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MARKOVIAN AND SEMI-MARKOV MODELS FOR AVAILABILITY EVALUATION OF NPP SUBSYSTEMS AND EQUIPMENT

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ABSTRACT

The behavior of many subsystems and equipment in industrial facilities, including the nuclear power plants (NPPs) need to be modeled by means of time variant models over their lifetime. As regards the availability of subsystems and equipment in an NPP, it has to be evaluated over an operational cycle. The random character of relevant parameters related to the system under consideration and its loading conditions makes necessary to use stochastic models. In this paper we adapt some methods to evaluate the availability of systems based upon Markov and semi-Markov type models. Time intervals for maintenance and repair of subsystems (in NPPs) are taken into account as subintervals of an operational cycle. Specific distributional assumptions are selected for the time lengths for the most typical states. Thus, we extend our earlier research concerns reported in our contributions to SMiRT 15 and other conferences [6], [7], [8].

Keywords: Markov models, transition matrices, semi-Markov systems

INTRODUCTION

Specific models and methods involving Markov and semi-Markov processes have been adapted for the time-dependent availability evaluation of the subsystems and equipment items in nuclear power plants (NPP). Markov processes have some special properties that make them appropriate for mathematically treating the reliability and availability analysis for NPP (sub)systems and equipment. A subsequent development of the proposed models is focused on an effective procedure to assemble the (sub)system transition matrices and to evaluate its availability along an operational cycle. In this paper the time intervals for maintenance and repair of subsystems in NPP are taken into account as subintervals of an operational cycle. The interval reliability for a repairable system which alternate between working and repair periods is defined as the probability of the system being functional. In this paper, Markov-type modeling is applied to the evaluation of availability and maintainability for (sub)systems of NPPs which can undergo failure and repair.

In many physical problems the probabilistic law determining the future state of a system can be determined if it's present state is known, irrespective of how the system arrived at the present state. This property is called *the Markov property*, and such processes are called *Markov processes*. Models for time-dependent reliability and availability evaluation (of structural systems and equipment in industrial facilities) using Markov processes and Markov chains were introduced and developed in the 70's. Y.K.Lin's monograph [1] of 1967/1976 is one of the significant examples for early probabilistic models in structural dynamics. A more recent presentation is Y.K.Lin's Chapter 4 in monograph [2] edited by G.I.Schuëller & M.Shinozuka. Probabilistic methods for the analysis of NPP components, involving Markov chains, were developed in the 80's; the paper [3] by K.Doliński & G.I.Schuëller (a contribution to the 8th SMiRT Conf.) employs a Markov chain model for the fatigue crack propagation in NPP components. Theoretical aspects but also applications for Markov / semi-Markov models can be found in [4] & [5], too.

It deserves mentioning that the classical notion of reliability for structural systems seems to cover only in part the analysis of (safe) behaviour of subsystems and equipment in industrial facilities, including the NPPs (nuclear power plants). The term *availability* has gained a rather wide use in recent references. It may cover a more general or narrower significance. The way we approach it is presented in the next section. We also consider several types of availability measures in the subsequent sections.

2. MARKOV AND SEMI-MARKOV MODELS FOR TIME INTERVAL AVAILABILITY EVALUATION

The *availability* of a system or equipment could be described as its capacity to remain in an operating state during a certain time interval. This capacity has to be probabilistically evaluated, and such an evaluation can be formulated with reference to a *set of states* of the system under analysis. The nature of this set is rather significant. In the simplest models, two states are sufficient, for instance *up & down* as in [6]. But a more realistic approach considers a general set (or space) of states S and this set is partitioned into two subsets of *up*-states and *down*-states. A more formal presentation of such an approach follows below. As regards the time interval over which the analysis is performed, it can be - for instance - the lifetime of a structural system or the designed operating time of an equipment between two capital repairs. The model becomes more realistic (but more complicated, too) if the time interval T includes subintervals for inspections / maintenance / current repairs which can be undertaken either under an operating state or with the system temporarily stopped.

The *interval reliability* for a repairable system which alternates between working and repair or maintenance periods is defined as the probability of the system being functional throughout a given time interval $[t, t+x]$, where $x > 0$.

A random process $X(t)$, $t \in T$ is said to be *Markov* if the conditional probability

$$P[X(t_n) = x_n \mid X(t_1) = x_1, \ldots , X(t_{n-1}) = \mathrm{x}_{n-1}] = \mathrm{P}[X(t_n) = x_n \mid X(t_{n-1}) = \mathrm{x}_{n-1}] \tag{1}$$

The set $\{X(t) : t \in T\} =_{not} S$ is called *the state set* (or *state space*) of the process.

The *semi-Markov* models are commonly used for the evaluation of reliability and availability characteristics of technical systems. In many applications, the model finite state space S is naturally partitioned into two disjoint subsets, that is

$$S = U \cup D, U \cap D = \varnothing \tag{2}$$

where U is the set of working (or *"up"*) states while D is the set of not-operating / repair (or *"down"*) states. A repairable system would (at least theoretically) alternate between U & D indefinitely. Finite semi-Markovian reliability models whose state space is partitioned as in Eq.(2) will be considered in what follows. Before entering into a more rigorous approach, let us denote by s(t) or s(τ) $\in S$ the state of the system at moment t / $\tau \in [0, +\infty)$. The probabilistic reliability / availability measure to be evaluated is

$$\mathrm{IA}(x, t) =_{\mathrm{def}} P(s(\tau) \in U : \tau \in [t, t+x]) \tag{3}$$

In terms of semi-Markovian models, a (semi-Markov) process has to be considered (as in [8]), $Y = (Y_t : t \geq 0)$, $\mathrm{Y}_t \in S$. In the set U of *up* states, the system is operational or *available*. No service is delivered if the system is in the set D of *down* states. To be more explicit, the *down* states may include the periods for repairs, inspection / testing and maintenance operations which cannot be accomplished when the system is operating. In more sophisticated models, even the set of *up* states is partitioned into several service / availability levels. With the semi-Markov process just considered (instead of a general state function $s(t)$), the interval reliability / availability can be rewritten as

$$\mathrm{IA}(x, t) =_{\mathrm{def}} P(Y_\tau \in U \quad \text{for all} \quad \tau \in [t, t+x] \,;\, x, t \geq 0) \tag{4}$$

It must be emphasized that the term of "interval reliability" (or "interval availability") just defined means a measure of safe operation *during the time interval* $[t, t+x]$ and not the evaluation of the reliability of a system in terms of a couple of lower - upper bounds, as the well-known Ditlevsen bounds (which must be as close as possible). The semi-Markovian feature of the model appears if the probability in Eq. (4) is replaced by a conditional probability with respect to the states of the system at earlier moments. Thus formula (4) allows to consider two special cases, namely: IA(0, t) = system's *pointwise availability* at time (or moment) t, and $\mathrm{IA}_u(x, 0)$ = the conditional *reliability* at time x, given that the system started in the working state $u \in U$ at time 0. In fact, the pointwise availability is *not* an interval function, that is a function depending on interval of length $x > 0$. It is rather an *instantaneous availability* function and it can be rewritten (by taking $x = 0$ in Eq.(4)) as

$$\mathrm{IA}(0, t) = \mathrm{A}(t) = P(Y_t \in U, t \geq 0) \tag{5}$$

The transition of the system from a state to another needs a cumulative distribution function (*cdf*) to be considered, namely

$$F_{s_1,s_2}(x) = P(Y_t = s_1 \mid Y_{t+x} = s_2). \tag{6}$$

More explicitly, $F_{s_1,s_2}(x)$ is the *cdf* of the conditional holding time of Y in $s_1 \in S$, given that the next state to be "visited" is $s_2 \in S$, $s_1 \neq s_2$. It is assumed (in [6]) that instantaneous transitions of Y are not possible, i.e. $F_{s_1,s_2}(0) = 0$.

The *transition probability matrix* of the system is defined, over the state space S with card $S = n$, as **R** $\in$ MATRIX$_{nxn}$ ([0,1]) **:** **R** = $[r_{s_1,s_2}]_{n\times n}$. Thus the entries of matrix **R** are of the form r_{s_1,s_2} = the probability for the system to pass from the state s_1 to state s_2. So far, the time argument is not involved in the definition of a transition matrix. The time dependent entries of a transition matrix are defined by

$$q_{s_1,s_2} = \begin{cases} r_{s_1,s_2} F_{s_1,s_2} & \text{for} \quad s_1 \neq s_2, \\ 0 & \text{for} \quad s_1 \neq s_2. \end{cases} \tag{7}$$

Some more mathematics on the semi-Markov processes follow in the next section.

3. SEMI-MARKOV PROCESSES, KERNEL AND TRANSITION MATRICES

A *Markov renewal process* (MRP), defined on a complete probability space, was also presented in [8]. It consists of the pair

$$(J, S) = (J_n, S_n)_{n \geq 0}, \tag{8}$$

where $(J_n)_{n\geq 0}$ is a *Markov chain* taking values in a finite set $\mathsf{E} = \{1, 2, \ldots, s\}$ = the state space of the process, and $(S_n)_{n\geq 0}$, $S_n \in \overline{\mathbf{R}}_+ = [0, +\infty]$ is a sequence of *jump times*, while $J_1, J_2, \ldots, J_n, \ldots$ are the consecutive states to be visited by the MRP. The random variables $X_1, X_2, \ldots, X_n, \ldots$, defined by $X_n = S_n - S_{n-1}$ for $n \geq 1$, are the *sojourn times* in these states taking values in $\overline{\mathbf{R}}_+$. A MRP can be completely determined by its *initial law* and its *semi-Markov kernel*, respectively defined by $\mathsf{P}(J_0 = k) = p(k)$ and

$$\mathsf{P}(J_{n+1} = k, X_{n+1} \leq x \mid J_0, J_1, \ldots, J_n) = Q_{J_n k}(x) \text{ a.s.}, \tag{9}$$

for all $x \in \overline{\mathbf{R}}_+$ and $1 \leq k \leq s$; a.s. means almost surely. The probabilities

$$p_{ij} = Q_{ij}(\infty) = \lim_{t\to\infty} Q_{ij}(t) \tag{10}$$

are the transition probabilities of the Markov chain $(J_n)_{n\geq 0}$.

A much simpler alternative to the semi-Markov processes are the failure processes, also presented in [8]. Two states only are considered **:** the failure state and the operating state. The times for repairs are ignored, and the times when a failure state occurs are also taken as points $T_1, T_2, \ldots$ on the time axis. Another restriction consists in the hypothesis that the time intervals between two successive failures, that is

$$X_1 = T_1, X_2 = T_2 - T_1, \ldots, X_n = T_n - T_{n-1}, \ldots \tag{11}$$

are independent and identically distributed (abbreviated i.i.d.). A numerical characteristic of such processes is $N(t)$ = the number of failures in interval $[0, t]$. The failure rate of the failure process is defined by

$$r(t) = \lim_{h\to 0} \frac{P(t \leq X < t + h \mid X > t)}{h} = \frac{f(t)}{\overline{F}(t)} = \frac{d}{dt}\left[-\ln \overline{F}(t)\right], \tag{12}$$

where $\overline{F}(t) = 1 - F(t) = P(X > t) = \exp[-\int_0^t r(x)\,dx].$ This latter "complementary" *cdf* equals the probability that a continuous operating cycle (without any failure) is longer than t. The conditional probability $\mathrm{P}(X - t \le y \mid X > t) = [F(t+y) - F(t)]/\overline{F}(t)$ is the residual cycle length ; in [8], this characteristic is called the *residual lifetime*. We have recalled these notions connected with the failure processes in order to see how some of them can be extended to corresponding characteristics of semi-Markov systems with several states.

Coming back to the MRPs, let us suppose that, for all i and j, $Q_{ij}(\cdot)$ is absolutely continuous with respect to the Lebesgue measure and $q_{ij}(\cdot)$ is the corresponding density. Then, for any two indices i and j $(1 \le i, j \le k)$, the instantaneous transition rate of the semi-Markov kernel is given by

$$\lambda_{ij}(t) = \begin{cases} \dfrac{dQ_{ij}(t)/dt}{1 - \sum_{i \ne j} Q_{ij}(t)} & \text{if } p_{ij} > 0 \text{ and } \sum_{i \ne j} Q_{ij}(t) < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

The cumulative hazard rate from state i to state j is defined by

$$\Lambda_{ij}(t) = \int_0^t \lambda_{ij}(u)\,du \tag{14}$$

and the total cumulative hazard rate of state i at time t by $\Lambda_i(t) = \sum_{i-1}^{s} \lambda_{ij}(t)$. For $t \in [0, \infty)$ the probabilities involved in Eqs. (9) and (10) are given by

$$Q_{ij}(t) = \int_0^t \exp(-\Lambda_i(u)\,\lambda_{ij}(u)\,du. \tag{15}$$

For a uniform division $(v_k)_{0 \le k \le m}$ of interval [0, T] , with step $D_m = T/m$ and m a positive integer, the transition rate $\lambda_{ij}(t)$ can be approximated by

$$\lambda^*_{ij}(t) = \sum_{k=0}^{m-1} \lambda_{ijk} \mathbf{1}_{[v_k, v_{k+1})}(t). \tag{16}$$

The maximum likelihood values of the transition rates λ_{ijk} are given by

$$\hat{\lambda}_{ijk} = \begin{cases} d_{ijk}/v_{ik} & \text{if } v_{ik} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

where v_{ik} is the sojourn time in state i on the time interval I_k given, for $N_T > 1$, by

$$v_{ik} = \sum_{\ell=1}^{N_T} \inf(X_{\ell+1}, v_{k+1} - v_k)\mathbf{1}_{\{J_\ell = i,\, X_{\ell+1} \ge v_k\}} + \inf(U_T, v_{k+1} - v_k)\mathbf{1}_{\{J_{N_T} = i,\, U_T \ge v_k\}} \tag{18}$$

and

$$d_{ijk} = \sum_{\ell=1}^{N_T} \mathbf{1}_{\{J_\ell = i,\, J_{\ell+1} = j,\, X_{\ell+1} \in I_k\}} \tag{19}$$

The functions $\mathbf{1}_{[\,,\,)}$ and $\mathbf{1}_{\{\,,\,\dots\}}$ are the indicator functions of the interval / set that appears as a subscript.

The problem approached in the next section is how to adapt these semi-Markov models to the availability evaluation of *repairable systems*.

4. AVAILABILITY ESTIMATION BY SEMI-MARKOV MODELS

As we have remarked in the Introduction, the two-state (*up* and *down*) models are inadequate for describing the possible states of a subsystem in a complex system like an industrial facility. As we suggested in Eq. (1), two subsets of states U and D are better suited for describing the evolution of a subsystem during its lifetime or operational cycle. We keep the notation suggesting "up" and "down" states, but a state in U will be – in fact – a state in which the subsystem is available but not necessarily operating ; a state in D will correspond to various situations when the system is unavailable like a state of failure, of repair, of preventive / periodical maintenance or off-service inspection, etc. A more detailed discussion on the possible states in U or D will be presented in the next section. The availability at a certain time t in $[0,T]$ of the subsystem means that it is either operating or (let us say) in a stand-by state allowing it to become immediately operational on demand at time t. We denote the availability function by $A(t)$ and it can be formally defined, for a MRP, by

$\mathrm{E} = \{e_1, \ldots, e_{s_1}, e_{s_1+1}, \ldots, e_s\}$.

$$A(t) = \mathrm{P}[J_{N_t} \in U] \tag{20}$$

where N_t is a positive integer indirectly defined by N_T of Eqs. (18) and (19). It follows from Eq. (18) that N_t equals the number of states of availability $J_{\ell+1}$ at t provided the preceding state was $J_\ell = i$ and the sojourn time in $J_{\ell+1}$ is $X_{\ell+1}$, at least equal to the left end of the division interval $I_k = [v_k, v_{k+1}] \subset [0,t]$. According to [8], the availability of a semi-Markov system at time t can be expressed in closed form as

$$A(t) = \sum_{i \in \mathcal{E}} p(i)\, A_i(t) \tag{21}$$

where $A_i(t)$ is the availability at t given that the system starts in state i at the initial time ; $\mathbf{p}(i)$ is the initial law for state i in E. Eq. (21) can be rewritten under the matrix form

$$A(t) = \mathbf{p} \cdot \mathrm{P}(t) \cdot \mathbf{1}_{s_1, s} \tag{22}$$

where $\mathbf{p} = [\,p(1) \ldots p(i) \ldots p(s)]$ is the initial law,

$$\mathrm{P}(t) = [p_{ij}(t)]_{s-\mathrm{by}-s} = [P(J_{N_t} = j \mid J_0 = i)]_{s-\mathrm{by}-s}$$

is the semi-Markov transition matrix, and

$$\mathbf{1}_{s_1, s} = [1\;1 \ldots 1\;0\;0 \ldots 0]^{\mathrm{T}}$$

with the first s_1 entries = 1 while the next $s - s_1$ are = 0. A more explicit form for Eq. (22) is

$$A(t) = [\,p(1) \cdots p(i) \cdots p(s)] \cdot \begin{bmatrix} p_{11}(t) & \cdots & p_{1j}(t) & \cdots & p_{1s}(t) \\ \vdots & & \vdots & & \vdots \\ p_{i1}(t) & \cdots & p_{ij}(t) & \cdots & p_{is}(t) \\ \vdots & & \vdots & & \vdots \\ p_{s1}(t) & \cdots & p_{sj}(t) & \cdots & p_{ss}(t) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{23}$$

It is possible to obtain, from Eqs. (22) – (23), several types of availability (at time t) depending on the initial state of the system. For instance, the availability at t provided the system started in state i is given by

$$A_i(t) = p(i) \cdot \sum_{j=1}^{s_1} p_{ij}(t). \tag{24}$$

A relevant availability measure is the so-called *interval availability* (also considered in our paper [6]). It expresses the probability that the system is available at time $t + x$ provided it has been available at t and on the whole interval $(t, t + x)$. . Formally, we may define it by

$$\mathrm{IA}(x,t) = \mathsf{P}(J(\tau) \in U : \tau \in [t, t + x]). \tag{25}$$

If the states of availability are grouped together as above (from 1 to s_1), then the interval availability in Eq. (25) can be equivalently written as

$$\mathrm{IA}(x,t) = \mathsf{P}(J(\tau) \le s_1 : \tau \in [t, t + x]). \tag{26}$$

Certainly, this grouping could be considered as being more or less arbitrary, and it is implicity assumed that the set of states E is – in fact – an ordered s-tuple.

We can also remark that Eqs. (25) & (26) express the probability that the system remains in an available state along the whole interval $[t, t + x]$, what does not exclude the possibility that it changes its state but without going out of U. The evaluation of the interval availability would clearly be of practical interest since it gives a measure of continuous availability over a given time interval. If $J(t) = j \in U$ and the successive jump times to other states of availability, following to t, pass over $t + x$ then it is clear that the system will be still in a state of availability at time $t + x$. However, this is only a sufficient and particular condition for the system to remain available over the whole interval in discussion. We have to look for a general formula to evaluate the probability in Eq. (26). Let us first remark that the interval availability generalizes what is called the reliability of a MRP (in [8]), given by

$$R(t) = \mathsf{P}\big(J(\tau) \in U \quad \text{for all} \quad \tau \le t\big). \tag{27}$$

In other words, the reliability in Eq. (21) equals the probability that the system remains available over the whole time interval [0, t]. It follows from Eqs. (26) and (27) that $R(t) = \mathrm{IA}(t,0)$. The reliability can be expressed in terms of the reliability conditional on the departure state i, by equations similar to Eqs. (21) and (22) **:**

$$R(t) = \sum_{i \in U} p(i)\, R_i(t) = \mathbf{p}_U \cdot \mathrm{P}^U(t).\mathbf{1} \tag{28}$$

where $\mathbf{p}_u$ is the subvector of the (initial) probability law restricted to the *up*-states, and 1 is the 1/entry vector $\mathbf{1} = [1\ 1\ \ldots\ 1]^{\mathrm{T}}$. Thus

$$\mathrm{P}^U(t) = [I - Q^U(t)]^{-1} * [I - \mathrm{diag}(Q(t) \cdot \mathbf{1})^U]. \tag{29}$$

with $*$ = the Stieltjes convolution product. However, it appears that these measures of the time dependent availability of a (sub)system, based on the MRP model, are less adequate for a system which can "visit" (or pass through) states when it is not available, yet such states are not actual failure states. This is the case with states of periodic maintenance / off-service inspections, for instance.

A couple of relevant parameters and functions for the availability of a semi-Markov system are the following**:** $N_j(t)$ = the number of visits paid to state j in the interval [0, t] ; $R_{ij}(t) = E[N_j(t) \mid J(0) = i]$, that is the expected number of vistits paid to state j (over the same interval) provided the initial state was i ; $p_{ij}(t) = \mathsf{P}[J(t) = j \mid J(0) = i]$; $G_{ij}(t)$ = the distribution function of the first jump time from i to j. These parameters can be evaluated provided certain assumptions of the distribution / density functions are adopted. Such assumptions will be presented in the next section.

Before approaching this problem of what must be modified in the model earlier presented when the (sub)system can be subjected to (preventive) maintenance, replacement & repair operations, let us also present other measures for the availability than the ones given in Eqs.(3),(4)&(5). The *average uptime availability* over a period T is defined as

$$\frac{1}{T}\int_0^T A(t)\,dt$$

$$A(T) = \qquad (30)$$

This availability is involved in defining the *steady state* (or *long term*) availability by

$$\mathrm{A}(\infty) = \lim_{\mathrm{T} \to \infty} A(T) = \lim_{\mathrm{T} \to \infty} \frac{1}{T} \int_0^T A(t)dt \qquad (31)$$

An important feature in time-dependent analysis of systems if MTBF = the *mean time between failures* and it is connected with the distribution of the number m of failures in time t which is widely accepted to follow a Poisson law**:**

$$P(m) = \frac{(\lambda t)^m}{m!} e^{-\lambda t} \quad \text{where} \quad \lambda = \frac{1}{\mathrm{MTBF}} \qquad (32)$$

In many references with a Markovian approach for availability analysis, the exponential distributions are often used for failure times and repair times. If the repair time follows an exponential distribution with a repair rate of μ, that is the probability density function (*pdf*) of the repair time is $f_{\mathrm{RT}}(t) = \mu \exp(-\mu t)$ with $\mu = 1 / \mathrm{MTTR}$, then the maintenability function is

$$M(t) = P(\mathrm{RT} \le t) = 1 - \mathrm{e}^{-\mu t} \qquad (33)$$

This more detailed model for the availability evaluation of repairable systems can be combined with the rather formal model presented in the former section by including in the subset D of *down states*, just when the system is under repair / maintenance, or during a "ready time" period, or during an MDT period, etc. The number of such *down* states may be larger or smaller; certainly, without excessively increasing it's analytic / probabilistic complexity and/or it's practical applicability.

We first present the state space for a component (among K components of the system). However, the following definitions of the states are also valid for the whole system under analysis. Let us consider a component (or subsystem) with six possible states such that ythe state space is $S = U \cup D$, with a single up state, that is $U = \{u\}$ in which the component is operational (or, in our terms, *available*, even if it is not effectively operating), but several *down* (or non-operational) states of the component / system, namely $D = \{s_1, s_2, s_3, s_4, s_5\}$ where **:**

$X(t) = d_1 \iff$ the system is in a failure state; (34.1)

$X(t) = d_2 \iff$ the system is in a (delay time) state; (34.2)

$X(t) = d_3 \iff$ the system is in a (R/M) state (repair/maintenance); (34.3)

$X(t) = d_4 \iff$ the system is in a (ready time) state; (34.4)

$X(t) = d_5 \iff$ the system is in a (preventive) maintenance state. (34.5)

The way the *down* states were defined logically induces a certain structure of the transition probability matrix **R**. Thus, it is quite likely that a "repair/maintenance" state will be followed by an *up*-state.

A transition diagram for this 6-state model is presented below (in Fig. 1) in terms of a directed graph, where the nodes denote the six states and the arcs correspond to "probable transitions" from a state to another.

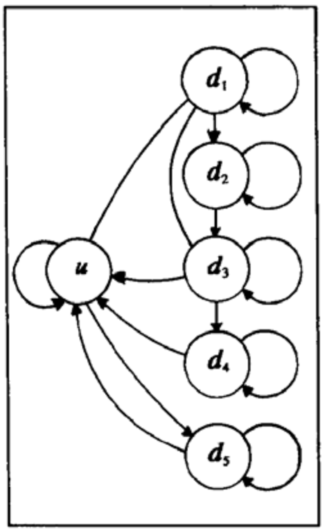

Fig.1 Transition diagram for a 6-state component / system

A possible transition matrix corresponding to the diagram in Fig. 1 can be the following**:**

$$\mathbf{R} = \begin{bmatrix} 0.75 & 0.05 & 0 & 0 & 0 & 0.20 \\ 0 & 0.40 & 0.30 & 0.30 & 0 & 0 \\ 0 & 0 & 0.20 & 0.80 & 0 & 0 \\ 0.15 & 0 & 0 & 0.25 & 0.60 & 0 \\ 0.85 & 0 & 0 & 0 & 0.15 & 0 \\ 0.65 & 0 & 0 & 0 & 0 & 0.35 \end{bmatrix}. \quad (35)$$

As it can be seen, the sums of the entries on each row are = 1 since whichever is the current state of the component / system, it can either enter one of the other states or it may remain in the current state, with a total probability of 1. Certainly, the zero entries in such a matrix could actually be "almost equal to zero". For instance, the zero entries on row 6 mean that the component / system would enter a state of preventive maintenance (or inspection) without any "delay time" and it can return to an *up*-state without any "ready time". The order of rows and columns of matrix R correspond to the (conventional) order

$$u < d_1 < d_2 < d_3 < d_4 < d_5. \quad (36)$$

It is possible to re-denote the states as $u = s_1$, $d_1 = s_2$, ... , $d_5 = s_6$. The links in the graph of Fig. 1 correspond to transition probabilities $r_{s_1,s_2} \geq 0.05$, for instance. But this threshold condition should be understood as corresponding to a sufficiently small time interval of length $\Delta t > 0$. The stochastic entries in a matrix as in Eq.(35) may change when Δt is smaller or larger. On another hand, they essentially depend on the distributional assumptions on the specific time intervals that occur in Eqs.(10)-(11).

The next step in the application of the semi-Markov model of the preceding section consists in proper selections for the cumulative distribution functions that occur in Eqs.(6)-(7). On the basis of available statistical evidence and/or engineering judgment, they have to be found so as to yield a "semi-Markov kernel" $Q(t)$ consistent with the evidence and regulatory standards. On another hand, the resulting interval availabilities should be close to those resulting from definitions in Eqs.(4)-(5). For instance, $IA_u(x,t)$ should be rather low if $X(t) = u$ but $x >$ MTBF. Similarly, an instantaneous availability IA(0, $t+\Delta t$) should be very high if $X(t) = d_5$ and $\Delta t >$ MDT.

As mentioned in the introduction, the structure of the system under analysis is essential in evaluating it's availability along specific time subintervals included in an operational cycle T. For instance, a series system consisting of two components remains in an U-state as long as both it's components are in an up-state. Instead, a parallel system of two components enters a *D*-state when both it's components are in a down-state. The high reliability requirements in NPP systems led to the design of rather highly redundant subsystems consisting of several nominally identical components operating under similar demand and maintenance conditions.

5. A SEMI-MARKOV APPROACH TO AVAILABILITY OF NPP SUBSYSTEMS

The general presentation of the up and down states (in the previous sections) can be more or less apropriate for the PRA (probabilistic risk assesment) of nuclear facilities. For instance, a component in a NPP system may be unavailable if it is either failed or only functionally unavailable; the later term covers cases in which the nonfunctioning is due to the lack of required input. A "classical" example is an event that happened at Peach Bottom Plant in June 1977, when three out of four diesel generators became inoperable: air start systems of DG's 3 and 4 were cross-tied with the air start of DG1, but the later was taken out of service for maintenance. Such situations are the subject matter of the well-known *common case failure* analysis (in the PRA of NPP's).

Without entering into more technical details here, let us try to adapt the classification of the possible states of a component in a NPP system. The component is in an ***up***-state when either

1 - it is in operating state (including in-service inspection states) or

2 - it is in a "cold stand-by" state, waiting for a possible demand consequent to a down state entered by a parallel component.

A ***down***-state occurs in one of the following situations:

3 - the component is failed (due to internal causes / improper maintenance / overpassing beyond the normal functioning time from the latest maintenance operation);

4 - the component is functionally unavailable (due to lack of required input or due to a human error);

5 - the component is in a maintenance / repair / replacement stage (consequent to a failure state);

6 - the component is under a preventive maintenance / replacement / off-service inspection - testing (periodical) routine.

Consequently, the state space of a component in a NPP system would be

$$S = U \cup D \quad \text{with} \quad U = \{s_1, s_2\}, \quad D = \{s_3, s_4, s_5, s_6\} \tag{37}$$

Clearly, the subscripts on the states in Eqs.(37) correspond with the numbers in the previous enumeration. Deterministic time intervals or probabilistic time distributions must be assigned to each of these states. The intervals between periodical off-service inspection / testing routines, followed (or not) by preventive maintenance / replacement operations, as well as their time lengths, can be considered as deterministic. Let us denote by PIT the standard (or mean) time for a periodical off-service inspection and by PMRT the duration of a preventive maintenance or replacement operation. If the cumulative time for scheduled operations of this type per operational cycle should amount to – for instance – at most 20% of the total operational time *T*, then the number of off-service inspections and preventive maintenance would be given by the integer part of

$$\frac{0.2T}{\mathrm{PIT}+\mathrm{PMRT}} = m_{\mathrm{PI+PMR}} \tag{38}$$

The (ready time) of the previous section (see Eq.(34.4)) may be included into s_5 periods. As regards the (delay time) intervals, they may be part of failure (i.e. s_3) down times. These residence times in s_3 states should normally be rather short: once a failure is detected, the component suddenly enters a down state and a delay time automatically occurs, followed by a repair / replacement (i.e. s_5 state). Certainly, the residence time in an s_3/s_4 state cannot be deterministic. It may depend on many factors and the PRA studies for NPP systems and equipment can yield realistic estimations for specific events. The same remark holds for the RT, i.e. repair / replacement time.

A more formalized discussion can be found in our paper [6]. As earlier mentioned, the problem of evaluating the random time lengths needs specific time distributions to be selected. For the maintenance / repair / replacement time consequent to a failure state, that is for $W_5 = DT_5$, it seems that a more appropriate choice than the *cdf* of the down-time in state 5 is given by a lognormal distribution which yields a corresponding repair / replacement rate**:**

$$f_{W_5}(w) = \frac{1}{\sqrt{2\pi} w\alpha} \exp\left[-\frac{1}{2} \left[\frac{\ln(\rho w)}{\alpha} \right]^2 \right] \Rightarrow \mu(t) = \frac{\frac{1}{\sqrt{2\pi} t\alpha} \exp\left[-\frac{1}{2} \left[\frac{\ln(\rho w)}{\alpha} \right]^2 \right]}{1 - \phi(\frac{1}{\alpha} \ln \rho t)} \quad (38)$$

where Φ(.) is the *standard normal cdf*. The use of a Markov model for a time-dependent transition matrix over the state set *S* needs, however, constant aproximations to be used for the transition rates. We cannot enter into more details here, but let us mention that a matrix as in Eq.(35) can thus be obtained. Next, a time-dependent transition matrix $Q_c(t) = [q_{ij}(t)]_{nxn}$ has to be determined using the distributional assumptions for the transition times from a state s_i to another state s_j.

Finally, semi-Markovian transition matrices (from an up state to a down state and conversely) $K_{UD}(t)$ & $K_{DU}(t)$ for $t \in T$ have to be assembled from the component transition matrices $Q_c(t)$: $c \in C$ = the set of components in the subsystem under analysis, according to available fault trees / event trees consistent with the plant logic.

6. CONCLUSIONS

Certain methods for the Markov and semi-Markov modeling have been reformulated and adapted for the time-dependent availability evaluation of the components in subsystems of nuclear power plants. Some more mathematics on the semi/Markov processses have been presented. A subsequent development of the models proposed will be focused on effective ways to assemble the (sub)system transition matrices and to evaluate it's availability along an operational cycle. In fact, we have continued our research of [6], [7] and [8], giving more details on the mathematical nature of the semi-Markov systems. We have also discussed possibilities to consider systems with a larger variety of possible states. Certainly, the availability evaluation for such systems would imply rather sophisticated and complex mathematical (stochastic) models. The semi-Markov models provide the important advantage that they take into account the *sojourn times* in the possible states, while the renewal models assume that any failing component is immediately replaced by a new one. It still remains to go further with the application of semi-Markov models and interval reliability in the stochastic evaluation of the availability of subsystems and systems.

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