

Modal Spectra Seismic Analysis for Non-Uniform Excitation

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ABSTRACT

The Independent Support Motion (ISM) method of Modal Spectra Seismic Analysis (MSSA) applies to structural systems subject to non-uniform excitation. This paper presents the merits of the ISM method over the Envelope Response Spectra (ERS) method of MSSA, discusses adequacy of the supports group definition, evaluates the effect of support motions correlation, and explains the response patterns and dependency on problem characteristics.

1 INTRODUCTION

Piping, equipment, and structural systems rarely experience uniform seismic excitation at all supports. In numerous investigations¹⁻⁴, seismic response of structural systems to non-uniform excitation or ISM represented by response spectra were studied. This paper examines the more general ISM method of which the ERS method is a special case.

2 MERITS OF THE ISM METHOD

The ERS and ISM methods examined herein for a uni-directional excitation can be easily extended to a three dimensional case.

The ERS method normally used to compute response of a structural system to non-uniform supports excitation is the conventional MSSA method that uses envelope spectra attempting to account for non-uniformity. It implicitly assumes in-phase spectra. Although envelope spectra are intended to compensate for phasing effects, no proof exists for the validity of this. The ERS method computes the i th mode normal coordinate q_i as an algebraic sum of the products of i th mode participation factor P_{ij} for j th support and i th mode spectral displacement d_{ij} for j th support, which, by assumption, is equal to envelope d_i . This yields:

$$q_i = \sum_{j:1}^N P_{ij} d_{ij} = d_i \sum_{j:1}^N P_{ij} \quad (1)$$

The ISM method accounts for differences in phasing and amplitudes of supports spectra. With this method, some judgement is necessary about phasing, or phasing information must be available. For motions in phase, equation (1) becomes:

$$q_i = \sum_{j=1}^N P_{ij} d_{ij} \quad (2)$$

The summation is algebraic (ALG). For motions in random phase, equation (1) becomes:

$$q_i = \left[\sum_{j=1}^N (P_{ij} d_{ij})^2 \right]^{1/2} \quad (3)$$

The Square Root Sum of Squares (SRSS) is used for summation. For the Absolute Sum (ABS) method, equation (1) becomes:

$$q_i = \sum_{j=1}^N |P_{ij} d_{ij}| \quad (4)$$

This conservative method does not realistically account for phasing. For mixed phasing (i.e., some motions in phase, some randomly phased and phasing of some unknown), a combination of equations (2), (3), and (4) is used.

The significance of phasing and amplitude differences is illustrated by three cases of numerical examples. Assuming three supports, corresponding P_{ij} , d_{ij} , and d_i are given in Table 1. From the Table 1 answers for $|q_i|$, it is seen that: ERS < ISM-ALG < ISM-SRSS < ISM-ABS for case 1, ISM-ALG < ISM-SRSS < ISM-ABS < ERS for case 2, and ISM-SRSS < ERS = ISM-ALG < ISM-ABS for case 3. Thus, no definite comparative pattern of answers exists for all cases except ISM-SRSS ≤ ISM-ABS and, in the unlikely situation of all participation factors with same signs, ISM-ALG = ISM-ABS ≤ ERS.

Table 1. Numerical Examples

Case	Pi1	Pi2	Pi3	di1	di2	di3	Envelope di	qi			
								Eq(1) ERS	Eq(2) ISM-ALG	Eq(3) ISM-SRSS	Eq(4) ISM-ABS
1	1.0	-1.5	1.1	2	3	0.5	3	1.8	1.95	4.96	7.05
2	1.0	-0.5	1.5	2	3	1	3	6.0	2.0	2.92	5.0
3	1.0	-0.5	1.5	2	1	3	3	6.0	6.0	4.95	7.0

3 SUPPORTS GROUP DEFINITION CONSIDERATIONS

Algebraic sum and SRSS for the ISM method are a function of phasing only. The group definition should not be a function of spectral amplitudes. The Subudhi^{4,5} and NUREG-1061⁶ definitions are impractical. No two spectra from a building seismic analysis are identical, so these definitions place every support in a distinct group. This can be unrealistic (conservative or unconservative) as shown by the numerical examples.

Mathematical analysis for phase correlation is the best way to determine in-phase, out-of-phase, and random phasing conditions for supports motions. In lieu of analysis, one can use careful examination of supports spectra and sound engineering judgement to establish phasing. This approach is not a mere assumption; it realistically reflects the nature of support motions and is superior to the assumption of the in-phase envelope spectra of the ERS method. On this basis, the algebraic sum for response from a set of supports spectra is applicable when it can be judged that these spectra are essentially proportionally related. For this, the motion for any of the supports can be obtained from a reference excitation. Therefore, this includes the uniform and 180-degree out-of-phase motions as special cases. Higher building elevations floor spectra, even with significantly different amplitudes, can be considered as proportionally related. Similarly, the SRSS combination for responses from a supports spectra set is applicable when it is judged that the spectra are essentially statistically independent (uncorrelated). Absolute sum should be used only as a last resort when it is not possible to make sound engineering judgement about phasing.

Based on sound engineering judgement, spectra in the Subudhi⁴ analysis can be considered proportionally related. This is further borne out by the ISM-ALG results, which are, on an overall basis, closer to time-history than results from other methods. An examination of Subudhi⁵ and Bezler⁷ studies for consistency between input spectra and results follows.

- **RHR Problem** - Nine spectra groups were used, but two can represent all X and Y spectra because groups 2 - 9 are almost identical. Three groups can be used for Z direction. Bezler⁷ figures 7 through 12 indicate that the ISM-ALG method is, in general, closest to time-history because the number of groups is two or three (not nine). This makes analysis more amenable for the ISM-ALG method rather than ISM-SRSS or ISM-ABS method.
- **AFW Problem** - Fifteen groups were used, but three or four can represent all X and Z spectra. Fewer than 15 groups can be used for Y, but we cannot specify the number because of the busy nature of the plots. Bezler⁷ figure 21 and Subudhi⁵ figures 5-5 through 5-8 favor the ISM-ALG method for the same reasons as for the RHR problem.
- **Z-Bend Problem** - Three groups were appropriately used; hence, Bezler⁷ figures 13 and 14 favor the ISM-SRSS method.

- ⊙ **BM1 Problem** - Five groups can possibly be reduced to four in X and Y directions and to three in Z direction. Trends in Bezler⁷ figures 15 and 16 are difficult to associate with the ISM-ALG or ISM-SRSS method. When spectra with different amplitudes but similar frequency content are grouped, the results generally favor the ISM-ALG method.
- ⊙ **BM2 Problem** - Four groups can possibly be reduced to three; hence, Bezler⁷ figures 17 and 18 favor the ISM-SRSS method.
- ⊙ **BM3 Problem** - Two groups can possibly be reduced to one; hence, Bezler⁷ figures 19 and 20 favor the ISM-ALG method.

The preceding proposal to reduce the number of groups is based on judgement of proportional spectra and is borne out by results.

4 SUPPORT MOTION CORRELATION EFFECTS

The support motion correlation effects are very important in the ISM method. Bezler⁷, Francis⁸, and Liu⁹ studied the ISM method coupled with ASME Code Case N-411 damping and phase correlation among the motions. Francis concluded that low correlation coefficients are not a prerequisite for the ISM-SRSS method even though many coefficients were quite high. He studied ISM-SRSS and ISM-ABS, but not ISM-ALG methods. Even though Francis' results are conservative with respect to time-history results, the lack of concern over the degree of similarity between motions for the validity of SRSS appears to be unwarranted. Apparently, this was recognized based on the recommendation for additional phase correlation data. Bezler found, in general, highly correlated motions in the RHR and AFW problems and could not assess the impact of correlation. ISM-SRSS and ISM-ABS results were conservative with respect to time-history results; however, based on sound engineering judgement to define "support group", the ISM-ALG results were more favorably comparable to time-history results for several problems. That is borne out by the highly correlated motions for those problems. Liu found the motions to be quite dependent. He concluded that support phasing is not important and that the SRSS produced acceptable results in spite of significant correlation. MS1 SRSS results with each support as a group are not significantly different from MS2 algebraic results with all supports in a group or from results using other groupings.

5 RESPONSE DEPENDENCY ON PROBLEM CHARACTERISTICS

Subudhi⁵ and Bezler⁷ reported that the ISM-ABS method provides higher responses than those by the ISM-SRSS, ISM-ALG, or the ERS methods, but Francis⁸ showed that the ISM-ABS method provides lower responses than the ERS method. These studies are examined to establish reasons for such opposite conclusions and dependency between response and problem characteristics.

Subudhi⁵ and Bezler⁷ applied the ISM method to six problems. Nine spectra groups were used for the RHR problem. Only one

differed significantly from the others (at the two Hertz peak, or about half the fundamental frequency of the piping). These differences were not of any consequence; therefore, all the spectra were essentially of comparable magnitudes. The RHR piping consists of a typical configuration between anchors with intermediate supports. Because of the complexity, it is difficult to identify any peculiarity or isolation zones, except that the highest spectra seem to be applied at the highest elevation. Fifteen spectra groups were used for the AFW problem. Two had amplitudes and shapes significantly different from the others. The AFW spectra and piping geometry are similar to those of the RHR problem. Several factors may have caused ISM-ABS results to be larger than ERS results. A combination of averages over 33 earthquakes, an unrealistic absolute sum because of the proportional spectra, interaction of responses (i.e., global nature of modes) caused by complex piping configurations, and consideration of closely-spaced modes may account for the variance. The number of spectra groups for the Z-bend problem appears to be appropriate, and the piping configuration is standard and rigid. Therefore, Bezler⁷ figures 13 and 14 do not exhibit the patterns of concern in a pronounced manner. In fact, some ERS results seem to be higher than ISM-ABS results. There is no obvious additional reason to explain why ISM-ABS results are higher than ERS results for the BM1, BM2, and BM3 problems. Francis⁸ applied the ISM method to two problems. For the main steam system, three groups of spectra were different from each other, which somewhat contradicts the high correlation reported. Mainly, for the recirculation loop, for which no spectra are provided, ISM-ABS provided lower results than the ERS method.

6 CONCLUSIONS AND RECOMMENDATIONS

- ⑥ The ISM method, which is technically, physically, and mathematically more rigorous than the ERS method, requires knowledge of the supports spectra phasing. A summation method appropriate to the phasing characteristics must be used for realistic response. Because the ERS method assumes supports spectra to be identical and in phase, the response obtained is valid only if this assumption is justifiable.
- ⑥ The "support group" definition for the ISM method needs improvement. It should be a function of phasing only. Mathematical analysis for phase correlation among motions, even though welcome, more rigorous, and expensive, is not absolutely necessary. Instead, careful examination of the supports spectra combined with sound engineering judgement should establish whether the spectra are in phase, out of phase, or randomly phased and then define "support group" more realistically.
- ⑥ Various studies have shown that motions are, in general, fairly strongly correlated. Even though SRSS produces acceptable or conservative results with respect to time history or test, it is not necessarily and inherently the best procedure, nor is it unrelated to phasing of input motions. Extensive studies on phase correlation and SRSS of group responses indicate that SRSS

should be used as an interim procedure. Meanwhile, further studies on phase correlation and algebraic combination should be performed.

- ⑥ The examination of problems in Subudhi⁵, Bezler⁷, and Francis⁸ indicated no significant anomaly to explain why the ISM-ABS method produces sometimes lower and sometimes higher results than ERS. Based on numerical examples and other observations, patterns of results from ISM and ERS methods are not independent of problem characteristics. The common belief that the ERS method always yields conservative results because of the envelope spectrum is not correct. Frequency content of input, system dynamic properties, and methodology differences between the ISM and ERS methods affect overall system response; thus, seemingly contradictory conclusions in Subudhi, Bezler, and Francis do not constitute a concern.

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