Effect of Cracks on the Dynamic Characteristic of Pipes

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ABSTRACT

This paper treats a pipe as a beam or as a thin shell. Then, the effect of cracks on the dynamic characteristics of pipes is calculated by means of the model of zero dimension element with crack. The results of the examples calculated show very good agreement with experimental results.

NOMENCLATURES

A—area of crack
E—young's modulus of the pipe material
μ—Poisson radio of the pipe material
G—energy release rate

$$E^* = \begin{cases} E\text{—in plane stress} \\ E/(1-\mu^2)\text{—in plane strain} \end{cases}$$

G^*—shear modulus of the pipe material
K—stress intensity factor (like K_{IP}, K_{IM}, K_{IIC}, K_{IIIQ})
R—radius of pipe
h—thinkness of pipe
m—number of nodes in axial mode shape
n—number of circumferential waves

1 INTRODUCTLON

Alomst all reported studies however considered ideal uncracked pipes. People, especially those who pay particular attention to safe factor, anxiously want to know the quantitative effect (not only qualitative concept) of cracks on the original design parameters of structures.

Piping systems are important parts of oil industry, nuclear power plant, etc. Vibration of pipes with cracks may be harmful to the safety of production. In this paper, the dynamic characteristics of the pipe of which the displacement vectors are not continuous everywhere due to the presence of cracks are studied theoretically and experimentally.

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In this paper, a pipe is treated as a beam when the ratio of its radius to its length is small ($R / L < 5$) or as a thin shell of revolution when $R/L > 5$ and $R / h > 20$.

2 MODEL OF NUMRICAL CALCULATION

For the investigation of static solution of structures with crack, the solution of this problem is obtained as superposition of two parts (Fig.1). Now the dynamic characteristics of pipes with cracks are studied following to the same principle. The effect of cracks on pipes is reflected by a special finite element in the calculation of general finite element method. It is called zero–dimension element with crack (i.e., the element length on the perpendicular direction of the crack surface is zero). This particular crack element is assumed to have the following properties: ①The nodes of the element are on the opposite surfaces of the crack. They have the standard nodal degrees of freedom associated with the general finite element, but the dimensional parameter for expressing the size of the element is zero. ②The crack causes no change on the distribution of structural density, so that the mass matrix of the element is a zero matrix.

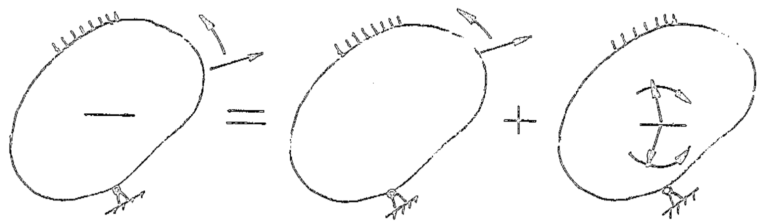


Fig.1 Division and superpoosition of problem with crack

The local flexibility of the structure will be increased and its local stiffness is decreased due to the presence of the crack. According to definition, flexibility coefficient λ_i is

$$\lambda_i = u_i / P_i \tag{1}$$

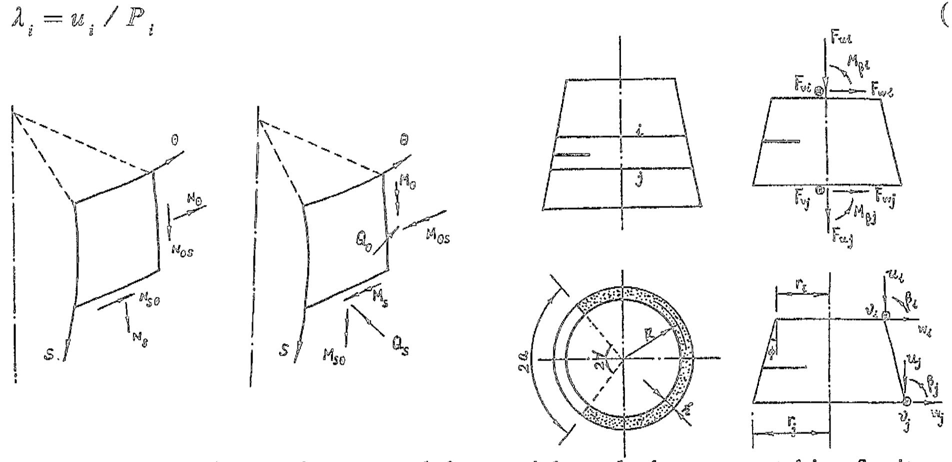


Fig.2 Truncated cone element and the special crack element matching for it.

As shown in Fig.2, the dimension between node i and node j of particular crack element of the pipe tends to zero. Considering static equilibrium, we have $F_{ui} = -F_{uj} = P$, $F_{vi} = -F_{vj} = Q$, $F_{wi} = -F_{wj} = \tau$, and $M_{\beta i} = -M_{\beta j} = M$. By means of equation (1):

$$
\begin{cases}
u = \lambda_{pp} P + \lambda_{mp} M \\
\beta = \lambda_{PM} P + \lambda_{mm} M \\
v = \lambda_{QQ} Q \\
w = \lambda_{\tau\tau} \tau
\end{cases}
\tag{2}
$$

Solution of equations (2) is:

$$
\begin{cases}
P = (u\lambda_{mm} - \beta\lambda_{mp}) / (\lambda_{pp}\lambda_{mm} - \lambda_{mp}^2) \\
M = (\beta\lambda_{pp} - u\lambda_{mp}) / (\lambda_{pp}\lambda_{mm} - \lambda_{mp}^2) \\
Q = v / \lambda_{QQ} \\
T = w / \lambda_{\tau\tau}
\end{cases}
\tag{3}
$$

Then, u,v, w, β are expressed in terms of the nodal displacements (u_i, v_i, w_i, β_i) and (u_j, v_j, w_j, β_j). The strain enegy of this special crack element is:

$$
U = \frac{1}{2D}[(u_j^2 - 2u_j u_i + u_i^2)\lambda_{mm} - 2(u_j\beta_j - u_j\beta_i - u_i\beta_j + u_i\beta_i)\lambda_{mp} + (\beta_j^2 - 2\beta_j\beta_i + \beta_i^2)\lambda_{pp}] + \frac{1}{2\lambda_{QQ}}(v_j^2 - 2v_j v_i + v_i^2) + \frac{1}{2\lambda_{\tau\tau}}(w_j^2 - 2w_j w_i + w_i^2)
\tag{4}
$$

where $\quad D = \lambda_{pp}\lambda_{mm} - \lambda_{mp}^2$

According to Castigliano's theorem and paying attention to expressions related finite element method, the stiffness matrix of the special crack element is obtained as following:

$$
[K^e] = \begin{bmatrix}
\lambda_{mm}/D & 0 & 0 & -\lambda_{mp}/D & -\lambda_{mm}/D & 0 & 0 & \lambda_{mp}/D \\
 & 1/\lambda_{QQ} & 0 & 0 & 0 & -1/\lambda_{QQ} & 0 & 0 \\
 & & 1/\lambda_{\tau\tau} & 0 & 0 & 0 & -1/\lambda_{\tau\tau} & 0 \\
 & & & \lambda_{pp}/D & \lambda_{mp}/D & 0 & 0 & -\lambda_{pp}/D \\
 & & & & \lambda_{mm}/D & 0 & 0 & -\lambda_{mp}/D \\
\textit{symmetry} & & & & & 1/\lambda_{QQ} & 0 & 0 \\
 & & & & & & 1/\lambda_{\tau\tau} & 0 \\
 & & & & & & & \lambda_{pp}/D
\end{bmatrix}
\tag{5}
$$

The flexibility coefficient λ_{ij} in equation (5) is obtained by means of fracture mechanics. Considering the relation between the energy release rate G and the stress intensity factor K:

$$
G = (K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2) / E^* + (K_{\mathrm{III}}^2) / (2G^*)
\tag{6}
$$

and Irwin–Kies equation:

$$
G_i = \frac{P_i^2}{2}\frac{d\lambda_i}{dA}
\tag{7}
$$

we have:

$$
\lambda_{pp} = \frac{2}{E}\int_o^A \left(\frac{K_{IP}}{P}\right)^2 dA, \qquad \lambda_{mm} = \frac{2}{E}\int_o^A \left(\frac{K_{IM}}{M}\right)^2 dA,
$$

$$
\lambda_{QQ} = \frac{2(1+\mu)}{E}\int_o^A \left(\frac{K_{\mathrm{III}Q}}{Q}\right)^2 dA, \qquad \lambda_{\tau\tau} = \frac{2}{E}\int_o^A \left(\frac{K_{II\tau}}{\tau}\right)^2 dA,
$$

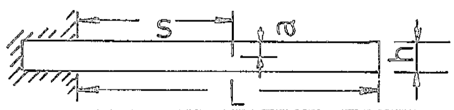
$$\lambda_{mp} = \lambda_{pm} = \frac{2}{E}\int_{o}^{A} (\frac{K_{IP}}{P})(\frac{K_{IM}}{M}) dA \tag{8}$$

where $dA = (R_e^2 - R_i^2)dr, and R_e = R + h/2,\ R_i = R - h/2.$

The stress intensity factors of a shell of revolution with circumferential crack (as shown in Fig.2) can be found from some literature. It is convenient for us to study the dynamic characteristics of pipes with cracks using the cracked–element modal together with the finite element method.

3 EXAMPLE NUMERICAL CALCULATION AND EXPERIMENTAL ANALYSES

Table 1: Comparision of results of a cantilever beam with crack from numerical calculation and experiment.

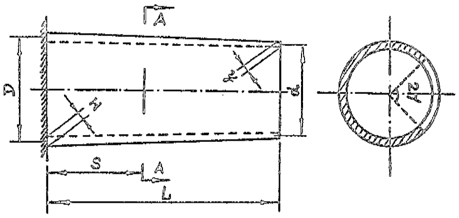


$E = 2.1 \times 10^6 \text{kgf} / \text{cm}^2$
$\mu = 0.34$
$\rho = 0.796 \times 10^{-2} \text{g.sec}^2 / \text{cm}^4$

Example f (Hz)		Example 1 (mm) L = 700, S = 350, h = 7.89, a = 4.25				Example 2 (mm) L = 700, S = 500, h = 7.59, a = 5.25			
		Calculation		Experiment		Calculation		Experiment	
		uncrack	crack	uncrack	crack	uncrack	crack	uncrack	crack
1	f	13.360	13.227	13.240	12.891	12.852	12.807	12.891	12.793
	δ	1.01%		2.34%		0.35%		0.76%	
2	f	83.738	80.270	82.424	79.305	80.544	76.237	80.078	75.781
	δ	4.13%		3.30%		5.35%		5.37%	
3	f	234.45	234.44	232.62	232.81	225.54	202.61	225.42	201.56
	δ	0.00%		0.00%		10.17%		10.42	
4	f	459.90	442.0	456.21	437.54	442.04	423.95	439.06	420.00
	δ	3.89%		4.05%		4.09%		4.34%	
5	f	759.90	759.88	753.12	753.12	731.01	730.33	725.31	724.51
	δ	0.00%		0.00%		0.09%		0.07%	
6	f	1136.0	1094.4	1119.0	1084.4	1092.8	1028.9	1081.2	1021.9
	δ	3.66%		3.09%		5.85%		5.48%	
7	f	1588.6	1588.5	1556.2	1562.5	1528.2	1440.5	1509.4	1421.9
	δ	0.00%		0.00%		5.74%		5.80%	
8	f	2118.9	2055.9	2078.1	1975.4	2038.4	2026.4	2005.6	2003.1
	δ	2.97%		4.96%		0.59%		0.12%	

Note: δ of table 1 is percent of decrease of every natural frequency.

Table 2. Comparision of results of a "clamped–free" truncated cone with an interior circumferential crack from calculation and experiment.



$L = 523.9mm$, $E = 0.96 \times 10^6 ks / cm^2$
$D = 98.95mm$, $\mu = 0.36$, $h = 1.70mm$
$d = 94.25mm$
$\rho = 0.888 \times 10^{-2} g.sec^2 / cm^4$
$H = 2.75mm$, $s = 199.0mm$

m	n	uncrack		crack			
				$2r = \pi / 2$		$2r = 2\pi / 3$	
		Calculation	Exper.	Calcul.	Exper.	Calcul.	Exper.
1	1	261.26	251.62	224.34	216.13	186.33	196.44
	2	376.73	369.99	371.12	366.71	367.43	365.62
	3	995.48	970.10	995.13	968.44	995.00	970.31
	4	1858.3	2075.7	1858.3	1912.4	1858.3	1808.6
2	1	1151.6	1180.7	1025.6	1177.3	942.60	967.19
	2	635.32	615.03	599.29	578.19	579.89	570.30
	3	1176.3	1227.5	1176.0	1190.6	1175.8	1184.4
	4	2108.1	2103.1	2107.2	2102.0	2106.9	2080.5
3	1	2524.9	2503.1	2447.5	2417.3	2394.0	2321.1
	2	1285.4	1240.0	1193.6	1202.3	1154.4	1168.7
	3	1390.4	1374.0	1352.7	1360.6	1342.0	1445.3
	4	2317.9	2325.0	2317.6	2318.2	2317.1	2205.0
4	1	4014.2	/	3861.1	/	3745.7	/
	2	2181.4	2200.0	2123.5	2145.3	2089.4	2097.7
	3	1783.6	1783.8	1780.4	1682.4	1776.6	1760.2
	4	2512.3	2503.1	2491.2	2487.0	2485.0	2471.1
5	1	5480.2	/	5152.2	/	4985.2	/
	2	3213.8	/	2921.0	/	2832.7	2713.3
	3	2386.7	2325.0	2197.6	2143.2	2150.3	2132.0
	4	2784.7	2812.5	2758.0	2731.0	2750.5	2661.7

4 CONCLUSIONS AND ANALYSES

4.1 The results of numerical calculation obtained using the model of zero dimension element with crack together with general finite element method show very good agreement with that from the experiments. It shows that the modal of zero dimension element with crack is rational and conformed. The physical and mathematical concepts of this modal are quite simple and clear. It offers us a way of determining quantitatively the effect of crack on the dynamic characteristics of a pipe.

4.2 The effect of surface crack on the dynamic characteristics of pipes also can be calculated by dealing with equivalent stress intensity factor. The surface crack is shown in Fig.3(b), the half length of its equivalent through crack (see Fig.3(c)) is: $\overline{C} = C(a / h)\pi / 4$ as given by P.Kuhu.

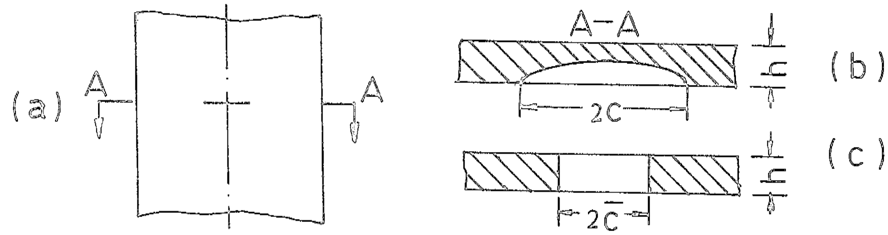


Fig.3 Surface crack and its equivalent through crack.

4.3 The difference of dynamic characteristics between uncracked pipe and cracked pipe resulting both from numerical calculations and experiments provides a scientific basis for detecting the defects in pipes, especial for detecting the Lead–Before–Break in on–line pipes, by using nondestructive testing techniques.

4.4 In the development of the theoretical modal in this paper, it is assumed that the cracks always remain open. It is the same research method as other authors adopted nowadays to study this kind of problems. The effects of closed cracks on the modal responses is not considered in this paper. It is possible to make the crack surfaces to close due to the external excitation and residual stresses which emerges during the process of the development of the crack. The closure effect will influence the response of the system, which is of nonlinear property. This influence will be studied later.

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