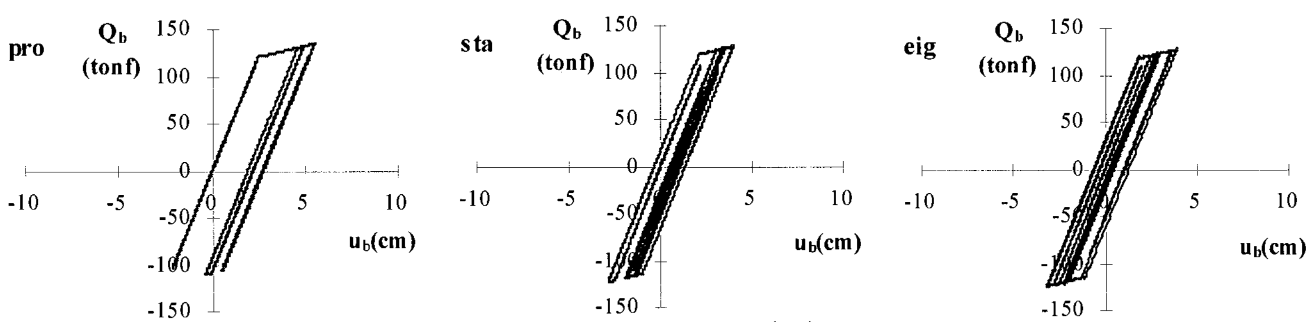
Table 5 Natural Frequency (Linear Case)

f_1	1.97Hz
f_2	4.04Hz

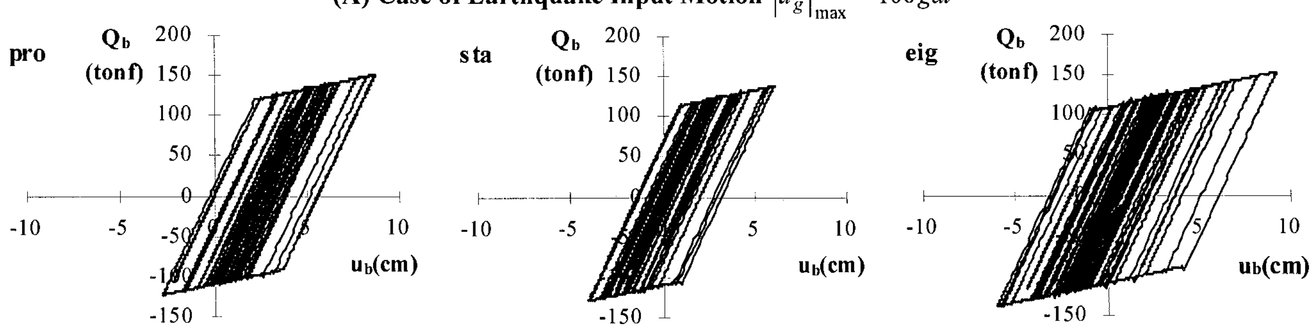


(A) Case of Earthquake Input Motion $|\ddot{u}_g|_{max} = 100gal$ (B) Case of Earthquake Input Motion $|\ddot{u}_g|_{max} = 200gal$

Fig. 8 Time Histories of Nonlinear Displacement Response of Superstructure u_b



(A) Case of Earthquake Input Motion $|\ddot{u}_g|_{max} = 100gal$



(B) Case of Earthquake Input Motion $|\ddot{u}_g|_{max} = 200gal$

Fig. 9 Hysteresis Loops of Superstructure Q_b-u_b