ABSTRACT

BATMAZ, EDİZ. Overall Heat Transfer Coefficients and Axial Temperature Distribution of Fluids in a Triple Tube Heat Exchanger. (Under the direction of K.P. Sandeep)

Computation of overall heat transfer coefficients in a triple tube heat exchanger (TTHE) is complicated when compared to a double tube heat exchanger (DTHE) since the two overall heat transfer coefficients are not independent of each other and must be solved for simultaneously. Previous methods established towards calculation of these parameters either include assumptions that are not valid for all flow conditions and fluid flow rates or use empirical correlations which may cause significant deviations from actual values of these parameters. A more generic technique was developed for calculation of overall heat transfer coefficients and axial temperature distribution of fluids in a triple tube heat exchanger. The developed procedure has been used for calculation of these parameters at various fluid flow rates and product inlet temperatures. Theoretical double tube heat exchanger results were also tabulated for comparison purposes. The advantages of using a TTHE over a DTHE has been both conceptually explained and demonstrated using the results obtained. However, it was also shown that design of TTHE experiments is critical, especially in the co-current flow arrangement, since the relative flow rates of the fluids may result in a decrease in the effectiveness. The effect of fluid flow rates, product inlet temperature, and flow arrangement on values of overall heat transfer coefficients, total amount of heat transferred, and effectiveness were also investigated. These results were analyzed using SAS and interpreted for the consistency of the obtained results with the literature.

OVERALL HEAT TRANSFER COEFFICIENTS AND AXIAL TEMPERATURE DISTRIBUTION OF FLUIDS IN A TRIPLE TUBE HEAT EXCHANGER

by

EDIZ BATMAZ

A thesis submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Master of Science

FOOD SCIENCE

Raleigh
2003

APPROVED BY:

_____ _____

_____
Chair of Advisory Committee

BIOGRAPHY

Ediz Batmaz was born in Tarsus, Turkey, a small town by the Mediterranean Sea, on June 19, 1977. After graduating from elementary and high schools in his home town, in 1995 he moved to Ankara, the capital of Turkey, to pursue a Bachelor of Science degree in the Department of Food Engineering at Hacettepe University. He graduated in June 2000, and in the summer of the same year was awarded a scholarship by the Higher Education Council to pursue his graduate studies in U.S. He started his graduate program at the University of Florida in the Agricultural and Biological Engineering Department. After completing one and a half years of his graduate work, he transferred to the Food Science program at the North Carolina State University. He is currently a research assistant in the same department.

ACKNOWLEDGMENTS

I believe in the fact that from elementary school to graduate school, the support and prayers of my parents helped me a lot emotionally and physically, especially at the moments when I felt I would never be able to accomplish this work. I am grateful to them.

My advisor has no idea what a great pleasure it is to talk about him at any chance I have: his support and understanding, his belief in me on my academic studies, and his wisdom was the key to the completion of this study. Being able to talk to him about anything whenever I needed it will be another good memory I will always remember and be thankful for.

I would like to thank my committee members, Drs. Brian E. Farkas and Andrey V. Kuznetsov. Their valuable input helped me to shape this thesis.

Special thanks go to Dr. Josip Simunovic for his continuous support and help during the experimental part of this study.

Gary Cartwright, Jack Canady, and the other staff in the dairy plant worked more than myself on the experimental setup. To me it was the most time consuming and most difficult part of this study. Many thanks to them for working on it as if it were their own responsibilities.

Finally, I would like to thank to the engineering group here: Tunc Koray Palazoglu, Pablo Coronel, Cristina Sabliov, Dorin Boldor, Qixin Zhong, Zeynep Tanyel, Aswini Kumar Singh Jasrotia, Jeff Resch, Brian Lloyd, Supriyo Ghosh, and Ming Li.

TABLE OF CONTENTS

LIST OF TABLES

MANUSCRIPT II

LIST OF FIGURES

INTRODUCTION

Understanding the fundamentals of heat transfer and applying it to real world scenarios is probably one of the topics that will always hold the attention of researchers. One of many applications of heat transfer in industry is heating and cooling of liquid, solid, and particulate foods. In the food industry, it is most desirable to heat or cool the product in the shortest possible time. The thermophysical, chemical, and sensory characteristics of foods tend to change with heat treatment, usually resulting in lower nutritional value and less acceptable sensory characteristics. Therefore, increasing the amount of energy transferred per unit area per unit time ($q"$, heat flux) is the key to accomplishing the heating or cooling treatment that results in the best quality product.

There are two ways to increase the heat flux: Increasing the temperature difference between the hot and cold fluids, and increasing the overall heat transfer coefficient. McCabe *et al.* (1985) defined overall heat transfer coefficient based on the following expression:

$$\frac{dq}{dA} = U\,\Delta T = U(T_h - T_c) \qquad (1)$$

where '$\frac{dq}{dA}$' is the heat flux, 'U' is the overall heat transfer coefficient, and 'ΔT' is the temperature difference between the hot and cold fluids. The temperature difference at any point can be modified to a limited extent by changing the directions in which fluids are flowing. For example, in a double tube heat exchanger (DTHE), there are two options for flow arrangement. Either the hot and cold fluids flow in the same direction (co-current), or they flow in opposite directions (counter-current). It has been proven experimentally and theoretically that the counter-current arrangement results in higher amounts of energy

transfer between the hot and cold fluids. Integrating equation (1) between the two end points of a DTHE yields the following expression:

$$q = U \cdot A_{lm} \cdot \Delta T_{lm} \tag{2}$$

where 'q' is the total amount of energy transferred, 'A_{lm}' is the logarithmic mean area, and 'ΔT_{lm}' is the logarithmic mean temperature difference.

The other factor that influences the amount of heat being transferred is the overall heat transfer coefficient. Overall heat transfer coefficient represents how fast energy can be transferred between two points. It is related to the total thermal resistance between the two points. For a DTHE, the resistance to heat transfer between the two fluids takes place in three steps: (1) resistance between the hot fluid and the tube wall, (2) resistance within the tube wall, and (3) resistance between the tube wall and the cold fluid. Overall heat transfer coefficient can be related to the overall effect of these resistances by the following expression:

$$\frac{1}{U \cdot A_{lm}} = \frac{1}{h_i A_i} + \frac{\Delta r}{k_w A_{lm}} + \frac{1}{h_o A_o} \tag{3}$$

where 'h_i' is the convective heat transfer coefficient between the fluid in the inner tube and the inner tube inner wall, 'A_i' is the inside surface area of the inner tube, 'k_w' is the thermal conductivity of the tube wall, 'h_o' is the convective heat transfer coefficient between the fluid in the annulus and the inner tube outer wall, and 'A_o' is the outside surface area of the inner tube.

A triple tube heat exchanger (TTHE) consists of three concentric tubes of equal length. The product to be heated or cooled flows in the inner annulus formed between the inner two tubes, and the heating or cooling medium flows in the inner tube, and outer

annulus formed between the outer two tubes of the TTHE. In a TTHE, heat transfer takes place in two different directions. Assuming that it is a cooling process, one direction for heat transfer is from the hot product in the inner annulus to the cooling medium in the inner tube and the other is from the hot product to the cooling medium in the outer annulus. An overall heat transfer coefficient has to be determined for each direction of heat transfer. Therefore, for a TTHE, two overall heat transfer coefficients are to be determined. In this case, integration of equation (1) is not as straightforward since the product temperature is affected by both of the cooling medium streams. Thus, equation (1) has to be written twice for the two U values and solved simultaneously to compute the overall heat transfer coefficients. For this purpose the heat capacities and inlet and outlet temperatures of all three streams have to be known.

The first part of this study deals with the derivation of equations that can be used to determine the overall heat transfer coefficients and axial temperature distribution of the fluids. Furthermore, a method for computation of an effective overall heat transfer coefficient representing the total heat transfer occurring in the TTHE is proposed. In the second part of the study, the developed procedure was used to compute the overall heat transfer coefficients and to generate the axial temperature distribution of all three fluids for each run conducted on a corrugated surface TTHE. SAS (SAS Institute Inc., Cary, NC) was used to analyze the data gathered from the experiments.

MANUSCRIPT I

Calculation of Overall Heat Transfer Coefficients in a Triple Tube Heat Exchanger

EDIZ BATMAZ

K. P. SANDEEP

Please address all correspondence to:

Dr. K. P. Sandeep

Department of Food Science, Box 7624

North Carolina State University

Raleigh, NC 27695-7624

E-mail: kp_sandeep@ncsu.edu

Phone: (1)(919) 515-2957

Fax: (1)(919) 515-7124

Abstract

In previous studies, calculation of overall heat transfer coefficients in a triple tube heat exchanger involved assumptions or approaches those are not valid in all cases. In this study a more generic way of calculating overall heat transfer coefficients in a triple tube heat exchanger has been developed. Consequently, temperature profiles of all streams in a triple tube heat exchanger in the axial direction were determined. An effective overall heat transfer coefficient that is related to the total resistance to heat transfer in the triple tube heat exchanger, was also determined to facilitate comparison of a triple tube heat exchanger to an equivalent double tube heat exchanger.

Keywords: triple tube heat exchanger, overall heat transfer coefficient, mathematical modeling, temperature profile

Abbreviations: DTHE, Double Tube Heat Exchanger;

TTHE, Triple Tube Heat Exchanger

Introduction

Heat exchangers have been used in various industries for a wide range of applications. Some of these applications may be found in space heating, air-conditioning, power production, waste heat recovery, and chemical processing. Heat exchangers have been categorized based on flow directions (parallel-flow, counter-flow, and cross-flow), type of construction of the heat exchanger (such as tubular or plate heat exchangers), or based on the contact between the fluids (direct or indirect) [1]. The most common tubular heat exchanger is the double tube heat exchanger (DTHE). A DTHE consists of two concentric tubes of the same length but different diameters. A triple tube heat exchanger (TTHE) is a slightly modified version of a DTHE where the number of concentric tubes is three instead of two. A TTHE has obvious advantages compared to double tube heat exchangers. These are the larger surface area for heat transfer per unit length and higher overall heat transfer coefficients due to higher fluid velocities in the annular regions [2].

Researchers have focused on increasing the fluid-to-wall heat transfer coefficients and therefore, the overall heat transfer coefficient in heat exchangers to transfer the targeted amount of energy in the shortest possible time [3]. For a DTHE, the overall heat transfer coefficient can be defined by an expression analogous to Newton's law of cooling. Accordingly,

$$q = U \cdot A_{lm} \cdot \Delta T_{lm} \tag{4}$$

where 'q' is the total heat transferred from the hot fluid to the cold fluid, 'A_{lm}' is the logarithmic mean surface area, and 'ΔT_{lm}' is the logarithmic mean temperature difference across the boundary where heat transfer is taking place. In a DTHE, the transfer of heat is in one direction only (from hot fluid to cold fluid). On the other hand, in a TTHE (Fig. 1), the energy of the product that flows in the inner annulus is transferred in two opposite directions (to the cold fluid in the inner tube and outer annulus). Therefore, we can define two separate overall heat transfer coefficients (U_1 and U_2) for each of these heat transfers taking place in a TTHE. Equation (4) is valid for a DTHE because the total energy lost by the hot fluid is the same as the energy gained by the cold fluid. However, we cannot use equation 1 to calculate U_1 and U_2 in a TTHE. The reason for this restriction is that, the 'ΔT_{lm}' term in this equation has the inlet and outlet temperatures of both the fluid in the inner annulus and the fluid in the inner tube (or outer annulus). Using this equation for a TTHE implies that, the temperature rise (or drop) of the fluid in the inner annulus is solely accomplished by the fluid in the inner tube (or outer annulus). However, the streams in the inner tube and outer annulus have an effect on the temperature rise (or drop) of the fluid in the inner annulus. Therefore, an alternative procedure has to be followed to calculate the overall heat transfer coefficients in a TTHE.

Despite its use in the food industry, limited studies have been performed examining the heat transfer phenomenon in a TTHE. Zuritz [2] and Unal [4, 5] developed mathematical models to determine the temperature profiles in the inner tube and annuli and performed case studies related to the design and performance of a TTHE. However, the temperature profile equations developed, as expected, depend on the overall heat

transfer coefficients. Zuritz [2] used equation (4) to calculate the overall heat transfer coefficients in a TTHE and Unal [4, 5] assumed that the heat fluxes from the hot fluid to the cold fluids (assuming a cooling process) were constant over the length of heat exchanger and then calculated U_1 and U_2 based on this assumption. Satyanarayana [6] and Sahoo [7] used correlations previously developed [8, 9, 10] to calculate local heat transfer coefficients (h) and used those values to calculate the overall heat transfer coefficients. These correlations are very specific to the fluids, range of parameters, and heat exchangers tested.

Therefore, a new method has to be developed to calculate the overall heat transfer coefficients in a TTHE without using correlations or making any of the assumptions in the studies cited above. These U values could then be used to determine the axial temperature distribution of the fluids in a TTHE.

Methodology

The analysis conducted in this study involves cooling of a product. A similar analysis could be performed for heating of a product. The cold fluid (cooling medium) in the inner tube and outer annulus enters the heat exchanger at a temperature of $T_{c(in)}$ and exits at temperatures $T_{c1(out)}$ and $T_{c2(out)}$ in the inner tube and outer annulus, respectively. The hot fluid (product to be cooled) enters the inner annulus of the TTHE at a temperature of $T_{h(in)}$ and exits at a temperature of $T_{h(out)}$. Modeling the heat transfer in a TTHE is not the same for the case where the hot fluid flows in the same direction as the

cold fluid (co-current) and the case where the hot fluid flows in the opposite direction as the cold fluid (counter-current). Therefore, the formulations for these two different arrangements are analyzed separately.

The following assumptions are made for simplicity:

1. The system is at steady state
2. Both fluids are incompressible
3. Fluid properties are constant
4. Phase change does not occur at any point in the heat exchanger
5. The heat exchanger is insulated from the surroundings

Overall heat transfer coefficients in a counter-current arrangement

The counter-current arrangement is shown in Figure 1. In this arrangement, the hot fluid in the inner annulus flows in the opposite direction of the cold fluid in the inner tube and outer annulus.

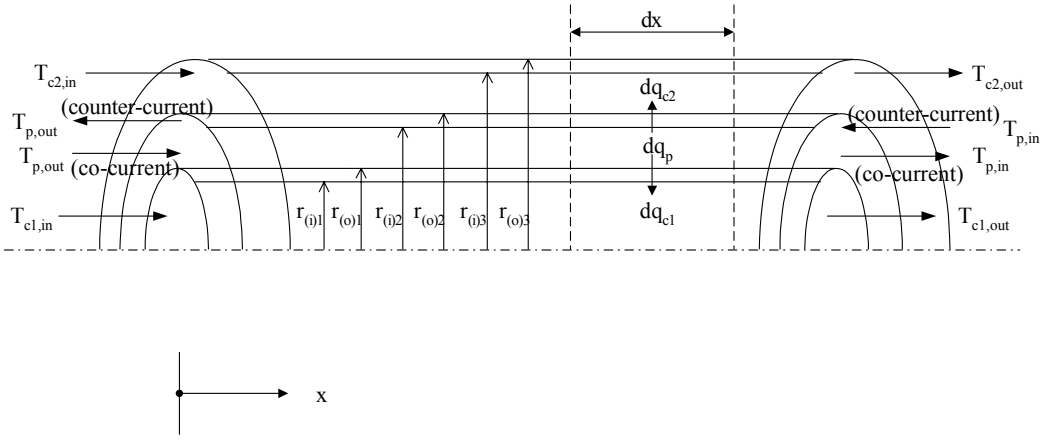

Figure 1. Counter-current and co-current arrangements in a TTHE

The energy balance equations written in differential form are as follows for each stream:

$$dq_{c1} = C_{c1} \cdot dT_{c1} \tag{5}$$

$$dq_{c2} = C_{c2} \cdot dT_{c2} \tag{6}$$

$$dq_{h} = C_{h} \cdot dT_{h} \tag{7}$$

The conservation of energy relates equation (7) to equations (5) and (6) as follows:

$$dq_{h} = dq_{c1} + dq_{c2} \tag{8}$$

The energy balance between the fluids can also be written as follows:

$$dq_{c1} = U_{1} \cdot \Delta T_{1} \cdot dA_{lm1} \tag{9}$$

and

$$dq_{c2} = U_{2} \cdot \Delta T_{2} \cdot dA_{lm2} \tag{10}$$

where,

$$\Delta T_{1} = T_{h} - T_{c1} \tag{11}$$

and

$$\Delta T_{2} = T_{h} - T_{c2} \tag{12}$$

The differential forms of these equations are as follows:

$$d\Delta T_1 = dT_h - dT_{c1} \tag{13}$$

and

$$d\Delta T_2 = dT_h - dT_{c2} \tag{14}$$

Substituting equations (5) and (7) into equation (13), and equations (6) and (7) into equation (14), yields:

$$d\Delta T_1 = \frac{dq_h}{C_h} - \frac{dq_{c1}}{C_{c1}} \tag{15}$$

and

$$d\Delta T_2 = \frac{dq_h}{C_h} - \frac{dq_{c2}}{C_{c2}} \tag{16}$$

Substituting equations (8), (9), and (10) into equations (15) and (16) yields:

$$d\Delta T_1 = \frac{U_1 \cdot \Delta T_1 \cdot dA_{lm1} + U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_h} - \frac{U_1 \cdot \Delta T_1 \cdot dA_{lm1}}{C_{c1}} \tag{17}$$

and

$$d\Delta T_2 = \frac{U_1 \cdot \Delta T_1 \cdot dA_{lm1} + U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_h} - \frac{U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_{c2}} \tag{18}$$

where $dA_{lm1} = 2\pi \cdot \frac{r_{(o)1} - r_{(i)1}}{\ln\left(\frac{r_{(o)1}}{r_{(i)1}}\right)} dx$ and $dA_{lm2} = 2\pi \cdot \frac{r_{(o)2} - r_{(i)2}}{\ln\left(\frac{r_{(o)2}}{r_{(i)2}}\right)} dx$

For simplicity, the following definitions are made:

$$P_1 = 2\pi \cdot \frac{r_{(o)1} - r_{(i)1}}{\ln\left(\frac{r_{(o)1}}{r_{(i)1}}\right)} \quad \text{and} \quad P_2 = 2\pi \cdot \frac{r_{(o)2} - r_{(i)2}}{\ln\left(\frac{r_{(o)2}}{r_{(i)2}}\right)}$$

Thus,

$$dA_{lm1} = P_1 \cdot dx \tag{19}$$

$$dA_{lm2} = P_2 \cdot dx \tag{20}$$

Equations (17) and (18) can be rewritten in the following form:

$$d\Delta T_1 = \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_h} dx + \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_h} dx - \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_{c1}} dx \tag{21}$$

and

$$d\Delta T_2 = \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_h} dx + \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_h} dx - \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_{c2}} dx \tag{22}$$

Dividing both sides of equation (21) by ΔT_1 and equation (22) by ΔT_2, we get:

$$\frac{d\Delta T_1}{\Delta T_1} = \frac{U_1 \cdot P_1}{C_h} dx + \frac{U_2 \cdot P_2}{C_h} \cdot \frac{\Delta T_2}{\Delta T_1} dx - \frac{U_1 \cdot P_1}{C_{c1}} dx \tag{23}$$

and

$$\frac{d\Delta T_2}{\Delta T_2} = \frac{U_1 \cdot P_1}{C_h} \cdot \frac{\Delta T_1}{\Delta T_2} dx + \frac{U_2 \cdot P_2}{C_h} dx - \frac{U_2 \cdot P_2}{C_{c2}} dx \tag{24}$$

Integrating both sides of equations (23) and (24) over the length of the TTHE and solving the integrals results in:

$$\ln\left[\frac{\Delta T_1(L)}{\Delta T_1(0)}\right] = U_1 \cdot A_{lm1}\left(\frac{1}{C_h} - \frac{1}{C_{c1}}\right) + \frac{U_2 \cdot P_2}{C_h} \cdot f(L) \tag{25}$$

and

$$\ln\left[\frac{\Delta T_2(L)}{\Delta T_2(0)}\right] = U_2 \cdot A_{lm2}\left(\frac{1}{C_h} - \frac{1}{C_{c2}}\right) + \frac{U_1 \cdot P_1}{C_h} \cdot g(L) \tag{26}$$

where,

$$f(L) = \int_0^L \frac{\Delta T_2}{\Delta T_1} dx \tag{27}$$

and

$$g(L) = \int_0^L \frac{\Delta T_1}{\Delta T_2} dx \tag{28}$$

At this point, if f(L) and g(L) could be written as functions of U_1 and U_2, it would be possible to solve equations (25) and (26) for the two unknowns, U_1 and U_2. Writing ΔT_1 and ΔT_2 as functions of U_1, U_2, and x (the length along the tube) and then integrating equations (27) and (28), would enable us to express f(L) and g(L) in the desired form.

Expressing ΔT_1 and ΔT_2 as functions of U_1, U_2, and x can be accomplished using the following procedure:

From equations (21) and (22), we get:

$$\frac{d\Delta T_1}{dx} = \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_h} + \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_h} - \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_{c1}} \tag{29}$$

and

$$\frac{d\Delta T_2}{dx} = \frac{U_1 \cdot P_1 \cdot \Delta T_1}{C_h} + \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_h} - \frac{U_2 \cdot P_2 \cdot \Delta T_2}{C_{c2}} \tag{30}$$

Differentiating with respect to x yields:

$$\frac{d^2\Delta T_1}{dx^2} = U_1 \cdot P_1 \cdot \left(\frac{1}{C_h} - \frac{1}{C_{c1}} \right) \cdot \frac{d\Delta T_1}{dx} + \frac{U_2 \cdot P_2}{C_h} \cdot \frac{d\Delta T_2}{dx} \tag{31}$$

and

$$\frac{d^2\Delta T_2}{dx^2} = U_2 \cdot P_2 \cdot \left(\frac{1}{C_h} - \frac{1}{C_{c2}} \right) \cdot \frac{d\Delta T_2}{dx} + \frac{U_1 \cdot P_1}{C_h} \cdot \frac{d\Delta T_1}{dx} \tag{32}$$

Substituting equations (29) and (30) into equations (31) and (32) and rearranging, the equations yield:

$$\frac{d^2\Delta T_1}{dx^2} + B \cdot \frac{d\Delta T_1}{dx} + C \cdot \Delta T_1 = 0 \tag{33}$$

and

$$\frac{d^2\Delta T_2}{dx^2} + B \cdot \frac{d\Delta T_2}{dx} + C \cdot \Delta T_2 = 0 \tag{34}$$

where $$B = U_1 \cdot P_1 \left(\frac{1}{C_{c1}} - \frac{1}{C_h} \right) + U_2 \cdot P_2 \left(\frac{1}{C_{c2}} - \frac{1}{C_h} \right)$$ and,

$$C = U_1 \cdot P_1 \cdot U_2 \cdot P_2 \left(\frac{1}{C_{c1} \cdot C_{c2}} - \frac{1}{C_h \cdot C_{c1}} - \frac{1}{C_h \cdot C_{c2}} \right)$$

The solutions to these second order differential equations depend on the value of B^2- 4C (positive, negative, or zero). Computing the value of B^2- 4C based on the above expressions for B and C yields:

$$B^2 - 4C = \left(U_1 \cdot P_1 \left(\frac{1}{C_{c1}} - \frac{1}{C_h} \right) - U_2 \cdot P_2 \left(\frac{1}{C_{c2}} - \frac{1}{C_h} \right) \right)^2 + 4 \cdot \frac{U_1 \cdot P_1 \cdot U_2 \cdot P_2}{C_h^2} \tag{35}$$

This is always a positive quantity. So, the solutions to equations (33) and (34) are in the following form:

$$\Delta T_1 = G_1 \cdot e^{\lambda_1 x} + G_2 \cdot e^{\lambda_2 x} \tag{36}$$

and

$$\Delta T_2 = G_3 \cdot e^{\lambda_1 x} + G_4 \cdot e^{\lambda_2 x} \tag{37}$$

where G_1, G_2, G_3, and G_4 are constants (independent of location) to be determined from the boundary conditions and λ_1 and λ_2 are the roots of the following equation:

$$\lambda^2 + B \cdot \lambda + C = 0 \tag{38}$$

Accordingly,

$$\lambda_1 = \frac{-B + \sqrt{B^2 - 4C}}{2} \quad \text{and} \quad \lambda_2 = \frac{-B - \sqrt{B^2 - 4C}}{2}$$

To solve equation (36) for G_1 and G_2, and equation (37) for G_3 and G_4, we need two boundary conditions. For equation (36), one of the boundary conditions is the temperature difference between the product and cooling medium in the inner tube at the inlet (x = 0) of the heat exchanger. From equation (36):

$$\Delta T_1(0) = G_1 + G_2$$

The second boundary condition can be one of the following boundary conditions:

1. The temperature difference between the product and cooling medium in the inner tube at the outlet (x = L) of the heat exchanger. Substituting x = L in equation (36) yields:

$$\Delta T_1(L) = G_1 \cdot e^{\lambda_1 L} + G_2 \cdot e^{\lambda_2 L}$$

2. The gradient of the temperature difference between the product and cooling medium in the inner tube at the inlet (x = 0) of the heat exchanger. From equation (29):

$$\left.\frac{d\Delta T_1}{dx}\right|_{x=0} = \frac{U_1 \cdot P_1 \cdot \Delta T_1(0)}{C_h} + \frac{U_2 \cdot P_2 \cdot \Delta T_2(0)}{C_h} - \frac{U_1 \cdot P_1 \cdot \Delta T_1(0)}{C_{c1}}$$

Similarly, the boundary conditions for equation (37) are as follows:

$$\Delta T_2(0) = G_3 + G_4$$

and one of the two following boundary conditions:

1. $$\Delta T_2(L) = G_3 \cdot e^{\lambda_1 L} + G_4 \cdot e^{\lambda_2 L}$$

2. $$\left.\frac{d\Delta T_2}{dx}\right|_{x=0} = \frac{U_1 \cdot P_1 \cdot \Delta T_1(0)}{C_h} + \frac{U_2 \cdot P_2 \cdot \Delta T_2(0)}{C_h} - \frac{U_2 \cdot P_2 \cdot \Delta T_2(0)}{C_{c2}}$$

Thus, ΔT_1 and ΔT_2 can be solved for, as functions of x.

Substituting these values of ΔT_1 and ΔT_2 in equations (27) and (28), we get:

$$f(L) = \int_0^L \frac{\Delta T_2}{\Delta T_1} dx = \int_0^L \frac{G_3 \cdot e^{\lambda_1 x} + G_4 \cdot e^{\lambda_2 x}}{G_1 \cdot e^{\lambda_1 x} + G_2 \cdot e^{\lambda_2 x}} dx \tag{39}$$

$$g(L) = \int_0^L \frac{\Delta T_1}{\Delta T_2} dx = \int_0^L \frac{G_1 \cdot e^{\lambda_1 x} + G_2 \cdot e^{\lambda_2 x}}{G_3 \cdot e^{\lambda_1 x} + G_4 \cdot e^{\lambda_2 x}} dx \tag{40}$$

Integrating the above, yields:

$$f(L) = \frac{L(G_1 \cdot G_4 \cdot \lambda_1 - G_2 \cdot G_3 \cdot \lambda_2) + (G_2 \cdot G_3 - G_1 \cdot G_4)\left[\ln\left(\frac{G_1 \cdot e^{\lambda_1 \cdot L} + G_2 \cdot e^{\lambda_2 \cdot L}}{G_1 + G_2}\right)\right]}{G_1 \cdot G_2 \cdot (\lambda_1 - \lambda_2)} \tag{41}$$

and

$$g(L) = \frac{L(G_2 \cdot G_3 \cdot \lambda_1 - G_1 \cdot G_4 \cdot \lambda_2) + (G_1 \cdot G_4 - G_2 \cdot G_3)\left[\ln\left(\frac{G_3 \cdot e^{\lambda_1 \cdot L} + G_4 \cdot e^{\lambda_2 \cdot L}}{G_3 + G_4}\right)\right]}{G_3 \cdot G_4 \cdot (\lambda_1 - \lambda_2)} \tag{42}$$

Substituting these values of f(L) and g(L) in equations (22) and (23) results in two equations having two unknowns (U_1 and U_2). These two equations can be solved for the

two unknowns, U_1 and U_2, by using a mathematical software such as Maple (Waterloo Maple Inc., Toronto, Canada) which has the capability to solve systems of non-linear equations.

Axial temperature distribution of fluids in the counter-current arrangement

Once the overall heat transfer coefficient values are calculated (by following the procedure outlined above), the axial temperature distribution of the fluids can also be determined. To do so, one needs to follow the procedure outlined below:

Substituting equations (7), (9), and (10) in equation (8), we get:

$$C_h \cdot dT_h = U_1 \cdot \Delta T_1 \cdot dA_{lm1} + U_2 \cdot \Delta T_2 \cdot dA_{lm2} \tag{43}$$

Substituting equations (19) and (20) into equation (43) and dividing each term by C_h, we get:

$$dT_h = \frac{U_1 \cdot P_1}{C_h} \Delta T_1 \cdot dx + \frac{U_2 \cdot P_2}{C_h} \Delta T_2 \cdot dx \tag{44}$$

Integrating equation (44) from 0 to x yields:

$$T_{h(x)} - T_{h(out)} = \frac{U_1 \cdot P_1}{C_h} \int_0^x \Delta T_1 \cdot dx + \frac{U_2 \cdot P_2}{C_h} \int_0^x \Delta T_2 \cdot dx \tag{45}$$

Substituting equations (36) and (37) into equation (45), we get:

$$T_{h(x)} - T_{h(out)} = \frac{U_1 \cdot P_1}{C_h} \int_0^x (G_1 \cdot e^{\lambda_1 x} + G_2 \cdot e^{\lambda_2 x}) dx + \frac{U_2 \cdot P_2}{C_h} \int_0^x (G_3 \cdot e^{\lambda_1 x} + G_4 \cdot e^{\lambda_2 x}) dx \quad (46)$$

Computing the integrals and rearranging the terms, the axial temperature distribution of the fluid in the inner annulus is obtained as follows:

$$T_{h(x)} = T_{h(out)} + \frac{U_1 \cdot P_1}{C_h} \left[\frac{G_1}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_2}{\lambda_2} (e^{\lambda_2 x} - 1) \right] + \frac{U_2 \cdot P_2}{C_h} \left[\frac{G_3}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_4}{\lambda_2} (e^{\lambda_2 x} - 1) \right] \quad (47)$$

Substituting equations (47) and (36) in equation (11), we get the temperature profile of the fluid in the inner tube as a function of axial distance as follows:

$$T_{c1(x)} = T_{h(out)} + \frac{U_1 \cdot P_1}{C_h} \left[\frac{G_1}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_2}{\lambda_2} (e^{\lambda_2 x} - 1) \right] + \frac{U_2 \cdot P_2}{C_h} \left[\frac{G_3}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_4}{\lambda_2} (e^{\lambda_2 x} - 1) \right] - (G_1 \cdot e^{\lambda_1 x} + G_2 \cdot e^{\lambda_2 x}) \quad (48)$$

The axial temperature distribution of the fluid in the outer annulus is obtained in a similar manner by substituting equations (47) and (37) in equation (12):

$$T_{c2(x)} = T_{h(out)} + \frac{U_1 \cdot P_1}{C_h} \left[\frac{G_1}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_2}{\lambda_2} (e^{\lambda_2 x} - 1) \right] + \frac{U_2 \cdot P_2}{C_h} \left[\frac{G_3}{\lambda_1} (e^{\lambda_1 x} - 1) + \frac{G_4}{\lambda_2} (e^{\lambda_2 x} - 1) \right] - (G_3 \cdot e^{\lambda_1 x} + G_4 \cdot e^{\lambda_2 x}) \quad (49)$$

Effective overall heat transfer coefficient in the counter-current arrangement

If we consider using the TTHE for a cooling application, in most cases the cold fluids in the inner tube and outer annulus would originate from the same source. So, the cold fluids would in fact have the same temperature as they enter the TTHE. At the exit of the TTHE, the cold fluid in the inner tube and outer annulus would most likely have different temperatures due to differences in C_{c1} and C_{c2}, and, P_1 and P_2. If these two cold fluid streams are combined with a tee type fitting, we will have only one cold fluid stream leaving the system (Figure 2).

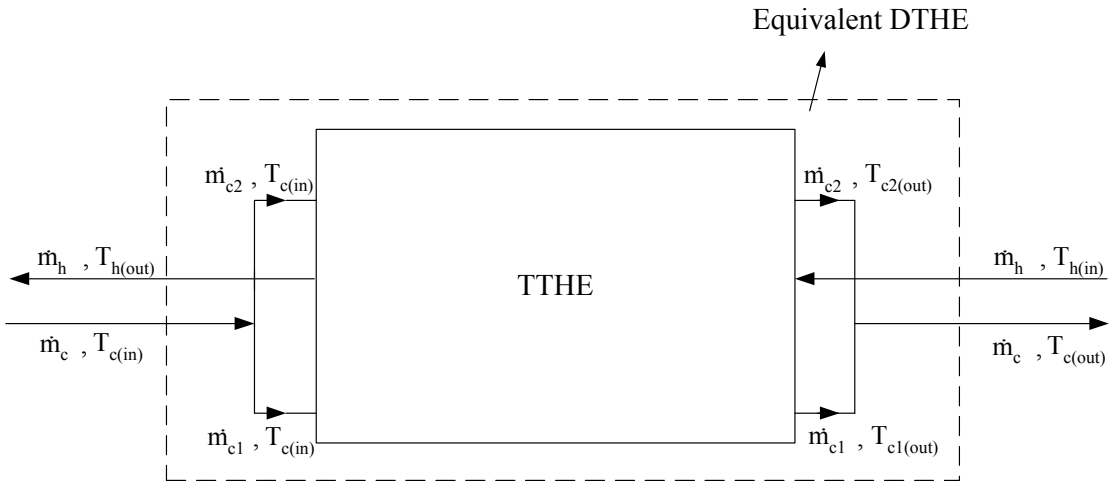

Figure 2. Converting a TTHE into an equivalent DTHE

The solid rectangle in Figure 2 represents the TTHE. There are three fluid streams entering and leaving the TTHE. On the other hand, there are only two fluid streams entering and leaving the dashed box, one being the hot fluid and the other being the cold fluid. This is similar to a DTHE with the flows being in a counter-current mode and

hence is an equivalent DTHE that can be used to replace the TTHE. To calculate the overall heat transfer coefficient in this equivalent DTHE, we use equation (4), where the logarithmic mean area of the DTHE is equal to the sum of the two logarithmic mean areas of the TTHE:

$$A_{lm} = A_{lm1} + A_{lm2} \tag{50}$$

and the logarithmic mean temperature difference is:

$$\Delta T_{lm} = \frac{(T_{h(in)} - T_{c(out)}) - (T_{h(out)} - T_{c(in)})}{\ln\left[\frac{(T_{h(in)} - T_{c(out)})}{(T_{h(out)} - T_{c(in)})}\right]} \tag{51}$$

The U value calculated for this equivalent DTHE combines the effect of the overall heat transfer coefficients U_1 and U_2. Thus, we refer to this value as the effective overall heat transfer coefficient (U_e). We can then use this value to compare a TTHE to a DTHE and make the choice of one over the other, depending on the process parameters.

Overall heat transfer coefficients in a co-current arrangement

The co-current arrangement is shown in Figure 1. Similar to the counter-current formulation, here the hot fluid flows in the inner annulus and the cold fluid in the inner tube and outer annulus with the only difference being that they all flow in the same direction.

Equations (5), (6), and (7) are valid for the co-current formulation also, but equation (8) takes the following form:

$$-dq_h = dq_{c1} + dq_{c2} \tag{52}$$

The negative sign in front of dq_h indicates that the product is losing energy along the tube length in the positive x direction. This is the only difference between the two flow arrangements and it yields the following equations:

$$d\Delta T_1 = \frac{-U_1 \cdot \Delta T_1 \cdot dA_{lm1} - U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_h} - \frac{U_1 \cdot \Delta T_1 \cdot dA_{lm1}}{C_{c1}} \tag{53}$$

and

$$d\Delta T_2 = \frac{-U_1 \cdot \Delta T_1 \cdot dA_{lm1} - U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_h} - \frac{U_2 \cdot \Delta T_2 \cdot dA_{lm2}}{C_{c2}} \tag{54}$$

Following the same steps as in the counter-current formulation, we get:

$$\ln\left[\frac{\Delta T_1(0)}{\Delta T_1(L)}\right] = U_1 \cdot A_{lm1}\left(\frac{1}{C_h} + \frac{1}{C_{c1}}\right) + \frac{U_2 \cdot P_2}{C_h} \cdot h(L) \tag{55}$$

and

$$\ln\left[\frac{\Delta T_2(0)}{\Delta T_2(L)}\right] = U_2 \cdot A_{lm2}\left(\frac{1}{C_h} + \frac{1}{C_{c2}}\right) + \frac{U_1 \cdot P_1}{C_h} \cdot j(L) \tag{56}$$

where,

$$h(L) = \int_0^L \frac{\Delta T_2}{\Delta T_1} dx \tag{57}$$

and

$$j(L) = \int_0^L \frac{\Delta T_1}{\Delta T_2} dx \tag{58}$$

Again, following a procedure similar to the one used for the counter-current formulation, h(L) and j(L) can be written as functions of U_1 and U_2. This time, the second order ordinary differential equations are as follows:

$$\frac{d^2\Delta T_1}{dx^2} + D \cdot \frac{d\Delta T_1}{dx} + E \cdot \Delta T_1 = 0 \tag{59}$$

and

$$\frac{d^2\Delta T_2}{dx^2} + D \cdot \frac{d\Delta T_2}{dx} + E \cdot \Delta T_2 = 0 \tag{60}$$

where $D = U_1 \cdot P_1 \left(\frac{1}{C_{c1}} + \frac{1}{C_h} \right) + U_2 \cdot P_2 \left(\frac{1}{C_{c2}} + \frac{1}{C_h} \right)$ and,

$$E = U_1 \cdot P_1 \cdot U_2 \cdot P_2 \left(\frac{1}{C_{c1} \cdot C_{c2}} + \frac{1}{C_h \cdot C_{c1}} + \frac{1}{C_h \cdot C_{c2}} \right)$$

Computing the value of D^2- 4E based on the above expressions for D and E, yields:

$$D^2 - 4E = \left(U_1 \cdot P_1 \left(\frac{1}{C_{c1}} + \frac{1}{C_h} \right) - U_2 \cdot P_2 \left(\frac{1}{C_{c2}} + \frac{1}{C_h} \right) \right)^2 + 4 \cdot \frac{U_1 \cdot P_1 \cdot U_2 \cdot P_2}{C_h^2} \tag{61}$$

As in the counter-current case, this quantity is also always positive. So, the solutions to equations (59) and (60) are in the following form:

$$\Delta T_1 = G_5 \cdot e^{\lambda_3 x} + G_6 \cdot e^{\lambda_4 x} \tag{62}$$

and

$$\Delta T_2 = G_7 \cdot e^{\lambda_3 x} + G_8 \cdot e^{\lambda_4 x} \tag{63}$$

where G_5, G_6, G_7, and G_8 are constants (independent of location) to be determined from the boundary conditions and λ_3 and λ_4 are the roots of the following equation:

$$\lambda^2 + D \cdot \lambda + E = 0 \tag{64}$$

Accordingly,

$$\lambda_3 = \frac{-D + \sqrt{D^2 - 4E}}{2} \quad \text{and} \quad \lambda_4 = \frac{-D - \sqrt{D^2 - 4E}}{2}$$

The boundary conditions for equations (62) and (63) can be defined in a manner similar to the case of counter-current flow. Solving these equations by using the boundary conditions and substituting the expressions for ΔT_1 and ΔT_2 [equations (62) and (63)] in equations (57) and (58), we get:

$$h(L) = \frac{L(G_5 \cdot G_8 \cdot \lambda_3 - G_6 \cdot G_7 \cdot \lambda_4) + (G_6 \cdot G_7 - G_5 \cdot G_8)\left[\ln\left(\frac{G_5 \cdot e^{\lambda_3 \cdot L} + G_6 \cdot e^{\lambda_4 \cdot L}}{G_5 + G_6}\right)\right]}{G_5 \cdot G_6 \cdot (\lambda_3 - \lambda_4)} \tag{65}$$

and

$$j(L) = \frac{L(G_6 \cdot G_7 \cdot \lambda_3 - G_5 \cdot G_8 \cdot \lambda_4) + (G_5 \cdot G_8 - G_6 \cdot G_7)\left[\ln\left(\frac{G_7 \cdot e^{\lambda_3 \cdot L} + G_8 \cdot e^{\lambda_4 \cdot L}}{G_7 + G_8}\right)\right]}{G_7 \cdot G_8 \cdot (\lambda_3 - \lambda_4)} \quad (66)$$

As in the counter-current case, substituting equations (65) and (66) in equations (55) and (56), we end up with two equations with two unknowns. These two equations can be solved again by using a mathematical software such as Maple (Waterloo Maple Inc., Toronto, Canada) to obtain U_1 and U_2.

Axial temperature distribution of fluids in the co-current arrangement

The axial temperature distribution of the fluids in the co-current arrangement are derived in a manner similar to the counter-current case.

Rewriting equation (52), we get:

$$-C_h \cdot dT_h = U_1 \cdot \Delta T_1 \cdot dA_{lm1} + U_2 \cdot \Delta T_2 \cdot dA_{lm2} \quad (67)$$

Starting of from this equation, and following the same steps as in the counter-current formulation yields the following axial temperature distribution equations for the co-current arrangement:

$$T_{h(x)} = T_{h(in)} - \frac{U_1 \cdot P_1}{C_h}\left[\frac{G_5}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_6}{\lambda_4}(e^{\lambda_4 x} - 1)\right] - \frac{U_2 \cdot P_2}{C_h}\left[\frac{G_7}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_8}{\lambda_4}(e^{\lambda_4 x} - 1)\right] \quad (68)$$

$$T_{c1(x)} = T_{h(in)} - \frac{U_1 \cdot P_1}{C_h}\left[\frac{G_5}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_6}{\lambda_4}(e^{\lambda_4 x} - 1)\right] - \frac{U_2 \cdot P_2}{C_h}\left[\frac{G_7}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_8}{\lambda_4}(e^{\lambda_4 x} - 1)\right] - (G_5 \cdot e^{\lambda_3 x} + G_6 \cdot e^{\lambda_4 x}) \quad (69)$$

$$T_{c2(x)} = T_{h(in)} - \frac{U_1 \cdot P_1}{C_h}\left[\frac{G_5}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_6}{\lambda_4}(e^{\lambda_4 x} - 1)\right] - \frac{U_2 \cdot P_2}{C_h}\left[\frac{G_7}{\lambda_3}(e^{\lambda_3 x} - 1) + \frac{G_8}{\lambda_4}(e^{\lambda_4 x} - 1)\right] - (G_7 \cdot e^{\lambda_3 x} + G_8 \cdot e^{\lambda_4 x}) \quad (70)$$

Effective overall heat transfer coefficient in the co-current arrangement

Determination of the effective overall heat transfer coefficient (U_e) for the co-current arrangement is accomplished in a very similar way to that in the counter-current arrangement. Equation (4) is still valid but in this case the logarithmic mean temperature difference term is written as follows:

$$\Delta T_{lm} = \frac{(T_{h(in)} - T_{c(in)}) - (T_{h(out)} - T_{c(out)})}{\ln\left[\frac{(T_{h(in)} - T_{c(in)})}{(T_{h(out)} - T_{c(out)})}\right]} \quad (71)$$

Discussion

The formulations in the previous sections made use of the temperature difference terms between the hot fluid and cold fluids to compute U_1 and U_2. Both in the co-current and counter-current formulations, to determine the constants of the temperature

difference equations [(36), (37), (62), and (63)], three boundary conditions were specified, from which two need to be used. Under ideal conditions, where there are no errors originating from measuring flow rate and temperature, and when the assumptions listed at the beginning of the methodology section are valid, both boundary conditions ($\Delta T_1(L)$ or $\left.\frac{d\Delta T_1}{dx}\right|_{x=0}$) should give exactly the same results for the overall heat transfer coefficient values and the temperature profiles of the fluids. However, it is very likely to have errors during collecting data and/or errors associated with the accuracy and sensitivity of the measuring devices (thermocouples and totalizers). Therefore, using the two different approaches may result in slightly different answers upon solution of the system of equations to determine the overall heat transfer coefficients and temperature profiles.

Conclusion

The overall heat transfer coefficients and axial temperature distribution of fluids in a TTHE were determined using the energy balance equations on a control volume. Calculating U_1, U_2, and the temperature profiles are useful for designing a heat exchanger to meet the process requirements. U_1 and U_2 values may also be useful for determining the convective heat transfer coefficient values (h). Future work will be devoted to calculating the convective heat transfer coefficients in a TTHE.

Acknowledgements

Support from the North Carolina Agricultural Research Service for the research study undertaken here, resulting in the publication of Paper No. FSR-03-12 of the Journal Series of the Department of Food Science, North Carolina State University, Raleigh, is gratefully acknowledged.

Symbols

A	Heat transfer area	m^2
C	Heat capacity rate	W/K
f-j(L)	Non-linear functions of U_1 and U_2	m
G_{1-8}	Constants appearing in temperature profiles	
L	Length of the tubes	m
P	Mean perimeter	m
q	Heat transfer rate	W
r	Tube radius	m
T	Absolute temperature	K
U	Overall heat transfer coefficient	W/m^2-K
x	Distance in the axial direction	m
Δ	Difference	
λ_{1-4}	Roots of equations written to solve ODEs	

Subscripts

c	Cold fluid
c1	Cold fluid in inner tube
c2	Cold fluid in outer annulus
e	Effective
h	Hot fluid

i	Inner
in	Inlet
o	Outer
out	Outlet
lm	Logarithmic mean
1	between the inner two tubes
2	between the outer two tubes

References

1. Incropera, F.P., DeWitt, D.P. 1990. Fundamentals of heat and mass transfer (3rd edition). John Wiley & Sons.

2. Zuritz, C.A. (1990). On the design of triple concentric-tube heat exchangers. Journal of Food Process Engineering. Vol. 12: 113-130.

3. Afgan, N., Carvalho, M., Bar-Cohen, A., Butterworth, D., Roetzel, W. 1996. New developments in heat exchangers. Overseas Publishers Association.

4. Unal, A. (1998). Theoretical analysis of triple concentric-tube heat exchangers. Part 1: Mathematical Modeling. International Communications in Heat and Mass Transfer. Vol. 25(7): 949-958.

5. Unal, A. (2001). Theoretical analysis of triple concentric-tube heat exchangers. Part 2: Case studies. International Communications in Heat and Mass Transfer. Vol. 28(2): 243-256.

6. Satyanarayana, V.V., Datta, A.K., Mishra, B.P. (1995). Design and testing of a small scale indirect type ultra high temperature (UHT) milk sterilizer. Journal of Food Engineering. Vol. 26: 379-387.

7. Sahoo, P.K., Ansari, I.A., Datta, A.K. (2002). Computer-aided design and performance evaluation of an indirect type helical tube ultra-high temperature (UHT) milk sterilizer. Journal of Food Engineering. Vol. 51: 13-19.

8. Sieder, E.N., Tate, G.E. (1936). Heat transfer and pressure drop of liquids in tubes. Industrial and Engineering Chemistry. Vol. 28(12): 1429-1435.

9. Chopey, N.P., Hicks, T.G. 1984. Hand book of chemical engineering calculations. Vol. 7: 33. McGraw Hill, New York.

10. Kern, D.Q. 1984. Process heat transfer. McGraw Hill, New York.

Ediz Batmaz (Research Assistant)
K. P. Sandeep (Assistant Professor)

Dr. K. P. Sandeep
Department of Food Science, Box 7624
North Carolina State University
Raleigh, NC 27695-7624
E-mail: kp_sandeep@ncsu.edu
Phone: (1) (919) 515-2957
Fax: (1) (919) 515-7124

Submitted to the Journal of Heat and Mass Transfer on Monday, June 02, 2003.

MANUSCRIPT II

Overall Heat Transfer Coefficients and Axial Temperature Distribution in a Triple Tube Heat Exchanger

Ediz Batmaz (Research Assistant)

K.P. Sandeep (Associate Professor)

Please address all correspondence to:

Dr. K. P. Sandeep

129 Schaub Hall

Department of Food Science, Box 7624

North Carolina State University

Raleigh, NC 27695-7624

Phone: (919) 515-2957

Fax: (919) 515-7124

E-mail: kp_sandeep@ncsu.edu

Abstract

Computation of overall heat transfer coefficients in a triple tube heat exchanger is more complicated than in the case of a double tube heat exchanger since the two overall heat transfer coefficients are not independent of one other. A new procedure was developed to calculate these overall heat transfer coefficients and an effective overall heat transfer coefficient value for known inlet and outlet temperatures, and heat capacities of the fluids (product and heating/cooling medium). In this study, this newly developed procedure was utilized and the overall heat transfer coefficients and axial temperature distribution of fluids were computed for a cooling process for different flow rates and inlet temperatures of the fluid streams. The effectiveness of the triple tube heat exchanger was compared to that of a DTHE of identical length. It was observed that when the fluids were flowing in a co-current manner, the temperature of the cooling medium with lower heat capacity exceeded the temperature of the product before the fluids exit the TTHE which caused a loss in effectiveness of the TTHE.

Keywords: Triple tube heat exchanger, double tube heat exchanger, overall heat transfer coefficients, effective overall heat transfer coefficients, axial temperature distribution

Introduction

Heat exchangers are an essential part of the food industry. Pasteurization, sterilization, drying, evaporation, cooling, and freezing are just a few of the purposes that they are being used for. The type of heat exchanger to be used is determined by the process and the product specifications. Nevertheless, tubular heat exchangers play a major role in accomplishing the heat exchange needs of the food industry. Tubular heat exchangers can have different configurations. The simplest one is the double tube heat exchanger (DTHE) which consists of two concentric tubes of equal length. In this configuration, two fluids exchange heat with one flowing in the inner tube and the other in the annulus formed between the two tubes. A triple tube heat exchanger (TTHE) is a modified form of a double tube heat exchanger. In this case, there are three concentric tubes of equal length and accordingly, three fluid streams exchanging heat. The fluid to be heated or cooled flows in the inner annulus formed between the inner and intermediate tubes, and the heating or cooling medium flows in the inner tube and the outer annulus formed between the intermediate and outer tubes. TTHEs have advantages over DTHEs when compared from a heat transfer standpoint (Zuritz, 1990).

Several design and performance studies have been carried out to increase the overall heat transfer coefficients in tubular heat exchangers while keeping the pumping power at acceptable levels. For these heat exchangers, the overall heat transfer coefficient can be described using the following expression:

$$\frac{dq}{dA} = U \cdot \Delta T \tag{1}$$

where $\frac{dq}{dA}$ is the local heat flux, U is the local overall heat transfer coefficient, and ΔT is the overall local temperature difference between the two fluids. From the above equation, U can be defined as the proportionality factor between $\frac{dq}{dA}$ and ΔT. For a constant value of ΔT, an increase in U would increase the amount of heat transferred across the differential area, dA (McCabe, Smith, and Harriott, 1985).

For a DTHE, U can be calculated by substituting the experimental parameters to the following equation:

$$q = U \cdot A_{lm} \cdot \Delta T_{lm} \tag{2}$$

where 'q' is the total heat transferred from the hot fluid to the cold fluid, 'A_{lm}' is the logarithmic mean surface area, and 'ΔT_{lm}' is the logarithmic mean temperature difference across the boundary where heat transfer is taking place. Using the inlet and outlet temperatures of a fluid stream, the energy gained or lost by that fluid stream can be computed using the following expression:

$$q = \dot{m} \cdot c_p \cdot (T_{(out)} - T_{(in)}) \tag{3}$$

where 'c_p' is the specific heat of the fluid, '$T_{(out)}$' is the temperature of the fluid at the outlet of the heat exchanger, and 'T_{in}' is the temperature of the fluid at the inlet of the heat exchanger.

In the case of a TTHE, there are two overall heat transfer coefficients to be calculated; one for the heat transfer between the fluid to be heated/cooled and the heating/cooling fluid in the inner tube (U_1), and one for the heat transfer between the fluid to be heated/cooled and the heating/cooling fluid in the annulus formed between the outer two tubes (U_2). Calculation of these two overall heat transfer coefficients in a TTHE is not as straightforward as it is for a DTHE. Zuritz (1990) and Unal (1998, 2001) solved the governing energy balance equations to determine the axial temperature distribution for the fluids in a TTHE. However, the axial temperature distribution equations strictly depend on the overall heat transfer coefficients and the authors either used existing empirical correlations for heat transfer coefficients (h) and then calculated the U values based on these correlations, or assumed that the heat flux was constant throughout the length of the TTHE. These approaches are limited to certain heat exchanger specifications such as smooth surface of the tubes and also to certain operating conditions such as turbulent flow of the fluids. Batmaz and Sandeep (2003), started with the energy balance equations and derived two equations in terms of U_1 and U_2 which have to be solved simultaneously. This study provided a generic and more accurate way of calculating overall heat transfer coefficients (U_1 and U_2) in a TTHE. They also derived equations to determine the axial temperature distribution of all three fluids. The third part of their study focused on computation of a single U value for the entire TTHE which was based on comparing the TTHE to an equivalent DTHE.

The purposes of this study were to:

(1) compute the overall heat transfer coefficients in a corrugated TTHE by using the procedure developed by Batmaz and Sandeep (2003) at various flow rates and inlet temperatures of the fluids;

(2) determine the effectiveness of a corrugated TTHE at different operating conditions;

(3) compare the co-current and counter-current arrangements in a corrugated TTHE from a heat transfer standpoint;

(4) compute the effective overall heat transfer coefficient along with the individual overall heat transfer coefficients;

(5) compare the effectiveness of a corrugated TTHE to that of a smooth surface DTHE.

Materials and Methods

A figure of the experimental setup is presented in figure 1. The setup consists of a shell and tube heat exchanger (STHE -- No Bac Unitherm XIV, Cherry-Burrell, Delavan, WI) to heat the product, a holding section, and a TTHE (Waukesha Cherry-Burrell, Delavan, WI) to cool the product. A continuous loop was established for both the product and the cooling medium. The system was at steady-state at the time of recording temperatures and flow rates. Data (inlet and outlet temperatures) was recorded using a datalogger (CR10X, Campbell Scientific, Inc., USA). A back pressure valve was placed at the exit of the product line on the TTHE and the pressure was set to 60 psi to avoid boiling of the product. City water was used as the product and it was pumped through the

tube side of shell-and-tube heat exchanger, the holding tube, and the inner annulus of the TTHE. Steam was used as the heating medium on the shell side of the STHE and a propylene glycol solution (30% propylene glycol, 70% city water) was used as the cooling medium in the TTHE.

City water was pumped from a 50 gallon tank to the STHE by a variable frequency pump (Reeves Motordrive, Reliance Electric Company, Columbus, IN). The heating process took place in two steps (two shell passes). The outlet of the heating section was connected to a straight holding tube of 0.0381 m O.D. and 3.048 m length.

From the holding tube, the product (water) passed on to the TTHE (Figure 2) where it was cooled. The TTHE had a corrugated surface on all tubes and consisted of four straight sections. The O.D.s of the three tubes were 0.0508 m, 0.0635 m, and 0.0762 m respectively. The wall thickness of all three tubes was the same (0.00165 m). The effective length where heat transfer occurred between the product (in the inner annulus) and the cooling medium in the inner tube and outer annulus was 5.65 m for each of the four straight sections.

To monitor the temperatures at various locations, type-T thermocouples (Omega Engineering, Inc., Stamford, CT) were used. The temperature of the product was measured at the inlet and outlet of the TTHE. The inlet temperature of the cooling medium was the same for both the inner tube and outermost annulus. So, one thermocouple was located where propylene glycol entered to the system, and one

thermocouple at the exit of each of these cooling mediums. These separate paths for the cooling medium were then combined as they exited the TTHE and another thermocouple was used to determine the temperature of this mixed stream of cooling medium.

The flow rates of all three streams of the cooling medium (inner tube, outer annulus, and mixed stream at exit) were measured using totalizers (Pittsburgh Equitable Meterco. Type 14, Pittsburgh, PA). The product flow rate was measured by determining the time it took to fill a bucket of known volume. For simplicity, all fluid properties except specific heat were assumed to be constant and the TTHE perfectly insulated from the surroundings.

Once the total heat transferred from the product to the cooling medium (q_h) was calculated, the effectiveness of the heat exchanger for a given set of process parameters was calculated as follows:

$$\varepsilon = \frac{q_h}{C_{min}(T_{p(in)} - T_{c(in)})} \tag{4}$$

The denominator of this equation represents the maximum amount of heat that could be gained or lost by the fluid that has the minimum heat capacity rate (Incropera and de Witt, 1990).

To calculate the overall heat transfer coefficients for the co-current runs, the following equations developed by Batmaz and Sandeep (2003) were used:

$$\ln\left[\frac{\Delta T_1(0)}{\Delta T_1(L)}\right] = U_1 \cdot A_{lm1}\left(\frac{1}{C_h}+\frac{1}{C_{c1}}\right)+\frac{U_2 \cdot P_2}{C_h}\cdot h(L) \tag{5}$$

and

$$\ln\left[\frac{\Delta T_2(0)}{\Delta T_2(L)}\right] = U_2 \cdot A_{lm2}\left(\frac{1}{C_h}+\frac{1}{C_{c2}}\right)+\frac{U_1 \cdot P_1}{C_h}\cdot j(L) \tag{6}$$

where,

$$h(L) = \int_0^L \frac{\Delta T_2}{\Delta T_1} dx \tag{7}$$

and

$$j(L) = \int_0^L \frac{\Delta T_1}{\Delta T_2} dx \tag{8}$$

All experimental results for the co-current configuration indicated that the temperature of the product at the exit of the TTHE was lower than the exit temperature of the cooling medium in the outer annulus. This seemingly erroneous observation required further analysis since it could potentially alter the technique used to compute the overall heat transfer coefficients. Since the annular gap between the outer two tubes was much smaller than the diameter of the inner tube, the cooling medium in the inner tube always had a significantly higher flow rate than the cooling medium in the outer annulus. As a

result of this, and the fact that all the three fluids flow in the TTHE in the same direction, the temperature of the cooling medium in the outer annulus increased rapidly while the temperature of the cooling medium in the inner tube increased at a relatively low rate. At some point along the length of the TTHE, the temperatures of the cooling medium in the outer annulus and the product became equal to one other. At this same point, the temperature of the cooling medium in the inner tube was lower than the other two. Therefore, this cooling medium continued cooling down the product, which caused the product to have a lower temperature than the cooling medium in the outer annulus from that point on. That point is referred to as the crossover point. After the crossover point, ΔT_2 takes negative values, and the left hand side of equation (6) becomes incomputable. Therefore, the equations derived by Batmaz and Sandeep (2003) needed to be modified to allow us to calculate the overall heat transfer coefficients for co-current runs where crossover takes place.

Equation (5) still holds at and beyond the crossover point. So, at this point, there is one equation available and two unknowns (U_1 and U_2). Hence, either one more equation must be provided without introducing any additional unknowns or one additional equation must be provided for each unknown that is going to be introduced into the system of equations to be solved. It can be easily seen that one equation would be $\Delta T_2 = 0$ at the point of crossover (v).

$$G_7 \cdot e^{\lambda_3 v} + G_8 \cdot e^{\lambda_4 v} = 0 \qquad (9)$$

However, the point of crossover (distance from inlet of TTHE) is an unknown. Thus, one more equation is needed. Prior to the point of crossover, the temperature of the outer cooling medium rises due to the heat transfer from product. Beyond the point of crossover, however, the temperature of outer cooling medium decreases since the temperature of product is lower than that of the cooling medium in the outer annulus. Therefore, the temperature of the cooling medium in the outer annulus would have a maximum value at the point of crossover and the derivative of the axial temperature distribution should be zero at this point. The temperature distribution for the outer cooling medium is as follows for the co-current arrangement (Batmaz and Sandeep, 2003):

$$\begin{aligned} T_{c2(x)} &= T_{h(in)} - \frac{U_1 \cdot P_1}{C_h}\left[\frac{G_5}{\lambda_3}(e^{\lambda_3 x}-1)+\frac{G_6}{\lambda_4}(e^{\lambda_4 x}-1)\right] \\ &-\frac{U_2 \cdot P_2}{C_h}\left[\frac{G_7}{\lambda_3}(e^{\lambda_3 x}-1)+\frac{G_8}{\lambda_4}(e^{\lambda_4 x}-1)\right]-(G_7 \cdot e^{\lambda_3 x}+G_8 \cdot e^{\lambda_4 x}) \end{aligned} \tag{10}$$

Taking the derivative of this equation with respect to x, yields:

$$\begin{aligned} \frac{dT_{c2(x)}}{dx} &= -\frac{U_1 \cdot P_1}{C_h}\left(G_5 \cdot e^{\lambda_3 x}+G_6 \cdot e^{\lambda_4 x}\right) \\ &-\frac{U_2 \cdot P_2}{C_h}\left(G_7 \cdot e^{\lambda_3 x}+G_8 \cdot e^{\lambda_4 x}\right)-(G_7 \cdot \lambda_3 \cdot e^{\lambda_3 x}+G_8 \cdot \lambda_4 \cdot e^{\lambda_4 x}) \end{aligned} \tag{11}$$

Setting this expression equal to zero at the point of crossover yields:

$$\begin{aligned} &\frac{U_1 \cdot P_1}{C_h}\left(G_5 \cdot e^{\lambda_3 v}+G_6 \cdot e^{\lambda_4 v}\right) \\ &+\frac{U_2 \cdot P_2}{C_h}\left(G_7 \cdot e^{\lambda_3 v}+G_8 \cdot e^{\lambda_4 v}\right)+(G_7 \cdot \lambda_3 \cdot e^{\lambda_3 v}+G_8 \cdot \lambda_4 \cdot e^{\lambda_4 v})=0 \end{aligned} \tag{12}$$

Thus, we have three equations [(5), (9), and (12)] and three unknowns (U_1, U_2, and v). Solving these equations simultaneously, one can calculate the two overall heat transfer coefficients and the point of crossover for co-current runs where crossover takes place.

To compute the effective overall heat transfer coefficient for each run, the procedure described by Batmaz and Sandeep (2003) was followed. The authors computed the effective overall heat transfer coefficient for a TTHE using equation (2), where q is the total amount of heat transferred across the two heat transfer surface areas, A_{lm} is the sum of the two areas across which heat transfer is taking place. The ΔT_{lm} term in this equation can be expanded to the following forms for the counter-current and co-current experiments, respectively:

Counter-current:
$$\Delta T_{lm} = \frac{(T_{h(in)} - T_{c(out)}) - (T_{h(out)} - T_{c(in)})}{\ln\left[\frac{(T_{h(in)} - T_{c(out)})}{(T_{h(out)} - T_{c(in)})}\right]} \tag{13}$$

Co-current:
$$\Delta T_{lm} = \frac{(T_{h(in)} - T_{c(in)}) - (T_{h(out)} - T_{c(out)})}{\ln\left[\frac{(T_{h(in)} - T_{c(in)})}{(T_{h(out)} - T_{c(out)})}\right]} \tag{14}$$

The $T_{c(out)}$ term in these equations is the outlet temperature of the cooling medium that is formed by combining the two separate cooling mediums at the exit of the TTHE.

The next part of the study was to compare the corrugated surface TTHE to a smooth surface double tube heat exchanger (DTHE) in order to determine the difference

between these two heat exchangers in terms of the total amount of energy transferred and the effectiveness. For the DTHE analysis, all fluid properties were assumed to be constant. The flow rate and inlet temperature of the product was set to the same values as the ones measured from the TTHE experiments. The inlet temperature of the cooling medium was also set to the same value as the one measured from the TTHE experiments, and the flow rate of the cooling medium in the DTHE was set to be the sum of the flow rates of the cooling medium in the TTHE. The DTHE under consideration was assumed to consist of two smooth tubes, having the same dimensions as any of the two tubes of the TTHE. In this case, for each experiment conducted in the TTHE, the result can be compared to the theoretical results generated from three different DTHEs. These are the DTHEs formed by two tubes having the same dimensions as (1) the inner two tubes of the TTHE; (2) the outer two tubes of the TTHE; and (3) the inner and outer tube of the TTHE. In addition to varying the tube dimensions, one can also vary the tube in which the product and the cooling medium are flowing. In a DTHE, there are two alternatives, either the product flows in the inner tube and the cooling medium in the annulus, or vice versa. Taking into account all of the possibilities, the results from a TTHE experiments can be compared to the results from six different DTHE runs.

To compare the results from the TTHE to that of a DTHE, a theoretical heat transfer analysis of a DTHE was performed based on the Nusselt number correlations for flow in a tube [Sieder and Tate, 1936 -- equation (15)] and in an annulus [Lee, 1968 -- equation (16)]:

$$N_{Nu} = 0.027(N_{Re})^{0.8}(N_{Pr})^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.53} \tag{15}$$

$$N_{Nu} = 0.020(N_{Re})^{0.8}(N_{Pr})^{0.33}\left(\frac{D_{(j)_{inner}}}{D_{(j-1)_{outer}}}\right)^{0.53} \tag{16}$$

In equation (16), the hydraulic diameter (D_h) was used for calculations, with the hydraulic diameter for an annulus given by:

$$D_h = D_{(j)_{inner}} - D_{(j-1)_{outer}} \tag{17}$$

where j is the number of tube (2 or 3) being used as the outer tube of the DTHE. Once N_{Nu} is computed, we can determine the convective heat transfer coefficient (h) using the following expression:

$$N_{Nu} = \frac{h \cdot D_h}{k} \tag{18}$$

Using the convective heat transfer coefficient values computed at the inner and outer surfaces (h_i and h_o), the overall heat transfer coefficient can be calculated as follows:

$$\frac{1}{U \cdot A_{lm}} = \frac{1}{h_i A_i} + \frac{\Delta r}{k_w A_{lm}} + \frac{1}{h_o A_o} \tag{19}$$

After calculating the overall heat transfer coefficient, three equations with three unknowns can be written for comparison with a theoretical DTHE. These are equation (3) written twice, once for the heat lost by product and once for the heat gained by the cooling medium, and equation (2). The three unknowns associated with these equations

are q, $T_{p(o)}$, and $T_{g(o)}$. Mathcad 7 (Mathsoft Enginnering & Education, Inc., Cambridge, MA) was used to solve the equations.

Results and Discussion

Data was gathered by running experiments each time by varying one or more of the following parameters: Glycol flow rates, product flow rate, product inlet temperature and the flow arrangement (counter-current, co–current).

The results for counter-current and co-current runs are summarized in Tables 1 and 2, respectively. The program developed using Maple for computation of the overall heat transfer coefficients did not yield U_1, U_2 and v values for some co-current experiments. These results are indicated as N/A in Table 2. Errors originating from inaccurate measurement of temperatures and flow rates might be cause of the above mentioned program execution error. Analyzing the rest of the results in Table 2, it is seen that crossover occurs towards the exit of the TTHE. It can be readily seen from Tables 1 and 2 that, for the same values of product flow rate and product inlet temperature, as the cooling medium flow rate increased, the amount of heat transferred ($p < 0.0001$), U_1 ($p = 0.0009$), U_2 ($p = 0.01$), effective overall heat transfer coefficient ($p < 0.0001$), and effectiveness ($p < 0.0001$) values increased. Also, as the product inlet temperature increased and the product and cooling medium flow rates were kept constant, the total amount of energy transferred increased ($p < 0.0001$), but overall heat transfer coefficients and effective overall heat transfer coefficients did not change significantly ($p > 0.1$). For both co-current and counter-current runs, increasing the flow rate of the

product and keeping the other parameters constant increased the total amount of energy transferred, overall heat transfer coefficients, and effective overall heat transfer coefficient values ($p < 0.0001$), but decreased the effectiveness ($p = 0.0004$). The decrease in effectiveness values for an increase in product flow rate is mainly due to the fact that the total amount of time heat transfer occurred decreased when the product flow rate was high and product temperature did not decrease as rapidly as in the case of lower product flow rates because of its high heat capacity. On the other hand, all three overall heat transfer coefficient values increased as the product flow rate and/or glycol flow rates increased. As the mean velocities of the fluids increased, the Reynolds numbers associated with these streams also increased. There is a positive correlation between Reynolds number and Nusselt number, and Nusselt number and overall heat transfer coefficients. Thus, higher velocity of fluids resulted in higher overall heat transfer coefficients. Similarly, higher cooling medium flow rates in the inner tube than in the outer annulus was the reason behind U_1 values being higher than U_2 values for all experiments.

Axial temperature distribution of fluids for experiments 9 and 26 are also presented in Figures 3 and 4, respectively. Temperature distribution of fluids for experiment 26 was chosen among all the co-current results because the crossover could be illustrated clearly on the graph for this experiment. Experiment 9 on the other hand has the same input parameters as experiment 26 and has been presented along with results from experiment 9 for comparison of fluid temperature profiles for different flow arrangements. Based on the computational results, crossover occurred 16.0 m from the

entrance of the fluids for experiment 26. For other co-current experiments, the point of crossover ranged from 14.2 m to 22.3 m from the entrance of the TTHE.

To compare results of the counter-current and co-current experiments, fluid flow rates and product inlet temperature were set to the same values for the corresponding co-current runs. It was observed that for all runs, the counter-current arrangement resulted in higher effectiveness values than the co-current runs. Counter-current runs also yielded higher U_1 ($p = 0.02$), U_2 ($p = 0.008$), U_e ($p < 0.0001$), and net amount of energy transferred (q_p) [$p < 0.0001$]. q_p was higher in counter-current runs for two reasons: the crossover phenomenon that occurred during co-current runs which reversed the direction of heat transfer and therefore reduced the net amount of heat transferred, and the effect of local temperature difference between the hot and cold fluids which resulted in higher amounts of heat transferred in counter-current runs.

For every co- and counter-current run in the TTHE, all the possible DTHE results were generated. It was observed that the DTHE is most effective when it consisted of two tubes that have the same dimensions as the inner two tubes of the TTHE. The DTHE results are presented in tables 3 and 4 for counter-current and co-current runs, respectively. For some cases, the product flowing in the inner tube and cooling medium in the annulus resulted in higher effectiveness values; whereas for other cases, the DTHE was more effective when the product was flowing in the annulus and cooling medium in the inner tube. In fact, for all cases where glycol flow rates are high, calculations indicated that the effectiveness of the DTHE was higher when product was flowing in the

annulus and cooling medium in the inner tube. This is also true for the cases where both the product and glycol flow rates were low.

When the counter-current results of the DTHE and TTHE were compared, it was seen that the TTHE is able to transfer more energy for all cases and thus is more effective. However, when the effective overall heat transfer coefficient calculated for the TTHE experiments were compared with the overall heat transfer coefficient values calculated for the DTHE experiments, exceptions were seen where overall heat transfer coefficients calculated for the DTHE are higher. Comparison of the co-current results did not follow the same trend. For all cases, the overall heat transfer coefficient calculated for the DTHE was higher than the effective overall heat transfer coefficient calculated for TTHE. This could be due to the very high cooling medium flow rates in the DTHE. However, the net amount of energy transferred (q_p) was still higher in the TTHE than in the DTHE. Effectiveness values were higher for the TTHE experiments at high product flow rates (4 and 6 gpm). On the other hand, at lower product flow rates (2 gpm), the effectiveness of the DTHE was higher. Therefore, using a TTHE instead of a DTHE may not always be advantageous.

The difference in the values of U_1 and U_2 calculated based on previously used methods from the values calculated based on the new procedure were also analyzed. When the U values were calculated using equation (2), the highest deviation from the values calculated with the new procedure was 74%, and the lowest deviation was 1%.

When the U values were calculated based on empirical correlations (equations (15) and (16)), the highest deviation was 75% and the lowest deviation was 1%.

Conclusions

All three overall heat transfer coefficient values calculated for the runs in the TTHE indicated that the counter-current arrangement is better than co-current arrangement in terms of effectiveness for the range of the parameters covered. For the counter-current runs, the effectiveness values in the TTHE were higher than those in the DTHE consisting of two tubes with the same dimensions as the inner two tubes of the TTHE. Co-current results showed that in some cases using a DTHE may result in higher effectiveness values. This effectiveness loss in TTHE is mainly due to the crossover occurring in co-current runs. The values of the overall heat transfer coefficients depend on the temperature and flow rate values. Therefore, analyzing the effect of deviation of each input parameter from its actual value on calculated overall heat transfer coefficient would be useful in determining the sensitivity of the U values on these parameters.

Acknowledgements

Support from the North Carolina Agricultural Research Service for the research study undertaken here, resulting in the publication of Paper No. FSR-03-13 of the Journal Series of the Department of Food Science, North Carolina State University, Raleigh, is gratefully acknowledged. Thanks are also due to Gary Cartwright and Jack Canady for their time and contribution to this project. The use of trade names in this publication does

not imply endorsement by the North Carolina Agricultural Research Service of the products named nor criticism of similar ones not mentioned.

Symbols

A	Heat transfer area	m^2
C	Heat capacity rate	W/K
D	Diameter	m
c_p	Specific heat at constant pressure	J/kg-K
$G_{5\text{-}8}$	Constants appearing in temperature profiles	
h	Convective heat transfer coefficient	W/m^2-K
h(L), j(L)	Non-linear functions of U_1 and U_2	m
k	Thermal conductivity	W/m-K
$\dot{m}$	Mass flow rate	kg/s
N_{Nu}	Nusselt number	
N_{Pr}	Prandtl number	
N_{Re}	Reynolds number	
L	Length of triple tube heat exchanger	m
P	Mean perimeter	m
q	Heat transfer rate	W
r	Tube radius	m
T	Absolute temperature	K
U	Overall heat transfer coefficient	W/m^2-K
v	Point of crossover	m
x	Distance in the axial direction	m
Δ	Difference	
ε	Effectiveness	
λ_3, λ_4	Roots of equations written to solve ODEs	
μ	Viscosity	Pa-s

Subscripts

1	Between the inner two tubes
2	Between the outer two tubes
b	Bulk
c	Combined cooling medium
c1	Cooling medium in the inner tube
c2	Cooling medium in the outer annulus
e	Effective
h	Hot product
i	Inner
in	Inlet
j	Tube number (2, 3)
lm	Logarithmic mean
min	Minimum
o	Outer
out	Outlet

ss	Stainless steel
w	Wall

Abbreviations

DTHE	Double Tube Heat Exchanger
TTHE	Triple Tube Heat Exchanger
STHE	Shell and Tube Heat Exchanger

References

Batmaz, E., Sandeep, K.P. 2003. Calculation of Overall Heat Transfer Coefficients in a Triple Tube Heat Exchanger. Submitted to the Journal of Heat and Mass Transfer.

Incropera, F.P., DeWitt, D.P. 1990. Fundamentals of heat and mass transfer (3rd edition). John Wiley & Sons.

Lee, Y. 1968. Turbulent heat transfer from the core tube in thermal entrance regions of concentric annuli. International Journal of Heat and Mass Transfer. Vol. 11: 509-522.

McCabe, W.L., Smith, J.C., Harriott, P. 1985. Unit operations of chemical engineering (4th edition). McGraw – Hill.

Sieder, E.N., Tate, G.E. 1936. Heat transfer and pressure drop of liquids in tubes. Industrial and Engineering Chemistry. Vol. 28(12): 1429-1435.

Unal, A. 1998. Theoretical analysis of triple concentric-tube heat exchangers. Part 1: Mathematical Modeling. International Communications in Heat and Mass Transfer. Vol. 25(7): 949-958.

Unal, A. 2001. Theoretical analysis of triple concentric-tube heat exchangers. Part 2: Case studies. International Communications in Heat and Mass Transfer. Vol. 28(2): 243-256.

Zuritz, C.A. 1990. On the design of triple concentric-tube heat exchangers. Journal of Food Process Engineering. Vol. 12: 113-150.

Exp	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
V_h	6.0	6.0	6.0	6.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	2.0	2.0	2.0	2.0	2.0	2.0
V_c	54.6	55.8	20.3	54.9	19.8	54.0	19.7	53.8	19.6	53.6	19.3	53.5	19.2	52.7	18.7	52.9	18.9
V_{c1}	41.0	41.2	15.5	41.2	15.6	40.3	15.4	40.5	15.3	40.4	15.2	40.2	15.2	39.6	15.1	39.7	15.0
V_{c2}	14.0	14.7	4.9	13.9	4.4	13.9	4.5	13.4	4.3	13.4	4.3	13.4	4.0	13.3	3.7	13.3	3.8
$T_{h(in)}$	119.7	100.0	100.3	80.0	80.0	120.0	120.2	100.0	100.1	80.0	79.8	120.1	120.1	100.4	100.0	80.0	80.2
$T_{h(out)}$	12.2	10.3	18.0	7.1	12.8	7.0	14.0	6.1	8.4	2.9	7.5	2.3	4.8	1.6	4.3	1.3	4.2
$T_{c(in)}$	12.0	9.9	14.1	6.8	7.6	6.8	11.8	5.9	5.0	2.6	4.1	2.0	3.3	1.2	3.0	1.1	3.0
$T_{c1(out)}$	19.9	16.9	33.7	12.3	24.0	12.5	28.9	10.7	20.2	6.7	16.3	5.0	12.7	3.8	10.8	3.1	9.1
$T_{c2(out)}$	33.9	28.9	55.0	22.0	43.3	23.7	51.6	19.9	40.9	14.6	33.1	11.9	29.3	10.0	25.4	7.8	20.5
$T_{c(out)}$	25.0	20.5	39.8	15.3	29.0	16.8	35.5	13.9	25.6	9.5	20.6	7.6	17.2	6.0	14.9	4.9	12.4
q	164.9	142.7	129.4	112.7	105.5	118.1	112.3	97.5	97.3	81.5	77.2	62.7	61.5	53.5	50.4	41.7	39.8
U_1	1702	1397	958	1479	830	1130	767	1126	653	993	598	592	411	528	415	587	398
U_2	1144	1035	699	1003	577	711	518	664	440	619	416	252	241	248	234	256	228
U_e	1371	1218	799	1200	671	913	634	878	539	808	504	422	327	397	321	421	308
ε	1.00	1.00	0.95	1.00	0.93	1.00	0.98	1.00	0.96	1.00	0.96	1.00	0.99	1.00	0.99	1.00	0.98

TABLE 1. Data for counter-current runs in TTHE

V_h, V_c, V_{c1}, V_{c2} in gpm
$T_{h(in)}$, $T_{h(out)}$, $T_{c(in)}$, $T_{c1(out)}$, $T_{c2(out)}$, $T_{c(out)}$ in °C
q in kW
U_1, U_2, U_e in W/m^2-K

Exp	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
V_h	6.0	6.0	6.0	6.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	2.0	2.0	2.0	2.0	2.0	2.0
V_c	55.0	54.5	20.1	54.1	20.0	54.1	19.8	53.7	19.6	53.7	19.6	53.7	19.6	50.1	17.4	50.3	17.6
V_{c1}	41.0	40.0	15.3	40.3	15.1	40.3	15.0	40.3	15.0	40.5	15.1	40.4	15.0	37.7	13.6	38.0	13.8
V_{c2}	14.4	14.5	4.9	14.3	4.8	14.0	4.8	13.4	4.7	13.7	4.7	13.6	4.7	12.6	3.8	12.5	3.9
$T_{h(in)}$	119.5	100.0	99.7	79.8	80.0	119.8	120.1	100.1	100.2	80.6	79.8	119.4	119.6	100.4	100.0	80.4	80.3
$T_{h(out)}$	26.2	23.7	36.3	17.2	24.9	15.3	27.9	10.7	21.4	7.1	16.1	7.3	14.4	6.8	14.7	5.9	13.0
$T_{c(in)}$	14.6	13.8	14.5	8.7	4.9	6.0	7.0	2.5	2.9	0.2	1.2	2.0	2.2	1.7	2.6	1.9	3.2
$T_{c1(out)}$	23.6	21.4	33.8	15.1	21.7	12.9	25.4	8.5	18.7	5.1	13.3	5.7	12.6	5.2	12.2	4.5	10.6
$T_{c2(out)}$	28.2	25.4	37.8	18.8	27.2	17.4	29.9	12.6	23.7	8.9	18.3	8.8	16.5	8.3	17.1	7.3	15.4
$T_{c(out)}$	24.8	22.5	34.8	16.1	23.1	14.0	26.5	9.5	19.9	6.1	14.5	6.5	13.5	6.0	13.3	5.2	11.6
q	145.4	120.1	102.3	100.2	89.1	110.1	97.4	94.3	83.7	78.3	66.5	59.7	55.7	51.5	45.8	40.4	36.5
U_1	971	928	930	900	790	-	755	-	681	-	606	-	453	-	371	-	331
U_2	638	613	474	591	399	-	424	-	378	-	336	-	292	-	226	-	203
U_e	773	771	630	761	579	558	488	549	465	552	429	326	297	323	258	312	247
v	20.7	21.9	16.6	22.3	19.4	-	14.6	-	16.0	-	18.0	-	14.2	-	16.0	-	18.0
ε	0.89	0.89	0.74	0.88	0.73	0.92	0.82	0.92	0.81	0.91	0.81	0.95	0.90	0.95	0.88	0.95	0.87

TABLE 2. Data for co-current runs in TTHE

V_h, V_c, V_{c1}, V_{c2} in gpm
$T_{h(in)}$, $T_{h(out)}$, $T_{c(in)}$, $T_{c1(out)}$, $T_{c2(out)}$, $T_{c(out)}$ in °C
q in kW
U_1, U_2, U_e in W/m^2-K
v in m

Exp	1†	2†	3‡	4†	5‡	6†	7‡	8†	9‡	10†	11‡	12†	13†	14†	15†	16†	17†
$T_{h(in)}$	119.7	100.0	100.3	80.0	80.0	120.0	120.2	100.0	100.1	80.0	79.8	120.1	120.1	100.4	100.0	80.0	80.2
$T_{h(out)}$	23.2	19.0	34.1	14.4	24.7	12.2	28.6	10.4	19.8	6.3	16.0	3.1	8.5	2.1	7.5	1.8	6.6
$T_{c(in)}$	12.0	9.9	14.1	6.8	7.6	6.8	11.8	5.9	5.0	2.6	4.1	2.0	3.3	1.2	3.0	1.1	3.0
$T_{c(out)}$	22.7	18.8	33.8	14.1	24.5	14.9	30.5	12.7	21.7	8.2	17.5	6.5	15.2	5.0	13.1	4.2	11.1
q	150.9	126.5	103.5	102.6	86.5	112.3	95.5	93.3	83.7	76.7	66.5	61.0	58.1	51.2	48.2	40.7	38.4
U	1041	1051	732	1044	727	915	601	914	601	912	598	694	479	691	473	692	475
ε	0.90	0.90	0.77	0.90	0.76	0.95	0.85	0.95	0.84	0.95	0.84	0.99	0.96	0.99	0.95	0.99	0.95

TABLE 3. Theoretical results for counter-current runs in a smooth DTHE

Exp	18†	19†	20‡	21†	22‡	23†	24‡	25†	26‡	27†	28‡	29†	30†	31†	32†	33†	34†
$T_{h(in)}$	119.5	100.0	99.7	79.8	80.0	119.8	120.1	100.1	100.2	80.6	79.8	119.4	119.6	100.4	100.0	80.4	80.3
$T_{h(out)}$	31.3	27.5	41.3	20.1	28.8	17.3	33.6	12.3	26.0	8.3	19.8	7.0	15.9	6.2	15.0	5.5	12.9
$T_{c(in)}$	14.6	13.8	14.5	8.7	4.9	6.0	7.0	2.5	2.9	0.2	1.2	2.0	2.2	1.7	2.6	1.9	3.2
$T_{c(out)}$	24.3	21.9	32.0	15.4	20.7	13.7	24.8	9.2	18.3	5.7	13.6	6.3	13.0	5.6	12.6	5.0	11.0
q	137.7	113.2	91.2	93.3	80.1	106.7	90.0	91.4	77.3	75.3	62.5	58.5	54.0	49.1	44.3	39.0	35.1
U	1041	1051	730	1044	729	915	602	914	601	912	601	694	479	691	473	692	475
ε	0.84	0.84	0.69	0.84	0.68	0.90	0.76	0.90	0.76	0.90	0.76	0.96	0.88	0.95	0.87	0.95	0.87

TABLE 4. Theoretical results for co-current runs in a smooth DTHE

$T_{h(in)}$, $T_{h(out)}$, $T_{c(in)}$, $T_{c(out)}$ in °C
q in kW
U in W/m^2-K
(†) product in the annulus and cooling medium in the inner tube
(‡) product in the inner tube and cooling medium in the annulus

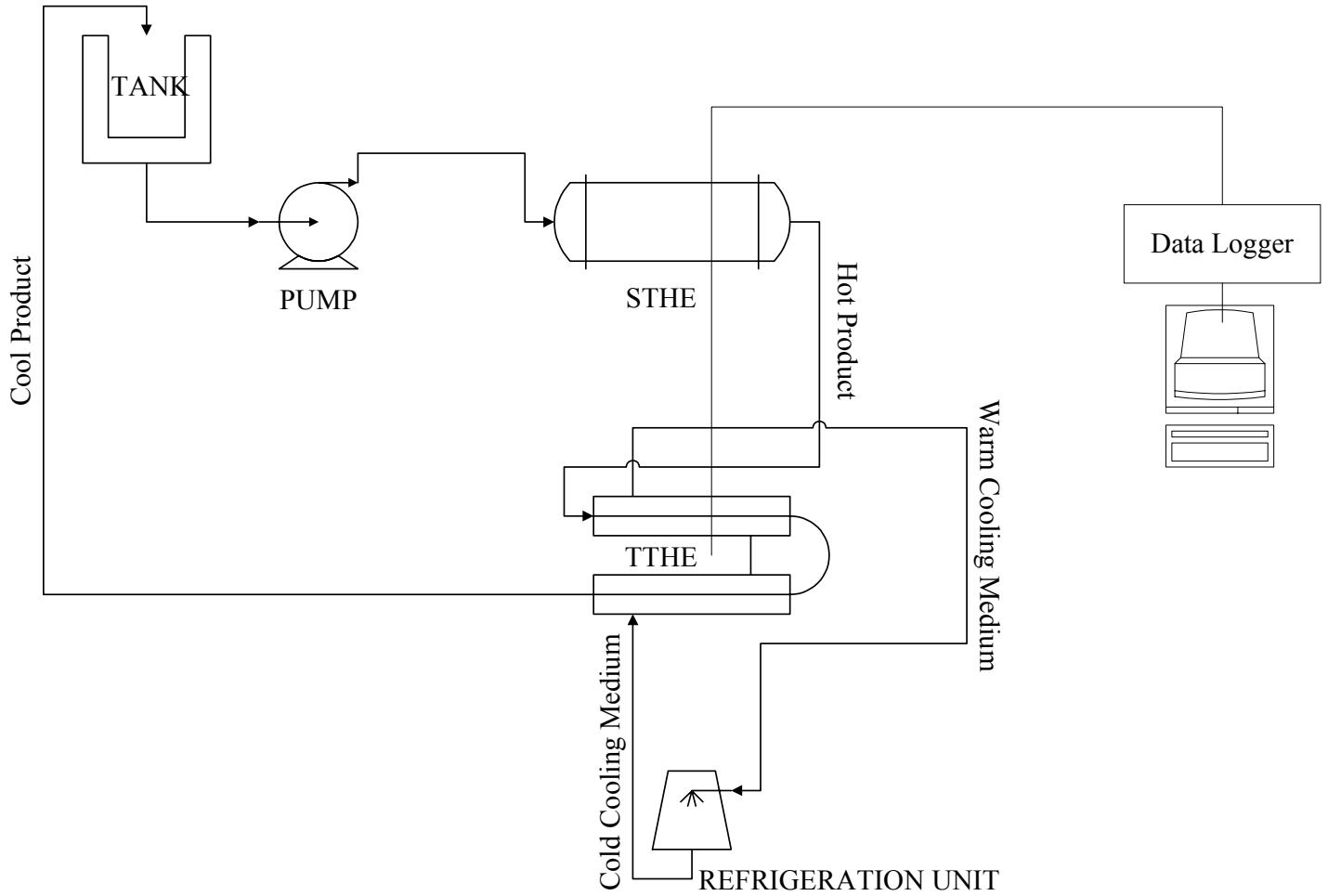

Figure 1. Exprerimental Setup

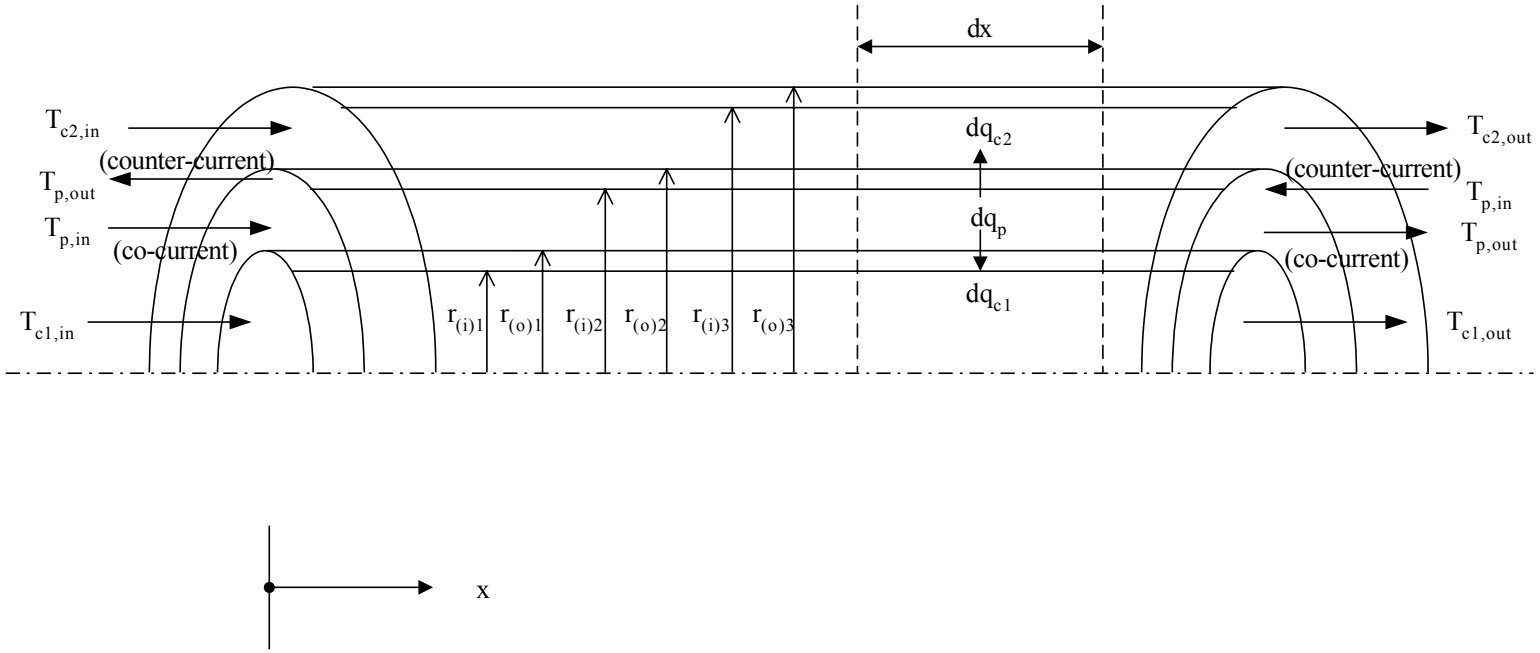

Figure 2. Counter-current and co-current arrangements in a TTHE

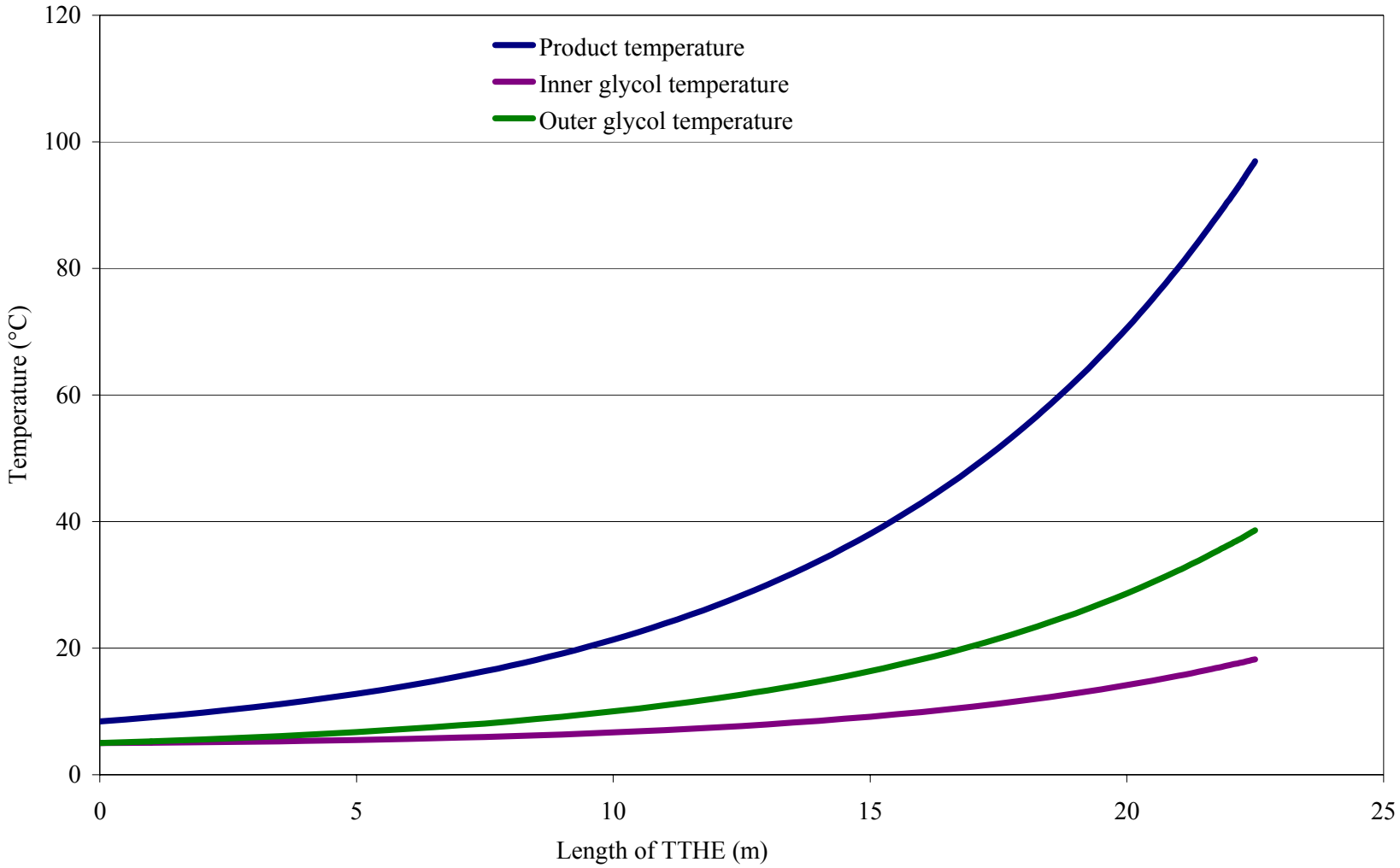

Figure 3. Axial temperature distribution of fluids in counter-current arrangement

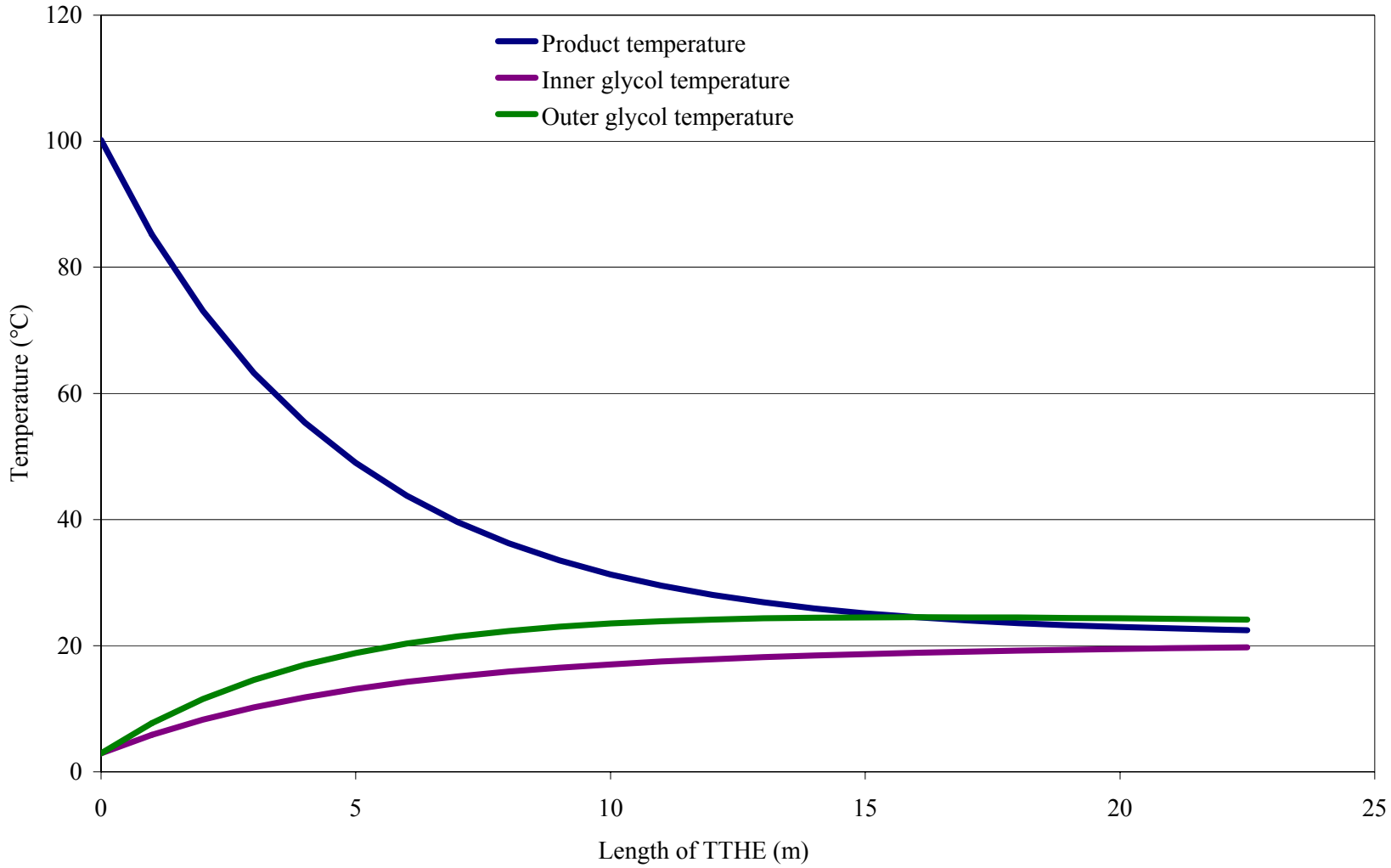

Figure 4. Axial temperature distribution of fluids in co-current arrangement

Ediz Batmaz (Research Assistant)
K. P. Sandeep (Associate Professor)

Dr. K. P. Sandeep
Department of Food Science, Box 7624
North Carolina State University
Raleigh, NC 27695-7624
E-mail: kp_sandeep@ncsu.edu
Phone: (1) (919) 515-2957
Fax: (1) (919) 515-7124

Submitted to the Journal of Food Process Engineering.

CONCLUDING REMARKS

This study was devoted to the analysis of the heat transfer phenomenon in a triple tube heat exchanger (TTHE). During the literature review on the subject, it was seen that no procedure is available for accurate calculation of overall heat transfer coefficients in a TTHE. Therefore, initial studies focused on developing a procedure for accurate computation of the overall heat transfer coefficients and temperature profiles of the fluids in a TTHE. An effective overall heat transfer coefficient concept was also established. Experiments were conducted in a corrugated surface TTHE and the data gathered was used to computer the overall heat transfer coefficients, axial temperature distribution of the fluids, and the effectiveness values for each run. The effective overall heat transfer coefficient and the effectiveness values were used to compare the TTHE with an equivalent (theoretical) double tube heat exchanger (DTHE). It was found that the effectiveness of the co-current runs in a TTHE was not always greater than the effectiveness of the theoretical co-current runs in a DTHE. The cause for a decrease in the effectiveness in the TTHE was attributed to the crossover phenomenon (pg. 45-46) that occurred during co-current runs. The developed procedure was modified for computation of the overall heat transfer coefficients for co-current runs where crossover occurred. The changes in calculated U values were also analyzed for changes in flow rates of fluids and product inlet temperatures. The results were in good agreement with the literature with respect to the factors affecting the U values; supporting the reliability of the developed method.