

Reliability Studies of Bored Concrete Pile Foundations

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Summary

There is a growing tendency to base the design of nuclear power plant structures on reliability criteria. In addition, even when conventional code specifications are followed, questions concerning the reliability of the system often arise. The paper presents results of ongoing studies aimed at the determination of the probability distribution of the global strength of bored pile foundations. The analysis resorts to a model in which the piles are represented as series of axially stressed elements, or "links". The parameters used in the model were determined from data available in an extensive experimental program that involved the measurement of the compression strength of about 3000 concrete core samples.

The first part of the study deals with the evaluation of various parameters, such as depth and age, on the strength of 'in situ' concrete. The properties of concrete are then used in conjunction with the pile cross-sectional dimensions to obtain the strength distribution of a pile "link". In a second stage, the global strength probability distribution of a foundation system with a large number of piles is calculated by simulation. The influence of instability effects, which could lead to tilting of the entire foundation, is examined.

1. Introduction

There is a growing tendency to base the design of nuclear power plant structures on reliability criteria. In addition, even when conventional code specifications are followed, questions concerning the reliability of the system often arise. In fact, application of conventional codes may lead, in case of highly redundant structures, to overdesign for certain excitations, without actually increasing the global reliability of the system.

This paper presents preliminary results of studies aimed at the determination of the probability distributions of the global strength of pile foundations. The parameters used in the theoretical model were evaluated from available data, and may be useful in reliability assessments of compressed concrete structures in general.

2. Description of the model

It is assumed that the pile head slab and the superstructure are rigid. The foundation consists of N concrete end-bearing piles, as schematically shown in Fig.1.

The piles are idealized as chains of axially stressed "links". The strength of a link depends on the distribution of the compressive strength of concrete f_c within its volume, according to the model of mechanical behavior introduced in Section 4. Thus, f_c is regarded as a random function of the age t of concrete and the spatial coordinates x , y , z , whose properties are investigated in Section 3. Bending and shear stresses near the pile-slab intersection are neglected at this stage.

3. Properties of concrete strength

It is herein admitted that $f_c = f(x, y, z) \phi(t)$, in which $\phi(t)$ is a smooth monotonically increasing function of time. The variation of concrete strength with age was determined for the same concrete type used in this study and differs very little from the recommendations of the German Standards DIN 1045. Results of compressive strength determinations on 3768 core samples recovered from 29 bored concrete piles were available. Using the function $\phi(t)$, the measured strength values were reduced to the standard 28 days strength, to which all ensuing results are referenced. It was further assumed that f_c was stationary in both horizontal coordinates x and y , i.e., that the mean \bar{f}_c and the standard deviation σ_{f_c} may vary with the vertical coordinate z only. Although in a massive concrete structure such assumption is not necessarily true, since it amounts to neglecting the influence of the boundaries, there was not sufficient data to attempt evaluating its influence.

3.1 Data description

The 3768 core samples tested were organized in individual data sets showing for each pile the measured strength at ordered depths. In addition, on the basis of field descriptions, each sample was assigned a "visual

quality" index: acceptable to good (c = 0) or "doubtful to poor" (c = 1). Although the amount of data is quite large, it must be observed that the basic principles of a statistically designed experiment were not followed in the selection of the samples. For example, the number of cores per pile varied from 14 to 238, and the percentage of pile lengths sampled varied from 10% to 80%. About 50% of the piles had only one bore. On the other hand, there were 7 piles with more than 4 borings, probably due to difficulties encountered in the drilling process.

3.2 Analysis of concrete strength

The concrete strength was initially evaluated, without regards for the influence of depth or other factors, for each of the 29 piles and for the two groups of visual quality, by means of program STATBAS 2, with the results indicated below.

Quantity	Global concrete properties		
	All Samples	Concrete visual quality	
		Acceptable to good	Doubtful to poor
Number of core samples	2768	2484 (90%)	284 (10%)
Minimum value (Mpa)	24	24	46
Mean (Mpa)	340	345	291
Maximum value (Mpa)	982	982	547
Coefficient of variation	30%	29.8%	33.9%

The tests of goodness of fit (Chi-Square, Kolmogorov-Smirnov, Wilk-Shapiro) show that the hypothesis of normality cannot be rejected at the 5% significance level in either case. Using the T and F tests it can be shown that a) the hypothesis of equality of the means should be rejected and b) the hypothesis of equality of the variance cannot be rejected, respectively. These results suggest a bi-modal distribution for the combined sample but, since the difference between the means is less than the common standard deviation, the bimodality will not be apparent in the combined frequency distribution.

3.3 Relation between concrete strength and depth

The combined data set for all the piles was organized in Sections 1 m long. The number of core samples n_i , the mean f_i and the standard deviation σ_i of each section were next determined. It was decided to introduce a weight of the form $W_i = \frac{n_i}{\sigma_i^2} \times 100$, associated with each depth, and to use a nonlinear weighted regressive method to fit the mean concrete strength with a function of the form:

$$f_c(z) = a - b e^{-cz} \quad (1)$$

with the following results: $a = 368.6$ Mpa, $b = 148.0$ Mpa, $c = +0.1217 \text{ m}^{-1}$, standard error of estimate $S_e = 99.0$ Mpa, coefficient of determination 6,5%, with residuals at each depth generally not normally distributed. The small coefficient of determination and the relatively large value of S_e indicate a relatively poor fit in spite of the fact that the coefficients of eq.(1) are

all significantly different from zero. A non-linear fit was also performed using ungrouped strength values in the combined set, with results similar to those referred to above. Fig.2 shows a graph of the variation of \bar{f}_{c_i} with depth. It is concluded that: a) the mean concrete strength increases with depth, b) the rate of increase is more pronounced near the top of the piles c) the standard deviation is not significantly affected by depth, although it strongly oscillates from one section to another.

3.4 Evaluation of the correlation length

The autocorrelation function of the random concrete strength along a vertical axis is defined as:

$$R_{f_c} = \lim_{\Delta z \rightarrow \infty} \frac{1}{\Delta z} \int_0^{\Delta z} [f_c(x, y, z + \Delta z) - \bar{f}_c] [f_c(x, y, z) - \bar{f}_c] dz \quad (2)$$

If the mean \bar{f}_c and the variance of f_c are constant, i.e. do not depend on the spatial coordinates, then the function R_f will also be independent of (x, y, z) , varying only with the vertical distance Δz . Observe that $R_{f_c}(0) = \sigma_{f_c}^2$. The normalized autocorrelation function is given by $\rho_{f_c} = R_{f_c}(\Delta z)/R_{f_c}(0)$. Now, since f_c is assumed to be isotropic, it follows that the separation Δz needs not be taken along a vertical axis, but in any direction. Denoting the distance as u , the "correlation length" L_c is defined as:

$$L_c = \int_0^{\infty} \rho_{f_c}(u) du \quad (3)$$

in which the integral is calculated along an arbitrarily oriented straight line. The strength of concrete cannot be measured at material points, but over a definite volume, for example, the volume of a core sample. Obviously, this prevents measuring strength values at distances smaller than a core length and suggests that L_c should be related to the size of the core samples and to the testing scheme. Since the coordinates of the top of each core sample are given in an integer number of centimeters, the correlation function R_{f_c} can be estimated at discrete points by replacing the integral in eq.(2) by the summation:

$$\hat{R}_{f_c}(n\Delta z) = \frac{1}{N} \sum_{j=1}^N [f_c(x, y, z_j + n\Delta z) - \bar{f}_c] \cdot [f_c(x, y, z_j) - \bar{f}_c] \quad (4)$$

in which $\Delta z = 1$ cm. The mean \bar{f}_c was calculated independently for each individual pile and any variation with depth was disregarded. Moreover, since strength observations were missing for many stations z_j , those terms were not included in the summation. The equation:

$$\hat{\rho} = a_0 + (1-a_0) e^{-\alpha n\Delta z} \quad (5)$$

was fitted to the estimated values of the normalized autocorrelation function by means of a nonlinear least-squares method, resulting $a_0=0.20$ and $\alpha=4.07m^{-1}$. The residual when $u \rightarrow \infty$ is due to pile to pile variation. It may be shown that the correlation length is $L_c = 1/\alpha = 0.245$ m.

4. Determination of the probability of failure of a compressed link

As pointed out in Section 3, the hypothesis of normality of the probability distribution of concrete strength could not be rejected for the combined data. On the other hand, such assumption is accepted by most Codes. In consequence, it is assumed that f_c is a normally distributed random function of the spatial coordinates with constant mean \bar{f}_c , standard deviation σf_c and correlation length L_c .

Cylindrical or prismatic bars subjected to pure compression usually fail, as schematically shown in Fig. 3 along failure surfaces S_i . The mean compressive stress f , at the section under consideration, is given by:

$$f = \frac{1}{A \cos \alpha} \iint_{S_1} \tau \, dA \quad (6)$$

in which A denotes the cross-sectional area and dA an element of area on the failure plane. The integral may be directly expressed in terms of the vertical stress p . Moreover it may be admitted that all points immediately above the failure surface experience the same vertical displacement, and consequently the same vertical deformation ϵ . Eq.(6) indicates that

$$f(\epsilon) = \frac{1}{A} \iint_{S_1} p(\epsilon) \, dA \quad (7)$$

For a given failure surface S_j , the bearing capacity $(f_u)_{S_j}$ of the cross section will be given by the peak value of $f(\epsilon)$, for any ϵ . For a perfectly elasto-plastic material, the next step would be the evaluation of the integral, along the failure surface S_j , of the random function f_c :

$$(f_u)_{S_j} = \frac{1}{A} \iint_{S_j} f_c(x, y, z) \, dA \quad (8)$$

This problem was studied by Matern[3], for the case in which f_c can be assumed to be "stationary". In this context, however, it is necessary to evaluate the strength f_u of the cross section, not the strength associated to a predetermined failure surface. In fact, for all possible failure modes, the strength of a link is $f_u = \min[(f_u)_{S_1}, (f_u)_{S_2}, \dots, (f_u)_{S_n}]$.

If f_c is normally distributed, when $(f_u)_{S_j}$ is given by eq.(8), the strength for any given failure mode will also be normally distributed. Under such conditions, and assuming independence between the $(f_u)_{S_j}$, f_u would present a type I distribution with parameters that may be determined in terms of \bar{f}_c , σf_c and n . However, a closed solution does not seem possible, because the correlation between the strengths $(f_u)_{S_j}$ is not null. On the contrary, for neighbouring failure surfaces, the associated strengths will be strongly correlated. In addition, the behavior of concrete differs significantly from that of a perfectly elasto-plastic material, and the ensuing difficulties render a closed solution unfeasible. Therefore, a Monte Carlo solution was attempted.

4.1 Solution by simulation

Let's consider a square cross-section, with width equal to $b = n L_c$, that is, equal to n times the correlation length of the concrete strength f_c . It is assumed that failure can occur in any of the modes shown in Fig. 3. In every case, the failure planes form a 45° angle with the column axis. In each failure mode n^2 cubic elements are intercepted. The integral of eq.(10) is now substituted by the sum:

$$f(\epsilon) = \frac{1}{n^2} \sum_i^{n^2} p_i(\epsilon) \quad (9)$$

in which i takes the values corresponding to all elements intercepted by S_j , and $p_i(\epsilon)$ denotes the stress-strain law of element i . Consequently:

$$(f_u)_{S_j} = \max \left[\frac{1}{n^2} \sum_i^{n^2} p_i(\epsilon) \right] ; \text{ for any } \epsilon \quad (10)$$

Obviously, other failure surfaces are possible, but in view of the strong correlation between corresponding ultimate loads, the results should not be significantly affected.

4.2 Stress-strain relation for concrete

In the standard test for the determination of the compressive strength f_c , the stress field is not uniaxial, and the resulting stress-strain diagrams reflect certain degree of lateral confinement, which is introduced in the sample through friction at both ends. This lateral confinement does not significantly affect the ascending branch, up to the value of f_c , but largely governs the behavior of the descending branch. Since in a compressed pile or column the degree of confinement is not easily quantifiable, two extreme cases were considered [1]:

(a) fully confined concrete:

$$p(\epsilon) = \frac{c^2}{4} (2\epsilon_c \epsilon - \epsilon^2) \quad \text{for } \epsilon < \epsilon_c, \text{ and } p(\epsilon) = f_c \text{ for } \epsilon \geq \epsilon_c \quad (11)$$

(b) confined concrete

$$p(\epsilon) = \frac{c^2}{4} (2\epsilon_c - \epsilon^2) \quad \text{for } \epsilon < 2\epsilon_c, \text{ and } p(\epsilon) = 0 \text{ for } \epsilon \geq 2\epsilon_c \quad (12)$$

in which ϵ_c denotes the strain corresponding to the peak of the stress-strain diagram and is related to f_c by means of $\epsilon_c = 2 \sqrt{f_c}/c$, in which $c^2 = 1,89^2 10^8$ dN/cm. The value of f_c completely defines the stress-strain curve.

4.3 Statistical processing of the simulated link strength

For given values of f_c and σ_{f_c} , series of 50 simulated values of the link strength were obtained for all possible combinations of the parameters: $n = 2, 3, 4$ and 5 and $cv_{f_c} = 0.20, 0.29$ and 0.40 . It was not possible to reject the hypothesis of normality for the distribution of f_y at the 90% confidence level. Next, the adimensional coefficients $\eta = \bar{f}_u / \bar{f}_c$ and

$\xi = \frac{\sigma_{f_u}}{\sigma_{f_c}}$, which relate the mean link strength \bar{f}_u and standard deviation of f_u to the concrete mean compressive strength and standard deviation, were introduced. Through standard regression techniques, the following expressions were fitted to the simulated strength data:

a) Confined concrete:

$$\eta = 0.99797 - 0.33536 \text{ cv}_{f_c} + 0.00794 \text{ cv}_{f_c}(n-1) \quad (13)$$

b) Unconfined concrete:

$$\eta = 1.01181 - 0.60274 \text{ cv}_{f_c} + 0.04775 \text{ cv}_{f_c}(n-1) \quad (14)$$

the equations above are valid for $n \geq 2$. The coefficient of multiple determination was larger than 0.9 and the standard error of estimate less than 0.009. It was also found that ξ does not depend on any significant degree on either cv_{f_c} or the degree of lateral confinement. A satisfactory approximation is given by:

$$\xi = c^{-b(n-1)} \quad (15)$$

in which b was evaluated through linear regression and found to be $b = 0.53$. Eq.(15) quantifies the rapid decay of the variability of the cross-sectional strength of a compressed concrete member as its size increases. Note that in case of circular member, n can be approximated by $n \approx \sqrt{\frac{\pi D}{4 L_c}}$.

4.3 Correlation length of the cross-sectional strength along the longitudinal axis.

The correlation length L_p of the cross-sectional strength should obviously exceed, for $n > 1$, the value of L_c . On the other hand, an upper bound for L_p is given by $L_p < nL_c = b$. The determination of L_p by simulation is straightforward but requires an enormous computational effort and will not be further discussed in this report.

5. Determination of the loading capacity of an axially compressed member.

Each pile is idealized as consisting of N independent "links". It is therefore a system connected in series, whose strength f_y is given by

$$f_y = \min [f_{u1}, f_{u2}, \dots, f_{uN}] \quad (16)$$

in which f_{ui} denotes the strength of each link. If the strengths of all the links are normally distributed with the same mean and standard deviation, then for large values of N the probability distribution F_y of f_y is type I, with parameters given by (Ref.4):

$$\bar{f}_y \approx \bar{f}_u - \sigma_{f_u} \sqrt{\ln N} ; \quad \sigma_{f_y} \approx \frac{\pi}{\sqrt{6 \ln N}} \sigma_{f_u} \quad (17)$$

$$\text{finally: } f_y(y_o) \approx 1 - \exp \left[-\exp \left(\frac{y_o - \bar{f}_y}{\sigma_{f_y}} \right) \right] \quad (18)$$

The results of eqs.(17-18) can be resorted to if the assumptions that the cross-sectional area is uniform throughout the length, and that

\bar{f}_c and σ_{fc} do not vary with depth are accepted. In a more general case, however, the distribution of f_y should also be obtained by simulation. Finally, to insure that the values of f_{uj} in eq.(16) are independent, the length of each link should be taken equal to the correlation length L_p . In what follows it will be admitted that $L_p \approx b$ in case of square sections, and to the diameter, in case of circular sections.

6. Evaluation of global strength of pile foundation.

In a first stage, only two degrees of freedom were considered: the vertical displacement q_1 and the rotation $q_2 = \theta a$ around a vertical plane of symmetry, as indicated in Fig.1. The generalized forces Q_1 and $Q_2 = 2 M/a$, in which M denotes the bending moment around 0 are given by:

$$Q_1 = \sum_{j=1} P_j \quad (19)$$

$$Q_2 = \sum_{j=1} P_j \beta_j \quad ; \quad \beta_j = y_j/a \quad (20)$$

y_j and P_j denoting the y -coordinate and the axial force of pile j , respectively. The incremental equations:

$$\Delta Q_j = \frac{\partial Q_j}{\partial q_1} \Delta q_1 + \frac{\partial Q_j}{\partial q_2} \Delta q_2 \quad ; \quad j = 1,2 \quad (21)$$

can be written in the form: $\Delta \vec{Q} = [k_{ij}] \Delta \vec{q}$, in which $k_{11} = \sum_{j=1} \frac{\partial P_j}{\partial u_j}$, $k_{12} =$

$k_{21} = \sum_{j=1} \frac{\partial P_j}{\partial u_j} \beta_j$ and $k_{22} = \sum_{j=1} \frac{\partial P_j}{\partial u_j} \beta_j^2$. The vertical displacement of each

pile is given by: $u_j = q_1 + q_2 \beta_j$. Eqs.(21) can be solved for the displacements:

$$\Delta q_1 = \left[(k_{22} - \frac{Q_1}{a^2} - k_{12}(e_o/a + q_2 L/a^2)) \right] \Delta Q_1 / D \quad (22)$$

$$\Delta q_2 = \left[-k_{12} + k_{11}(e_o/a + q_2 L/a^2) \right] \Delta Q_1 / D \quad (23)$$

$$D = k_{11} (k_{22} - Q_1 L/a^2) - k_{12}^2 \quad (24)$$

After each load increment, the displacements are corrected by iteration, until satisfactory convergence is achieved. Instability occurs when D changes sign.

6.1 Load-displacement relation for a single pile.

This item is subject of ongoing research. In the analysis reported in Section 7, the pile stress was related to the top displacement u_j by means of the equation:

$$p(u_j) = \frac{\epsilon_o u_j}{l_j \left[1 + \left(\frac{u_j}{l_j \epsilon_c} \right)^2 \right]} \quad (25)$$

in which l_j denotes the pile length.

7. Sample analysis

The pile foundation shown in Fig.1, with $N = 202$ endbearing piles, $L=20m$, and $a=30m$ was analyzed using the model described above. The length of the piles is variable, ranging from 30 m to 48 m. The interior and peripheral piles are 1.10 m and 1.30 m in diameter respectively. The variation of the mean concrete strength \bar{f}_c with depth was neglected, uniform values $\bar{f}_c=340dN/cm^2$ and $cv_{f_c} = 0.30$ being adopted. The correlation length L_c for concrete strength was taken equal to 0.25 m. The strengths of the piles were obtained by simulation, by means of eqs.(21) to (31) and the global stability analysis of the foundation performed as described in Section 7. The vertical displacements of the foundation plate at points along the O_z axis in terms of the total vertical load for a typical run are depicted in Fig.4, in which the global strength, equal to 5.88 GN, can be identified. The mean of five simulations was 5.93 GN, with a coefficient of variation of 0.77%. The small variability of the global strength is in this case due to the large degree of redundancy of the system, fact that permitted an important reduction in the number of necessary simulations.

Introduction of an excentric load, with $e_o = 2.65$ m, leads to a reduction of approximately 10% in the global strength, accompanied of a slight increase of its variability. It was also verified that, as might be expected, tilting of the foundation was significant only at load levels very close to the carrying capacity and therefore, approximate results can be obtained with a 1DOF model if the loads are centric.

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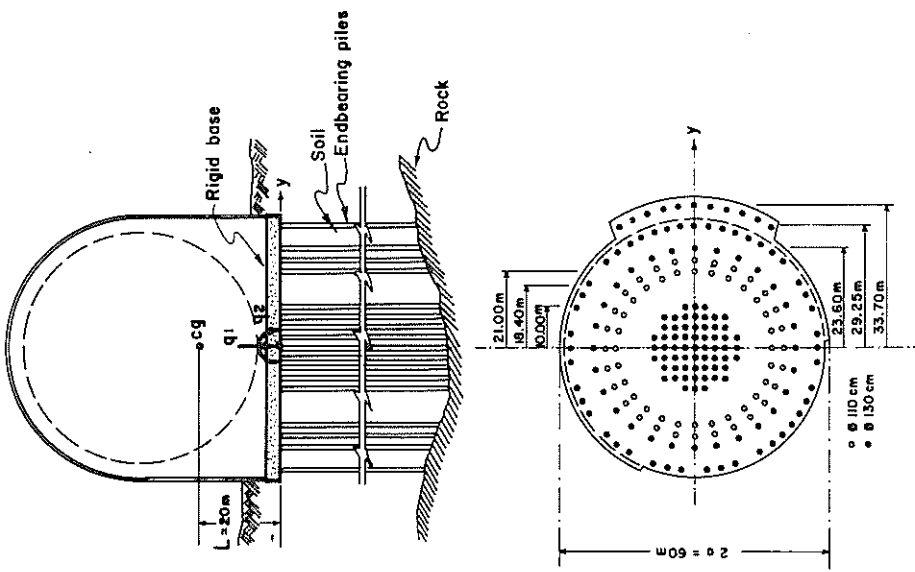


Fig. 1. Schematic view of foundation model, coordinate system and basic dimensions.

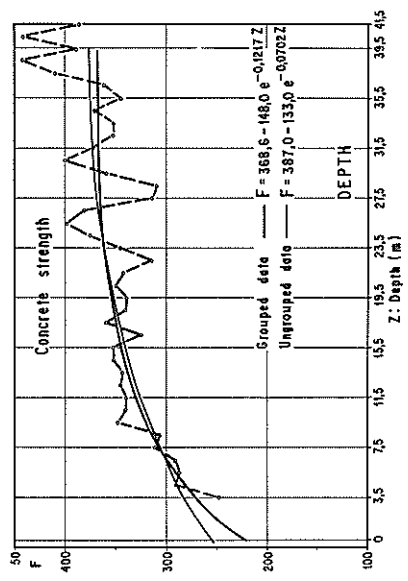


Fig. 2. Variation of mean 28 days concrete strength \bar{f}_c with depth for bored concrete piles.

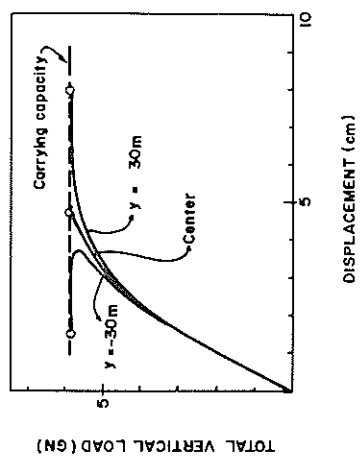


Fig. 4. Typical vertical load-displacement curve for pile foundation.

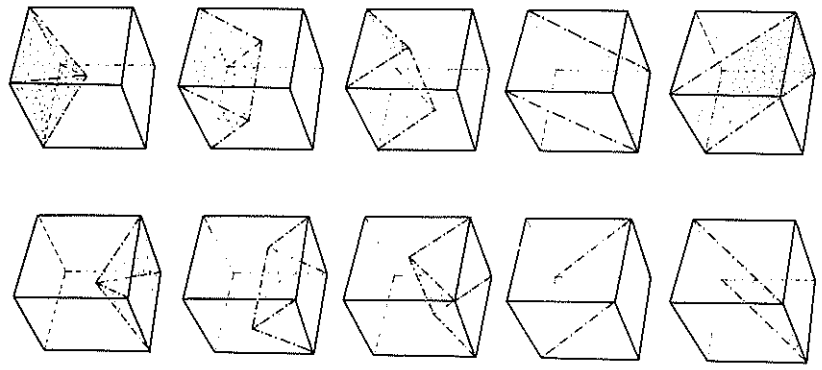


Fig. 3. Possible failure modes for prismatic concrete element subjected to axial compression.