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MATERIAL EQUATIONS FOR THE CALCULATIONS OF STEEL FIBER REINFORCED CONCRETE MEMBERS

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1. Abstract

Steel fiber reinforced concrete (SFRC) is made by the addition of steel fibers to fresh concrete. Usually the fibers are about 0.4-0.8mm in diameter and 25-80mm long. The addition of about 50-120 kg/m^3 is a practical and useful amount. That is about 0.6-1.5% by volume. The fibers are uniformly dispersed with a suitable concrete mix, so that clusters and uneven concentrations are prevented.

The tensile strength of steel fiber reinforced concrete is scarely better compared to that of plain concrete, but the fibers are very effective at preventing the propagation of tensile cracks. Thereby the tensile strength of fiber reinforced concrete is a reliable value. The addition of steel fibers also leads to a considerable increase of plastic deformations in the post cracking region, in comparison to plain concrete members. For nuclear power plant construction the use of steel fiber concrete with additional reinforcement of normal or prestressing steel is of special interest. The finished members exhibit good crack behaviour, increased shear strength and a considerable ability to absorb mechanical energy. These are valuable properties for members providing protection against extreme load cases (e.g. aircraft crash, earthquake, blast caused by explosion, debris due to hurricane, internal pressure loads or debris due to bursting of vessels or pipes). The Fig.1 shows the behaviour of a reinforced concrete beam with steel fiber reinforced concrete against that of a reinforced beam without.

Until now the use of steel fiber reinforced concrete in civil engineering has been restricted because of the lack of design rules. For the preparation of fundamental principles and for the development of design rules HOCHTIEF has undertaken a series of tests on steel fiber reinforced concrete members with and without additional bar reinforcement. For this purpose HOCHTIEF has carried out several series of tests using either static, impact or cyclic loadings. In section 2 of this paper the elements of a mathematical model are built up upon the principles based on the generally recognized behaviour of concrete.

In section 3 these elements are used in the description on the non-linear behaviour of SFRC. In sub-section 3.1. the incremental theory is applied, in sub-section 3.2 the model used in the Total Deformation Theory (HENCKY-model). A detailed examination is included in report [1].

2. Yield Criteria in the Stress Space

2.1 Some notations used:

The invariants of the stress tensors are described with

$$\sigma_1 = \sigma_{ii} = \sigma : I, \quad \sigma_{II} = \sigma_{ij}\sigma_{ji} = \sigma^2 : I \text{ ...etc..}$$

The stress deviator σ' is defined by $\sigma' = \sigma - \frac{1}{3}\sigma_I I$.

With the help of its second invariant the v.MISES-stress is defined:

$$\sigma_\upsilon^2 = \frac{3}{2}\sigma' : \sigma'$$

The principle stresses are σ_{hj}, j=1,2,3. They are arranged in the following order: $\sigma_{h_1} \geq \sigma_{h_2} \geq \sigma_{h_3}$. Their Eigenvectors nj are assumed as in standard practice. Especially valid is:

$$\sigma_{h_1} = \sigma . n_1 n_1^T, \quad n_1^T n_1 = 1,$$

In the construction of the material equations for SFRC the values $\sigma_\upsilon, \sigma_I, \sigma_{h_1}$ and the Eigenvector n_1 are employed. The description of the concrete constants are chosen following Eurocode No.2, Part 1[2].

2.2 Description of the Yield Condition

The degree of stress intensity is defined in the following terms:

$$\varphi = \sigma_v + c_1\sigma_{h_1} + c_2\sigma_I - c_3 \qquad (1)$$

It is required that

$$\varphi \leq 0. \qquad (2)$$

In the keeping of this requirement the plastic strain ε^p is needed.

The coefficients c_1, c_2, c_3 are not constant, but depend on the momentary plastic condition. This condition is defined by the parameters ε_υ^p and ε_H^p. The properties of SFRC are considered with the functions $h_2(\varepsilon_\upsilon^p)$ and $h_1(\varepsilon_H^p)$ .The function $h_2(\varepsilon_\upsilon^p)$ describes the nonlinear behaviour of concrete in the compression zone. The function $h_1(\varepsilon_H^p)$ describes the softening behaviour of SFRC after the onset of the development of tensile cracks. Fig. 2 shows the behaviour under single axial loading. The linear-elastic region of the concrete occures when $-f_{y,0} \leq \sigma_1 \leq f_t$. After the development of tensile cracks only the stresses $\sigma_1 \leq f_z = h_1(\varepsilon_H^p)$are possible In the compression zone $\sigma_1 \geq -f_y$, $f_y = h_2(\varepsilon_\upsilon^p)$is valid. The chosen functions h_1, h_2 and the accompanying $\sigma_1 / \varepsilon_1 -$ plots are reproduced in Fig.2.

The determination of the coefficients c_1 to c_3 are found with the following approximation points, every φ = o is valid (Fig.3).

Pkt. σ_1	σ_2	σ_I	σ_υ	σ_{h_1}	
(1): f_z	0	f_z	f_z	f_z	
(2): $-f_y$	0	$-f_y$	f_y	0	(3)
(3): $-\beta_{2p}f_y$	$-\beta_{2p}f_y$	$-2\beta_{2p}f_y$	$\beta_{2p}f_y$	0	

The coefficient $\beta_{2p}$is taken as a constant.In this paper the value $\beta_{2p} = 1{,}17$ is used[3]. In Fig. 3 the two boundary curves $\varphi = 0$ in the σ_1 / σ_2-planes are reproduced.

Graph (1): $f_y = f_{y,0}$, $f_z = f_t$ "Start of Yielding"

Graph (2): $f_y = f_c$, $f_z = f_t$ "Ultimate Conditions"

With $\sigma_{h_1} \geq 0$ coefficients c_1 to c_3 are obtained. Therefore only the principle stress $\sigma_{h_1} > 0$ is considered with respect to the stress intensity (1).

The condition $\varphi \leq 0$ can be produced in the stress-space (Fig.4). That is

$$\sigma_\upsilon^2 = \frac{3}{2} r^2, \sigma_{h_1} = \sqrt{\frac{2}{3}} |a_1'| + \frac{1}{3}\sigma_I \tag{4}$$

Consequently the equations (1) and (2) are equivalent to the equation

$$\sqrt{\frac{3}{2}}(r + \frac{2}{3} c_1 |a_1'|) \leq c_3 - (c_2 + \frac{1}{3} c_1)\sigma_I \tag{5}$$

On the left hand side of the equation is the stress which is related to the momentary strength of SFRC on the right hand side. .

3. Elastic-Plastic Material Behaviour

3.1 Incremental Description

The basis of this theory is built upon the classic material description

$$\dot{\sigma} = 2G\left\{\dot{\varepsilon} - \dot{\varepsilon}^p + \frac{\upsilon}{1-2\upsilon}(\dot{\varepsilon}_I - \dot{\varepsilon}_I^p)I - \alpha\frac{1+\upsilon}{1-2\upsilon}\dot{\Theta} I\right\} \tag{6}$$

To obtain the rate of plastic strain, the associated flow rule is used, which has been slightly modified..

$$\dot{\varepsilon}^p = \lambda\left\{\frac{3\sigma'}{2\sigma_\upsilon} + c_1 s_1 n_1 n_1^T + c_2 s_2 I\right\}, \tag{7}$$

With the associated condition $\lambda \geq 0$ the dissipation rate is $D = \sigma : \dot{\varepsilon}^p \geq 0$. The plastic parameter ε_υ^p and ε_H^p follow from (7):

$$\dot{\varepsilon}_\upsilon^p = \sqrt{\frac{2}{3}\dot{\varepsilon}^p : \dot{\varepsilon}^p}$$
$$\dot{\varepsilon}_H^p = \dot{\varepsilon}^p : n_1 n_1^T \tag{8}$$

The parameter ε_H^p is used for the measurement of the distributed crack widths..

3.2 Total Deformation Theory (Hencky-Model):

The equations shown are also suitable to describe the plastic behaviour of SFRC in the sense of HENCKY. The equations (1) and (2) serve as the boundary conditions in the stress-space. The stress is calculated from

$$\sigma = 2G\left\{\varepsilon - \varepsilon^p + \frac{\upsilon}{1-2\upsilon}(\varepsilon_I - \varepsilon_I^p)I - \alpha\frac{1+\upsilon}{1-2\upsilon}\Theta I\right\} \tag{9}$$

with

$$\varepsilon^p = \lambda\left\{\frac{3\sigma'}{2\sigma_\upsilon} + s_1 n_1 n_1^T + s_2 I\right\} \tag{10}$$

From (8) and (10) the plastic parameters ε_υ^p and ε_H^p are found. Numerical calculations show no significant deviation in behaviour as compared to the incremental description.

4. Comparitive Calculations of Selected Tests.

4.1. Beam-Test (Fig.5)

Plots of Load-Deformation are shown

Graph (1) Measured Reading
Graph (2) Calculation with a beam-element
Graph (3) Calculation with a plane element (9 nodes)

4.2. Test on Plate (Fig.6)

The Load-Deformation plots are shown:

Graph (1) Measured Reading
Graph (2) Calculation with the assumption of a cylindrical system (9 nodes)

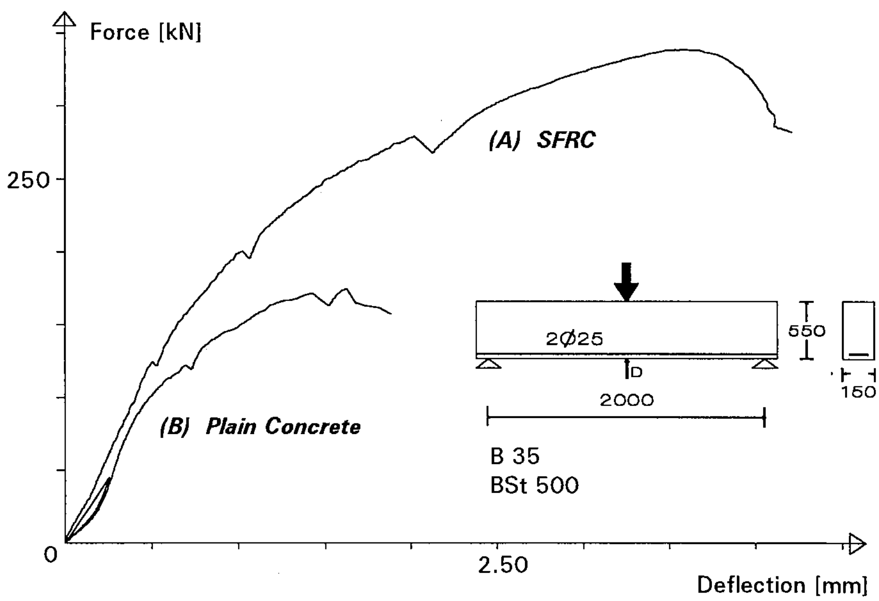


Fig. 1:Load-Deflection curves

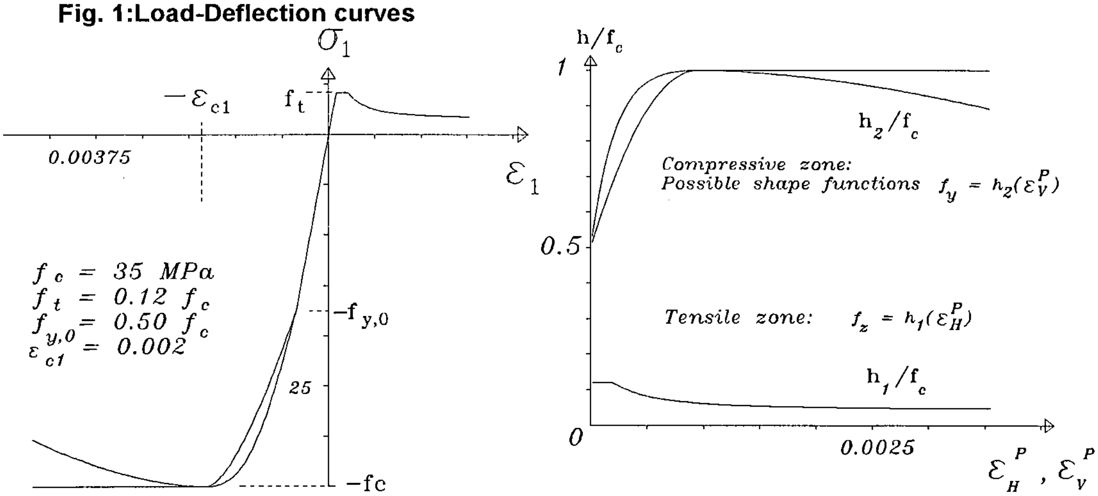


Fig. 2: Uniaxial behaviour and shape functions h_1 and h_2

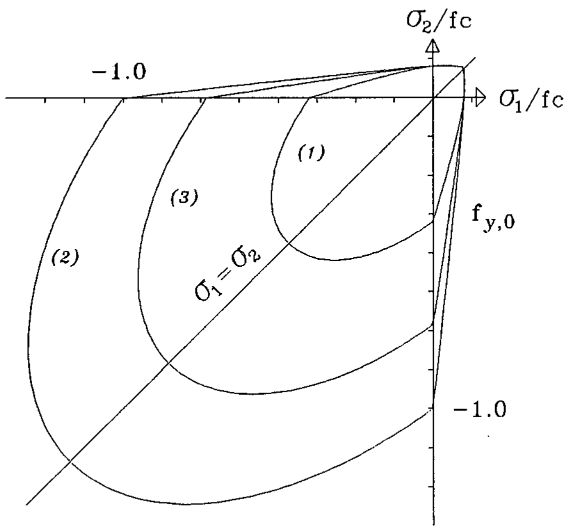


Fig. 3: Plots of φ=0 in σ_1/σ_2 plane

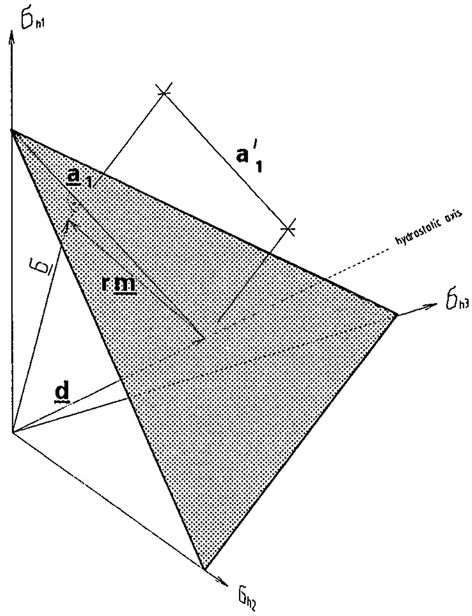


Fig. 4: Principal stress space

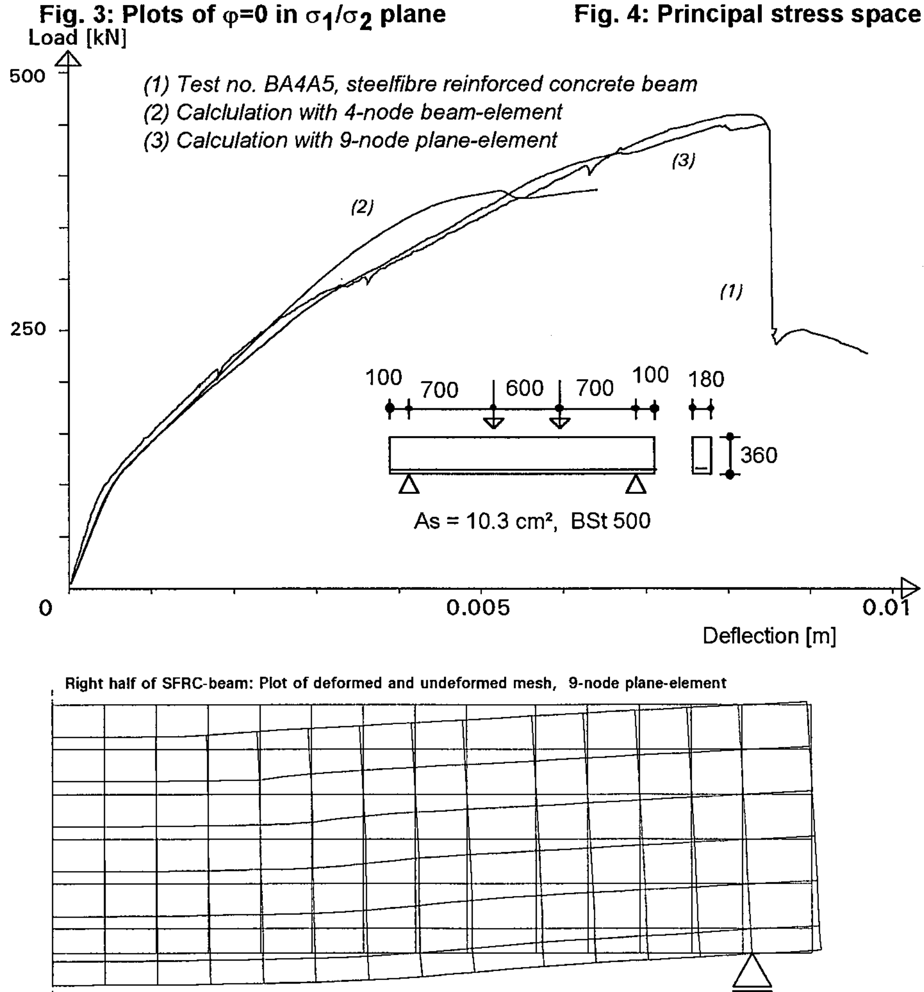
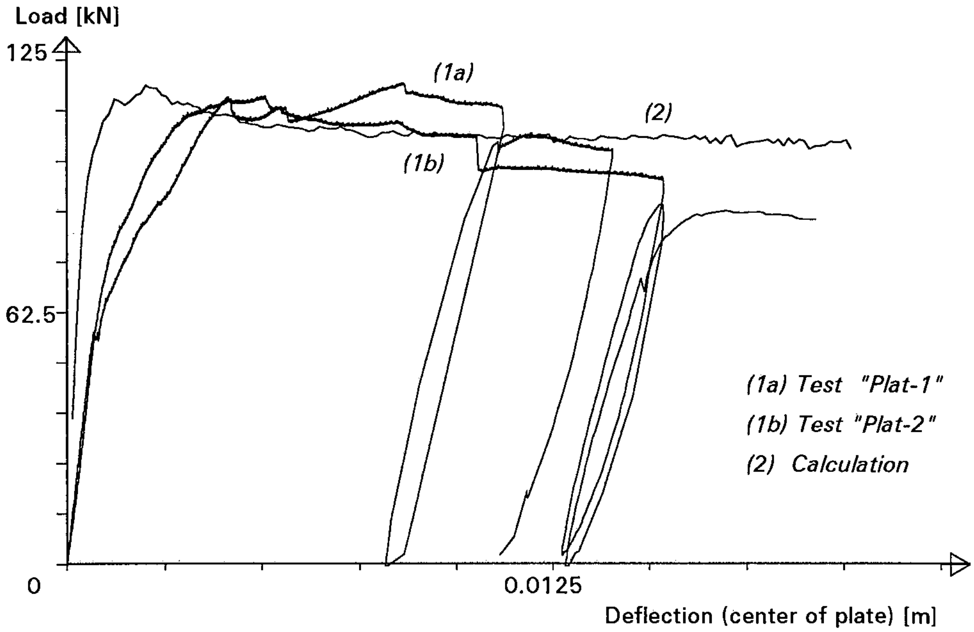


Fig. 5: Test of SFRC-beam: Measurement, calculations and deformed mesh plot



Steelfibre reinforced concrete plate (100/100/10 cm) with concentrated load in the center

Fig. 6: Test of SFRC-plate, measurements and calculation

Literature:

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