RESPONSE OF A NONLINEAR SYSTEM TO VARIOUS SPECTRAL EXCITATION TIME DECOMPOSITIONS

J. CURRERI, P. BEZLER, B. KOPLIK, H. GORADIA

Brookhaven National Laboratory, Structural Analysis Group, Department of Nuclear Energy, Upton, New York 11973, U.S.A.

Abstract

Many methods are available for determining the seismic response of equipment and structures. The analysis could be done either by the response spectrum approach or by applying an appropriate time history of the excitation. However, the only acceptable description of a seismic event is the site response spectra. An appropriate time history is obtained from the response spectra. [1]

For linear systems, either characterization of the seismic event is acceptable. For nonlinear systems, there is generally no choice and a time history must be derived to investigate the response.

Spectral excitations are routinely decomposed for real time integration of linear and nonlinear structures. Different approaches are being used to generate the time histories. The system's response to each of the time histories is intended to be conservative. There are, however, certain types of nonlinearities for which a linear treatment may not be conservative. The paper shows that the nonlinear response could be many times greater than the linear response in some systems. This depends upon factors associated with the width, shape, and decomposition of the spectrum as well as on the characteristics of the system.

This paper discusses the effects of different acceptable time histories that have been applied to a linear and a nonlinear system. The time histories have been obtained from a spectral description of an earthquake event. The nonlinear system is taken as a linear one to which a cubic hardening term has been added.

The SIMEAR code has been modified to decompose a given spectra to the time domain. By successive iterations, acceptable time functions are generated. The paper reports on the maximum response variations that are obtained for excitation of a linear and a nonlinear degree of freedom system. It is shown that for some system characteristics and time decompositions, the linear response is greater than the nonlinear response. In these cases, the linear natural frequency is favorably located in the excitation spectrum. As the natural frequency is shifted with respect to the excitation spectrum, the relative magnitudes of linear and nonlinear response change. Under some conditions the nonlinear response becomes sizably bigger than the linear response. It appears that this type of response characteristic is possible only for a nonlinear system with a jump phenomena, like the HTGR.

The cubic hardening system that is investigated in the paper is similar to a bilinear one. Both have multiroot possibilities. Both have jump phenomena. The same type of critical composition to the exciting time function may also affect the bilinear system response. For these cases, certain qualifications would have to be imposed on the conditions under which the linear solution is used as a conservative case for the maximum response.

Work performed under the auspices of the U.S. Nuclear Regulatory Commission.

Spectral excitations are routinely decomposed for real time integration of linear and nonlinear structures. The decomposition to the time domain is not unique and different response magnitudes can be obtained. For an elastic-plastic type of nonlinearity, a linear analysis gives a maximum response. There are, however, certain types of nonlinearities for which a linear treatment may not be conservative such as those associated with the HTGR core. The paper shows that the nonlinear response could be many times greater than the linear response. This depends upon factors associated with the width, shape, and decomposition of the spectrum as well as on the characteristics of the system.

This paper discusses the effects of different acceptable time histories that have been applied to a linear and a nonlinear system. The time histories have been obtained from a spectral description of an earthquake event. The nonlinear system is taken as a linear one degree of freedom system to which a cubic hardening term has been added.

Previous papers have shown the different types of response amplitudes that could develop in a nonlinear system under different harmonic compositions in the forcing function [9]. For those cases, the forcing terms were pre-selected. The study has been extended to include forcing functions that have been decomposed from spectral descriptions of an earthquake.

Many different time functions could be obtained from a spectral description of a seismic event. [2] Each of these time functions will produce different responses in linear as well as nonlinear systems. The first part of this study was done to obtain some comparative information about the extent of the variations in the response of a linear system. Then, a cubic type of nonlinearity was added to the linear system. The same time excitations were applied. The responses of the nonlinear and linear systems were compared.

A system with known charateristics was used. The equation under investigation is of the form

$$\ddot{y} + c\dot{y} + \alpha y + \beta y^3 = f(t) \quad (1)$$

where f(t) is an earthquake derived time function. The coefficient of the cubic hardening term is β. For the linear case, β is set equal to zero.

It has been previously shown [8,9] that there is a discontinuity in the response of a nonlinear system for certain ranges of multiple sine wave excitations. The discontinuity occurs in addition to the ordinary jump phenomena associated with nonlinear systems. In particular, when $f(t) = F_1 \cos (w_1 t + \phi_1) + F_2 \cos (w_2 t + \phi_2)$ the discontinuity in the response appears when $w_2 = 3w_1$ and for values of F_1/F_2 above a certain threshold level. A small change in F_1 at this level produces a substantial change in the response. As the coefficient β is increased, the threshold magnitude of F_1/F_2 is decreased. The question whether the same type of discontinuity could be induced in the nonlinear system by different time decompositions of the same spectral description of an earthquake is examined.

The fundamental natural frequency of the system described by eq. (1) was taken as 3 Hz. The excitation was 0.2g maximum ground acceleration, as shown in Fig. 1. The value of α was taken as 100 and β as 10000. Two percent of critical damping was used.

The SIMEAR code has been modified to decompose a given spectra to the time domain. By successive iterations, acceptable time functions are generated. A time function is considered acceptable when it envelops the required excitation spectrum. The quality of the final match between the target excitation spectrum and the enveloping spectrum is a function of the particular procedure used [2,5,6,7]. In any case, regardless of the individual method, many different time functions can be determined which will satisfy the target spectrum. To generate an individual time function, a random ISET number is selected and processed by the code. The resulting time function was then applied to the system described by Eq. (1).

Table I shows the maximum acceleration response that was obtained from four different acceptable time functions which were used as input excitations in Eq. (1).

The largest response was 0.3091 in. and least was 0.1857 in. This gives a factor of 1.6 between the maximum and minimum responses that were obtained in a linear system due to different, but acceptable, time functions.

The cases selected are not intended to show maximum possible differences. The cases chosen are simply reference results which will be used as a basis for comparison.

The same time decompositions, as defined by the previous ISET numbers, were applied to the nonlinear systems. Table II shows the results. Column 1 lists the various ISET numbers used. Column 2 shows the maximum response in the nonlinear system. Column 3 gives the maximum response if only frequency components above 5Hz are retained in the excitation. Column 4 shows the maximum response if frequency compoents above 5Hz are trained. Columns 5, 6 and 7 give the maximum response if the nonlinear coefficient β is taken as zero. This is the linear system response.

Columns 2, 3 and 4 show the nonlinear response. Columns 5, 6 and 7 show the comparable linear response. When all frequencies are retained in the excitation, column 2 shows that the nonlinear response is less than the linear response for all four time decompositions, as indentified by the four ISET numbers. Note that the nonlinear response is comparable to the linear response even after all frequencies below 5 Hz were filtered out of the time excitation. This is seen by comparing columns 3 and 5. The same information is obtained if all frequencies below 6 Hz are deleted. Recall that the natural frequency is 3 Hz. With exciting frequencies above 6 Hz, resonance in the linear system is not possible. Even so, the nonlinear response is no greater than the linear response.

It might be prematurely concluded that the linear system could be taken as a conservative case in solving nonlinear problems. Further examination shows that this is not so.

The results obtained in Table II were for a particular level of input. Would the same comparative evaluation be obtained if the overall magnitude of the excitation were changed? For the level of .2g input, it is clear that only the lower roots were obtained in the non-linear response. This is known since the level of responses in the nonlinear and linear systems are about equal even though the forcing frequency components are above the resonant frequency. The input excitation level was increased and the comparison repeated. Table III shows the results for five different multiplication factors. Note that when the input is doubled, the nonlinear response is greater than the linear response by a factor of more than 2. This is because the signal now contains magnitudes which exceed the threshold levels required for the nonlinear system to attain upper root response. However, a maximum is reached in the ratio of nonlinear to linear response when the multiplication factor is about 2 for the conditions used. Above and below this value, the ratio is reduced, although still greater than one.

In brief, Table II shows that the differences between linear responses for the different ISET numbers is 1.6. Table III shows that the differences between linear and nonlinear is 2.0. The overall difference that could be obtained for just the few cases listed amounts to a factor of 3.2. This shows that under some conditions, the nonlinear response could be sizably bigger than the linear response. This type of response behavior is possible only for a nonlinear system with a jump phenomena, like the HTGR core. Both our previous tests and the analytical work that was done with one and two sine waves showed that the HTGR has this type of characteristic. The cubic hardening system that is investigated in the paper is similar to a bilinear one.[4] Both have multiroot possibilities. Both have jump phenomena. The same type of critical composition to the exciting time function may also affect the bilinear system response. Previous information in the literature has concluded that a linearized treatment for both bilinear as well as an elastic plastic system gives rise to conservative results. The present investigation indicates that the conclusion is not correct for all cases of bilinearity. For these cases, certain qualifications would have to be imposed on the conditions under which the linear solution is used as a conservative case for the maximum response.

It does not appear that the conclusions for the elastic-plastic response curve are affected.[3] This is because the elastic-plastic response curve is single valued and the softening characteristic in this case is associated with large damping. There is no jump phenomena in the response curve of this type of system. A small change in the relative magnitudes of the exciting components cannot be expected to change the response to a different level of stability at the same frequency. This is because there is only a single value of response at that frequency. However, for other types of softening systems which have multiroot possibilities, the conclusions associated with the hardening characteristics would also appear to apply.

References

[1] NEWMARK, N.M., BLUME, J.A., KAPUR, K. K., "Seismic Design Spectra for Nuclear Power Plants," J. Power Division, ASCE, 99:P02, 1973.

[2] LIN, C. W., "Criteria for the Generation of Spectra Consistent Time Histories," Trans. 4th International Conference on SMiRT, Vol., K(a), p. K1/11, 1977.

[3] NEWMARK, N.M., "A Response Spectrum Approach for Inelastic Seismic Design of Nuclear Reactor Facilities," Trans. 3rd International Conference on SMiRT, Vol. K, p. K1/5, 1975.

[4] YEH, H. Y., YAO, J. T. P., "Response of Bilinear Structural Systems to Earthquake Loads," ASME paper 69-VIBR-20, Vibrations Conference, Philadèlphia, PA, April 1969.

[5] LEVY, S., WILKINSON, J. P. D., "Generation of Artificial Time-Histories, Rich in All Frequencies, From Given Response Spectra," Trans. 3rd International Conference on SMiRT, Vol. K, p. 1/7, 1975.

[6] SCANLAN, R. H., SACKS, K., "Floor Response Spectra for Multidegree of Freedom Systems by Fourier Transform," Trans. 4th International Conference on SMiRT, Vol. K, p. 5/5, 1977.

[7] TSAI, N. C., "Spectrum-Compatible Motions for Design Purposes," J. Eng. Mech. Div. Proc. ASCE, 98, No. EM2, p. 345-356, 1972.

[8] CURRERI, J. R., BEZLER, P., "Effect of Damping on the Response of a Nonlinear System with Multiple Sine Wave Excitation," Trans. 4th International Conference on SMiRT, Vol. K(b), p. K8/9, 1977.

[9] BEZLER, P., CURRERI, J. R., "Multiple Sine Wave Excitation of a Hard Spring Oscillator," 47th Shock and Vibration Bulletin, Summaries, October 1976.

TABLE I

RANDOM NO. ISET	MAXIMUM RESPONSE
234	0.2178
456	0.2330
567	0.3081
789	0.1857

TABLE II

1	2	3	4	5	6	7
ISET	NONLINEAR			LINEAR		
	ALL CYCLES	ALL CYCLES AFTER 5 Hz	ALL CYCLES AFTER 6 Hz	ALL CYCLES	ALL CYCLES AFTER 5 HZ	ALL CYCLES AFTER 6 Hz
234	0.1214	0.1142	0.04723	0.2178	0.1216	0.04669
456	0.1409	0.1365	0.04919	0.2330	0.1230	0.04525
567	0.1576	0.1274	0.04838	0.3081	0.1297	0.04449
789	0.1386	0.1147	0.04116	0.1857	0.1294	0.04050

TABLE III

INPUT CUT-OFF FREQ-6 HZ

MUL.	LINEAR	NONLINEAR
1.5	.07007	.063
2.0	.0934	.1858
2.1	.0980	.1967
2.5	.1167	.2115
3.0	.1401	.1898

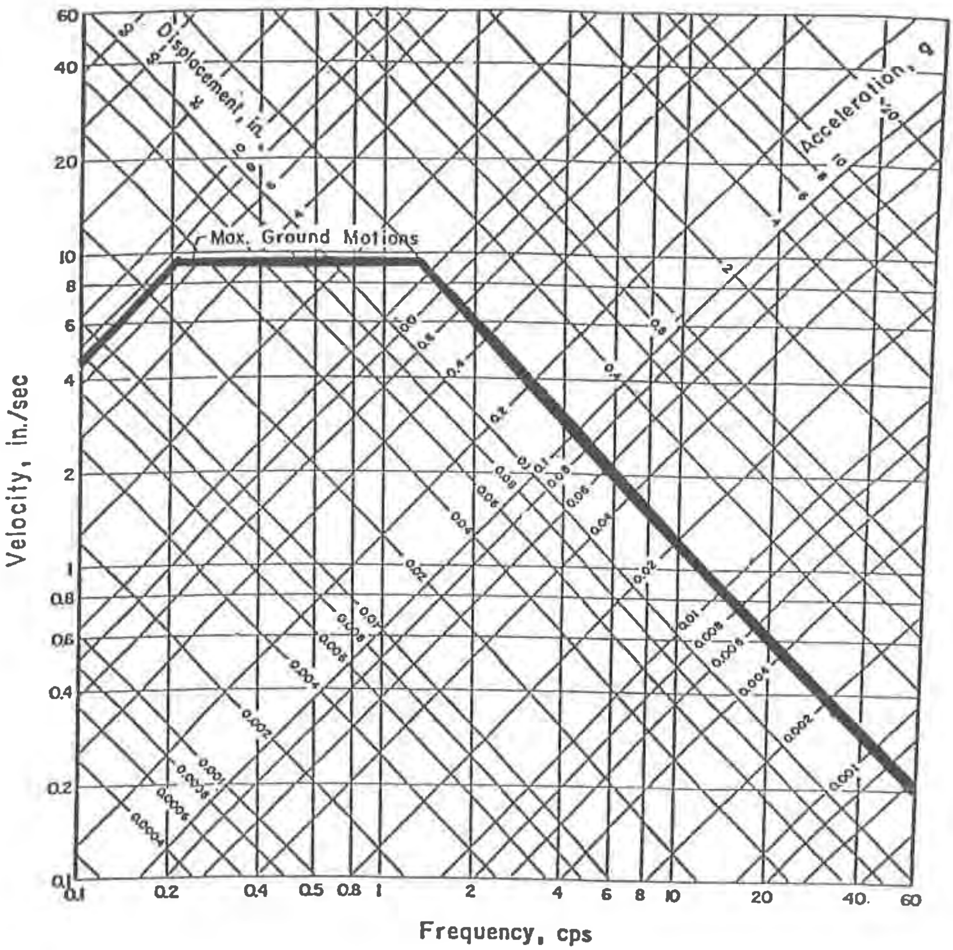

Figure 1 Ground Motion Design Spectrum