DYNAMIC BEHAVIOR OF A PWR-CORE BARREL ANALYTICAL INTEGRATION OF THE CYLINDRICAL SHELL EQUATIONS

A. LUDWIG, R. KRIEG

Institut für Reaktorentwicklung, Kernforschungszentrum Karlsruhe, Postfach 3640, D-7500 Karlsruhe, Germany

Summary

The knowledge of the dynamic behaviour of a PWR-core barrel is an essential prerequisite for reliable assessments of the consequences due to postulated blowdown accidents in PWRs. Therefore as part of the HDR program the core barrel dynamics is investigated both experimentally and theoretically. For this reason the computer code CYLDY3 for the structural dynamics of the core barrel has been developed, which will be described in this paper.

Usual core barrel represent thin-walled circular cylindrical shells. For linear-elastic material and small deformations such a shell may be described by Flügge's equations, extended by inertia terms. However, to the authors' knowledge mathematically exact solutions of these partial differential equations are restricted to very special boundary conditions, which do not match with the situation at PWR-core barrels. For general conditions only approximate solutions are available. Though the applied solution procedures may converge with increasing discretization effort, significant discrepancies occur between the results of some codes.

The computer code CYLDY3 represents a quasi-exact solution procedure. Since the inertia terms which extend Flügge's shell equations include only second time derivatives, introduction of harmonic time functions with unknown frequencies is allowed. Then the remaining partial differential equations depend only on the space coordinates in azimuthal and axial directions. Since the circular shell is closed, the sine and cosine terms of a Fourier expansion in azimuthal direction represent eigenfunctions. Thus the problem splits up into a set of uncoupled ordinary differential equation systems, where the axial coordinate is the independent variable. Each system is of eigth order and corresponds to one Fourier term. Since odd-numbered derivatives do not occur exact homogeneous solutions can be given with an unknown frequency as parameter. Now introducing the appropriate boundary conditions yields a homogeneous system of eight linear equations for the eight integration constants. The element of the coefficient matrix are transcendental functions of the unknown frequency of the harmonic time function. In order to obtain non-trivial solutions the determinant of the coefficient matrix must vanish. This can be achieved only at discrete values of the frequency which then are eigenfrequencies. To find them, the roots of a highly nonlinear algebraic equation have to be determined. Approximate solutions with any required accuracy can be obtained easily by appropriate iterative methods. For this reason the above method is called quasi-exact. Now the integration constants which describe the axial shapes of the eigenfunctions can be determined.

First results have been calculated for the HDR-core barrel and include some eigenfrequencies and corresponding eigenfunctions for low Fourier orders. They are compared with results from usual methods.

In order to calculate the response to prescribed shell loadings besides the above homogeneous solutions appropriate particular solutions must be superimposed.

1. Introduction

In nuclear reactor safety a sudden failure of the primary coolant circuit of PWR's is considered as a design basis accident. The rapid pressure drop occuring in such an event causes dynamic deformations of the pressure vessel internals. It must be proven that these deformations are within tolerable limits in order to guarantee proper function of both the shut down and the emergency cooling system. Here the dynamic behaviour of the core barrel is of special interest; if the core barrel deformations are small enough the integrity of other pressure vessel internals may be conjectured.

One of the main topics of the HDR program /‾1_/ is the investigation of the core barrel dynamics, both experimentally and theoretically. For the structural dynamics part of the theoretical analyses the core barrel represents a thin-walled circular cylindrical shell which may be described by the appropriate equations derived by Flügge, provided the material behaves linear-elastically and the deformations are limited to small values in comparison to the mean shell dimensions. Exact solutions /‾5_/ of these partial differential equations are restricted to very special boundary conditions, which do not coincide with the situation at core barrels. Approximative solution methods like the formerly developed model CYLDY2 /‾2_/ or finite element methods suffer from modelling deficiences on principle: either ansatz functions are used which are a matter of subjective judgements, or no appropriate measure exists for the discretization effort which is necessary for converging simulations. Consequently the results of various calculations of the same matter may differ from each other considerably, as will be shown below. Furthermore, usual finite element codes take too much computer time and storage capacity if the requirements of coupled fluid-structural analysis /‾3_/ are regarded. The CYLDY2 model, in particular, is restricted to a certain set of boundary conditions defined by its ansatz functions.

In order to overcome these difficulties, for the computer code CYLDY3 another solution method has been developed. It allows for varying the boundary conditions by minor program changes. Its results, however, are considered to be "quasi-exact", since Flügge's partial differential equations here are solved by analytical integration as far as possible, and numerical approximations are employed merely on a low level of mathematical complexity. The solution procedure which is used in CYLDY3 will be described in this paper, and some results concerning the eigensolutions will be given.

2. Solution of the differential equations

Consider a thin walled circular cylindrical shell with radius R of the middle surface, wall thickness h, and shell length d. x is the axial coordinate, whereas the angle θ destinates the azimuthal position. The material may behave linear-elastically with Young's modulus E, Poisson's ratio ν, and mass density ρ. Boundary conditions are prescribed for the edges $x = 0$ and $x = d$.

Then the equilibrium conditions for a infinitesimal shell element result in Flügge's differential equations /‾4_/ for thin-walled cylindrical shells, extended by inertia terms as done by Seide /‾5_/:

$$L_1 u + L_2 v + L_3 w = K \cdot p_x \qquad (1.a)$$

$$L_2 u + L_4 v + L_5 w = K \cdot p_\theta \qquad (1.b)$$

$$L_3 u + L_5 v + L_6 w = K \cdot p_r \qquad (1.c)$$

Here u, v, w are the displacement components in axial, azimuthal, and inward radial direction, resp., and p_x, p_θ, p_r are the components of the loading in these directions. K is a constant implying material and geometry properties. The coefficients L_1, L_2,... represent linear partial differential operators:

$$\begin{aligned}
L_1 &= \frac{\partial^2}{\partial x^2} + \frac{1-\nu}{2}(1+k)\frac{\partial^2}{\partial \theta^2} - \frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2} \\
L_2 &= \frac{1+\nu}{2}\frac{\partial^2}{\partial x \partial \theta} \\
L_3 &= \left[- \nu + k \left(\frac{\partial^2}{\partial x^2} - \frac{1-\nu}{2}\frac{\partial^2}{\partial \theta^2} - \frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2}\right)\right]\frac{\partial}{\partial x} \\
L_4 &= \frac{1-\nu}{2}(1+3k)\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \theta^2} - (1+3k)\frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2} \\
L_5 &= \left[- 1 + \frac{3-\nu}{2} k \frac{\partial^2}{\partial x^2} - 2k\frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2}\right]\frac{\partial}{\partial \theta} \\
L_6 &= 1 + \frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2} + k\left[1+2\frac{\partial^2}{\partial \theta^2} + \left(\nabla^2 - \frac{(1-\nu^2)\rho R^2}{E}\frac{\partial^2}{\partial t^2}\right)\nabla^2\right]
\end{aligned} \qquad (2)$$

with the abbreviations

$$k = h^2/(12R^2), \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \theta^2} .$$

As eq. (1) represents linear differential equations, its general solution may be written as a superposition of the complete homogeneous solution and any particular solution. The homogeneous solution may be composed of the eigensolutions of the system. Such an eigensolution separates into a shape function which depends on the spatial coordinates, and a time function. The complete solution is then obtained by superposition of all modes.

The main task, therefore, consists in the evaluation of the eigensolutions, which is the subject of this paper. The determination of particular solutions then follows well known procedures [5].

For boundary conditions prescribed along edges x = constant or θ = constant, the shape functions split up into an axial shape function depending on x, and an azimuthal shape function depending on θ. Thus the general eigensolution for the geometry under consideration may be written as

$$\begin{aligned}
u_{mn}(x,\theta,t) &= U_{mn}(x) \cdot \cos n\theta \cdot C_{mn} \cos \omega_{mn} t \\
v_{mn}(x,\theta,t) &= V_{mn}(x) \cdot \sin n\theta \cdot C_{mn} \cos \omega_{mn} t \\
w_{mn}(x,\theta,t) &= W_{mn}(x) \cdot \cos n\theta \cdot C_{mn} \cos \omega_{mn} t,
\end{aligned} \qquad (3)$$

where ω_{mn} is the yet unknown circular eigenfrequency and C_{mn} is a constant of integration (unessential integration constants are omitted). The subscript n is used for the distinction of different azimuthal eigenshapes, whereas m counts the axial shape functions and corresponding eigenfrequencies of each azimuthal shape. U_{mn}, V_{mn}, and W_{mn} are the axial shape functions for the corresonding displacement components.

Introduction of eq. (3) into the left-hand side of eq. (1) yields a new system of coupled ordinary differential equations for the axial shape functions:

$$\begin{aligned} L'_1\, U_{mn} + L'_2\, V_{mn} + L'_3\, W_{mn} &= 0 \\ L'_2\, U_{mn} - L'_4\, V_{mn} + L'_5\, W_{mn} &= 0 \\ L'_3\, U_{mn} + L'_5\, V_{mn} + L'_6\, W_{mn} &= 0 \end{aligned} \tag{4}$$

with the ordinary differential operators $L'_1, L'_2, \ldots$, which result from the partial operators $L_1, L_2, \ldots$ in eq. (2) by replacing $\partial/\partial\theta$ by n, $\partial^2/\partial\theta^2$ by $-n^2$, and $\partial^2/\partial t^2$ by $-\omega_{mn}^2$. Eq. (4) consists of ordinary differential equations with constant, though partially still unknown coefficients. Its general solution may be written as

$$\begin{aligned} U_{mn}(x) &= \sum_{j=1}^{J} a_{mnj}\, c_{mnj}\, e^{\lambda_{mnj} x} \\ V_{mn}(x) &= \sum_{j=1}^{J} b_{mnj}\, c_{mnj}\, e^{\lambda_{mnj} x} \\ W_{mn}(x) &= \sum_{j=1}^{J} c_{mnj}\, e^{\lambda_{mnj} x} \end{aligned} \tag{5}$$

Here a_{mnj}, b_{mnj}, c_{mnj} are constants of integration. The summations have to be extended over all independent solutions of eq. (4), marked by different "characteristic numbers" λ_{mnj}; consequently the upper summation bound J will be a result of the analysis. Each of the J independent solutions in eq. (5) must satisfy the differential equations (4). Substituting one of them, numbered j, into eq. (4), one gets a homogeneous system of algebraic equations for the integration constants a_{mnj}, b_{mnj}, c_{mnj}:

$$\begin{pmatrix} L_1^{(m,n,j)} & L_2^{(m,n,j)} & L_3^{(m,n,j)} \\ L_2^{(m,n,j)} & -L_4^{(m,n,j)} & L_5^{(m,n,j)} \\ L_3^{(m,n,j)} & L_5^{(m,n,j)} & L_6^{(m,n,j)} \end{pmatrix} \begin{pmatrix} a_{mnj}\, c_{mnj} \\ b_{mnj}\, c_{mnj} \\ c_{mnj} \end{pmatrix} = \underline{\underline{L}}^{(m,n,j)} \begin{pmatrix} a_{mnj}\, c_{mnj} \\ b_{mnj}\, c_{mnj} \\ c_{mnj} \end{pmatrix} = \underline{0} \tag{6}$$

where the elements $L_i^{(m,n,j)}$ of the matrix $\underline{\underline{L}}^{(m,n,j)}$ equal L'_i with d/dx replaced by λ_{mnj}. Evaluation of the condition Det $(\underline{\underline{L}}^{(m,n,j)}) = 0$ yields a polynomial of eigth degree in λ_{mnj}, where the coefficients still contain the frequency ω_{mn} as a parameter:

$$\lambda_{mnj}^8 + K_{6mn}(\omega_{mn}) \cdot \lambda_{mnj}^6 + K_{4mn}(\omega_{mn}) \cdot \lambda_{mnj}^4 + K_{2mn}(\omega_{mn}) \cdot \lambda_{mnj}^2 + K_{0mn}(\omega_{mn}) = 0 \tag{7}$$

Eq. (7) is called a "characteristic equation". It has eight independent solutions which may be real or conjugate complex in pairs. Consequently the upper summation bound in eq. (5) must be J=8. Due to the absence of odd-numbered powers of λ_{mnj}, eq. (7) can be solved by means of analytical formulas and gives the values of all λ_{mnj} as functions of ω_{mn}. Substituting these relations into eq. (6) allows the determination of a_{mnj} and b_{mnj} in terms of ω_{mn}, too.

In order to evaluate the eigenfrequency ω_{mn} and the remaining integration constants c_{mnj}, the boundary conditions at x=0 and x=d have to be considered. For each edge there exist four conditions; the boundary conditions for the HDR-core barrel are given in the next section as examples. Their general form reads

$$\sum_{j=1}^{8} R_{ij}^{(m,n)} \cdot c_{mnj} = 0; \quad i = 1, 2, \ldots, 8 \tag{8}$$

where i counts the number of the condition and $R_{ij}^{(m,n)}$ is a transcendental function of λ_{mnj} and, consequently, of ω_{mn}. Again non-trivial solutions require

$$\mathrm{Det}\ (\underline{\underline{R}}^{(m,n)}) = 0. \tag{9}$$

This is a highly non-linear implicit equation for the eigenfrequencies ω_{mn}. It has to be solved by means of a numerical iterative procedure, which is, however, not a substantial shortcoming. Any required accuracy of the approximate solutions can be obtained easily by appropriate methods. If ω_{mn} is found from eq. (9), seven of the constants c_{mnj} may be expressed in terms of the eigth by means of eq. (8).

By repeating this procedure with altered starting values for ω_{mn} as many eigensolutions as wanted may be determined for the azimuthal mode numbers n = 1, 2,...,N and axial mode numbers m = 1, 2,...,M. N and M are the highest mode numbers which have to be taken into account.

With n=0 advantage is taken from the fact that the system of eq. (6) splits up into a sixth order system of two equations describing the axisymmetric vibrations of the shell, called "breathing modes", and in a single independent second order equation describing torsional oscillations around the shell axis. Without regard to these peculiarities the solution method is similar as for n > 0.

Thus the mathematical problem of solving a system of coupled partial differential equations of higher order is reduced to the determination of the zeros of a non-linear function of one variable. The complexity of the latter problem is of essentially lower order than that of the first. Therefore we claim the solution obtained by the present method to be "quasi-exact".

3. Eigensolutions for the HDR-core barrel

The procedure described in section 2 has been applied to the core barrel of the planned HDR-experiments. This core barrel (middle surface radius R = 1.3185 m, wall thickness h = 0.023 m, shell length d = 7.57 m, Young's modulus $E = 1.7 \cdot 10^{11}$ N/m^2, Poisson's ratio r = 0.3, mass density $\rho = 7.8 \cdot 10^3$ kg/m^3) is clamped in the pressure vessel at the upper end (x=0) and stiffened by a rigid, heavy ring at the lower end (x=d). Details of the design are given in [1, 3]. If the pressure vessel is considered to be rigid and fixed, the following kinematic boundary conditions apply:

$$x=0, \quad u(0,\theta,t) = v(0,\theta,t) = w(0,\theta,t) = \frac{\partial}{\partial x} w(0,\theta,t) = 0 \tag{10}$$

$$\begin{aligned} x=d, \quad & u(d,\theta,t) = H_0(t) + H_1(t) \cos\theta \\ & v(d,\theta,t) = G_0(t) + G_1(t) \sin\theta \\ & w(d,\theta,t) = G_1(t) \cos\theta \\ & \frac{\partial}{\partial x} w(d,\theta,t) = H_1(t) \cos\theta \end{aligned} \tag{11}$$

Here the time functions G_0, G_1, H_0, H_1 are determined implicitely by dynamical equilibrium conditions: The inertia forces and moments of the end ring have to be balanced by forces and moments acting upon the ring by the shell. These may be expressed in terms of the displacement components by means of Flügge's theory of thin walled circular cylindrical shells [4]. Thus all the eight conditions are homogeneous linear equations combining the displacement components and their spatial and timewise derivatives, in general. Substitution of eq. (3) into eqs. (10) and (11) results in boundary conditions which are expressed in terms of the axial shape functions and their derivatives. For n=0 or n=1, however, these conditions differ from those with n>1 due to eq. (11). Finally introducing eq. (5), one gets equations in the form of eq. (8).

Fig. 1 to 6 show some axial eigenvibration shape functions for the HDR-core barrel as calculated by CLYDY3. The ordinates represent the displacement components, normalized by the maximum value of all three (or two, for n=0) components. The abscissa is the axial distance from the clamping, normalized by R.

The first breathing mode (fig. 1) mainly consists of axial vibrations of the end ring. Radial displacements are caused exclusively by the transverse compression. Due to their suppression by the edge clamping, strong bending of the shell arises near the edges. Fig. 2 shows the first beam mode. Here the difference between radial and azimuthal displacement functions near x=0 is striking, which means that the cross section of the shell does not remain circular. This finding, which is due to transverse compression, too, and causes likewise strong curvatures of the shell, is true for all beam mode vibrations. It may be seen very clearly from the seventh and eighth beam mode order in fig. 3 and 4. Fig. 5 and fig. 6 show the third axial order of the second and third azimuthal vibration mode, resp. They indicate that the zeros of U_{mn} approach the extrema of W_{mn} and V_{mn} with increasing azimuthal order n. Also here the rigid edge clamping causes strong bendings near both ends.

Table I gives some of the lowest eigenfrequencies of the HDR-core barrel for the azimuthal orders n=0 to 6 (column "CYLDY3"). They are compared to the results of several approximative calculations. It is seen that the simple vibration shapes (first breathing mode, first beam mode) are well described by the finite element models. For higher vibration orders, however, increasingly significant discrepancies occur. They are considered to arise from different discretization efforts rather than from differences of the employed codes. For the calculations with STRUDL/DYNAL [1], for instance, one half of the core barrel was represented by 384 triangular elements, whereas for the ASKA calculations [6] only 128 triangular elements were used. In the SAP IV calculations [7] 64 elements of a different type were employed.

CYLDY2 [2] uses modal superposition in a similar form as CYLDY2. Its modal axial shape functions, however, are given by ansatz functions instead of integrating the differential equations; the eigenfrequencies then are determined by means of a variational principle. Table I shows that for the first breathing mode, which is dominated by axial oscillations of the end ring (s. fig. 1), no corresponding mode is found by CYLDY2, and the calculated eigenfrequencies of beam mode oscillations are significantly too high. This is due to the fact that the ansatz functions imply additional constraints which are not justified by physical arguments. In case of shell vibrations with $n \geq 2$, however, the discrepancies seem to be of minor degree and decrease with increasing azimuthal order of vibration.

4. Conclusions

The computer code CYLDY3 follows a procedure which gives quasi-exact eigensolutions of Flügge's differential equations for circular cylindrical shells for a wide class of boundary conditions. Thus an objective measure for approximative methods has been gained. Furthermore, since "true" eigensolutions are computed, the mode shape functions form an orthogonal system. Therefore transient analyses may be performed in a very efficient matter using purely diagonal mass and stiffness matrices. This is an essential advantage for coupled fluid-structural dynamics calculations.

Application of CYLDY3 to the HDR-core barrel has shown that most of the modes involve strong curvatures of the shell near the ends. Hence numerical approximations would require high resolution of these regions, resulting in high computation efforts.

The reliability of the solution method realized in CYLDY3 will be assessed by comparison with some full-scale experiments at the HDR experimental facility, which are scheduled for the near future.

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Table I: Eigenfrequencies (Hz) of the HDR-core barrel; results of various codes

code:		CYLDY3	CYLDY2	finite element calculations:		
n	m			STRUDL/DYNAL [1]	SAP [7]	ASKA [6]
0, breathing	1	82.4	-	84.7	85.4	-
	2	328.6	-	-	-	-
1	1	16.2	18.9	16.4	17.1	17.3
	2	104.2	160.8	101.2	101.9	-
	3	220.7	394.8	208.5	191.9	-
2	1	62.0	65.8	62.9	67.5	57.0
	2	135.8	147.2	-	145.5	-
	3	218.7	229.3	-	-	-
3	1	41.4	42.8	44.7	72.8	44.8
	2	85.2	90.7	93.5	115.0	-
	3	141.3	149.2	-	184.5	-
4	1	49.0	49.4	50.7	127.7	55.1
	2	70.5	72.8	78.6	149.8	57.8
	3	106.3	110.7	120.9	206.3	-
5	1	72.4	72.6	72.5	-	-
	2	82.2	83.1	87.6	-	57.8
	3	102.4	104.5	116.0	-	-
6	1	104.1	104.3	102.8	-	-
	2	109.7	110.2	112.2	-	57.2

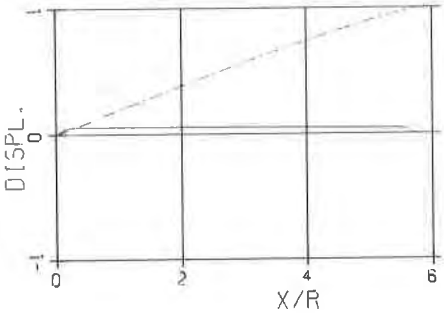

Fig. 1: n=0 (breathing), m=1.
Eigenfrequency f_{10}= 82.4 Hz.

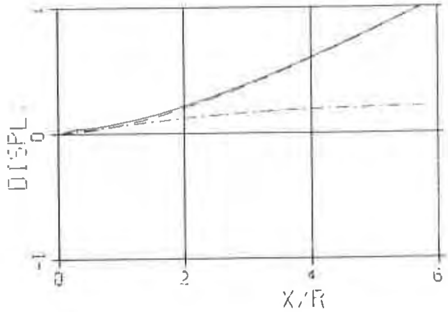

Fig. 2: n=1, m=1.
Eigenfrequency f_{11}= 16.2 Hz.

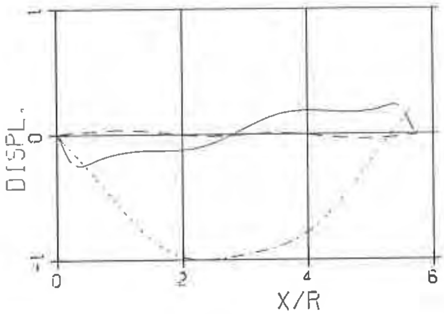

Fig. 3: n=1, m=7.
Eigenfrequency f_{71}= 460.7 Hz.

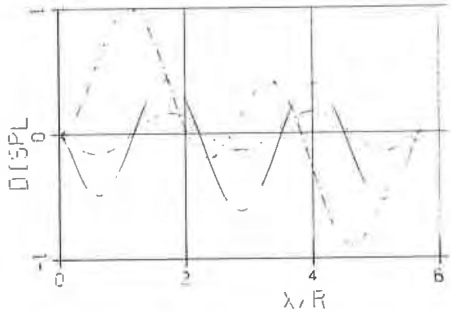

Fig. 4: n=1, m=8.
Eigenfrequency f_{81}= 481.1 Hz.

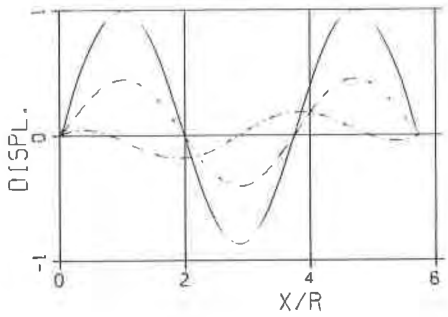

Fig. 5: n=2, m=3.
Eigenfrequency f_{32}= 218.7 Hz.

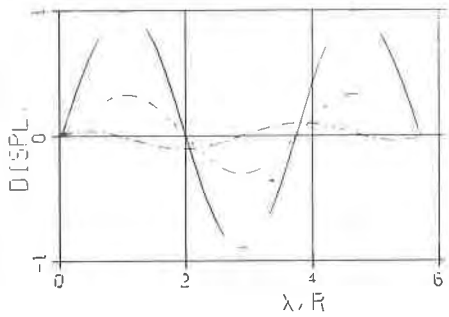

Fig. 6: n=3, m=3.
Eigenfrequency f_{33}= 141.3 Hz.

Fig. 1 to 6: Axial shape functions for selected eigenvibration modes of the HDR-core barrel.
-·-: $U_{mn}(x)$; ---: $V_{mn}(x)$; ——: $W_{mn}(x)$