RECONCILIATION OF CIVIL AND MECHANICAL ENGINEERING APPROACHES OF SEISMICALLY INDUCED STRESSES

P.B. Labbé[1]
[1]Nuclear Engineering Division, EDF, Saint-Denis, France
E-mail: pierre.labbe@edf.fr

ABSTRACT

Both civil and mechanical engineers have to deal with loads of the force-controlled type (qualified as “primary”) or of the displacement-controlled type (qualified as “secondary”). Question about of which type is a seismic load has been discussed for decades by different authors with either civil or mechanical backgrounds, but without apparent mutual understanding or mutual benefit so far. The purpose of this paper is to bring some clarity on the subject by considering a family of oscillators, introducing a τ parameter that enables to cover all cases from “perfectly damaging” to “perfectly plastic” systems. It is concluded that a seismic input cannot be regarded a priori as primary or secondary. Both the τ value of the structure or component and the slope, p, of the input response spectrum in the vicinity of its (dominant) eigenfrequency should be considered concurrently when making decision about stress classification. A formula is given that provides the primary part of a seismically induced stress as a function of these two parameters.

INTRODUCTION

In the general frame of nuclear industry design practices, seismic input motion is taken into account in such a way that, basically, structures or components do not undergo plastic strains, or only to a very limited extent. However, principally for evaluation of margins in existing facilities, issue of response in the plastic domain has been extensively discussed.

In this regard, both civil and mechanical engineers have to deal with loads of the force-controlled type (qualified as “primary”, as well as the corresponding induced stresses in the mechanical engineering terminology) or of the displacement control type (qualified as “secondary”). Question about of which type a seismic load should be classified has been discussed for decades by different authors with either civil or mechanical background, but without apparent mutual understanding or mutual benefit so far. For instance, the inelastic response spectrum established by Newmark [1] in the seventies can be regarded as a classification of seismic load as secondary for flexible structures and primary for stiff ones, but this result has not yet been acknowledged by mechanical engineers.

The purpose of this paper is to bring some clarity on the subject by considering the seismic response of a family of single degree of freedom system, with a range of features capable to cover both typical concrete structures and typical mechanical components.

FRAMEWORK AND ASSUMPTIONS

A family of oscillators

In this paper a family of single degree of freedom (SDOF) systems, also designated as oscillators, is considered, which is described as follows: As presented on Fig 1, for a displacement that does not exceed Xe, the force-displacement relationship is of elasticity type, with a k_0 stiffness. Then, there is a plateau ($F=F_e$) until an ultimate displacement X_u. $\mu=X_u/X_e$ is the ductility. The oscillator mass is m. When exited by a seismic input motion, the dynamic response (relative displacement) is denoted x(t) and its maximum absolute value X.

- In case X does not exceed X_e the oscillator is in elastic regime, with an f_0 frequency, and a ξ_0.damping.
- In case X is larger than X_e, the oscillator is in plastic regime. It is assumed that an effective frequency $f\leq f_0$, and an effective viscous damping $\xi\geq\xi_0$.can be determined. f and ξ possible values are discussed hereunder.
- In case X is larger than X_u, the oscillator is deemed collapsed.

The family of SDOF systems consists of the variety of f and ξ values considered in plastic regime, as introduced just below.

Effective eignenfrequency, effective stiffness

Regarding effective stiffness, k (and consequently effective frequency, f, so that $(2\pi f)^2=k/m$), it is assumed that k lies between k* and k_0, with $k^* = F_e/X$, as illustrated on Fig 1. In order to cover the whole range of possibilities, a parameter τ is introduced so that:

$$k=(1-\tau)k^*+\tau k_0 \; ; \; 0\leq\tau\leq 1.$$

The case $\tau=0$ corresponds to an "elastic perfectly damaging" system while $\tau=1$ corresponds to an "elastic perfectly plastic" system. Reality is somewhere in between. Some concrete structures are close to the first type while piping systems are close to the second type.

Regarding piping systems, it has been observed that under high seismic input motion, and in spite of very large plastic drifts, the frequency shift is limited. On the basis of random vibration theory developments by Vanmarke [2], a theoretical frequency shift was estimated by Labbé [3] for a perfectly plastic oscillator. For instance it was concluded that $\mu=3$ leads to a theoretical shift of only –7%, while $\mu=6$ leads to –30%. However it was experimentally observed [4] that the actual shift is significantly smaller, not discernable for $\mu=3$ for, around –15% for $\mu=6$ and around -30% for $\mu=10$, which cases correspond respectively to $\tau=1$, τ around 0.7 and τ around 0.5.

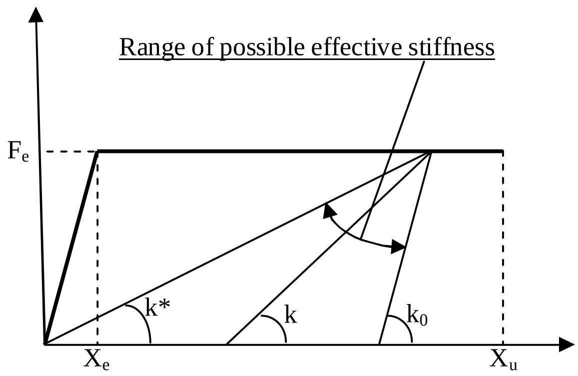

Fig. 1: Force-displacement relationship of the considered family of oscillators.

Effective viscous damping

It is clear that damping is strongly related with the area of the cycle exhibited by the relationship under cyclic loading conditions. Regarding the family of relationships considered in this paper, the cycle area is nil for $\tau=0$ (perfectly damaging oscillator) while it is maximum for $\tau=1$.

In this regard, it has been observed experimentally that equivalent damping of shear walls submitted to increasing seismic input levels is practically not dependant on this input level and not significantly different from the conventional $\xi_0=0.05$ (5%) value. Consequently it is assumed in this paper that, regardless μ value, damping of a perfectly damaging oscillator is limited to ξ_0.

Regarding piping systems, also on the basis of random vibration approach and for a perfectly plastic oscillator, Labbé [3] has proposed an effective viscous damping estimate, which reasonably fits experimentally observed values. For the sake of simplicity, and consistently with observed values, this relation can be expressed as: $\xi=\xi_0 + 0.02\mu$.

As the simplest possible interpolation between the two above-mentioned extreme cases ($\tau=0$ and $\tau=1$), the following empirical relationship is assumed in this paper:

$$\xi=\xi_0 + 0.02\tau\mu.$$

In case the reader has at his or her disposal other documented damping values or is willing to test other options, he or she can easily replace the above formula by his or her own formula in the following of the rationale. Neglecting any additional damping effect is also possible by selecting q=0 in the next section.

Seismic Input motion

The considered input motion is described by its pseudo absolute acceleration response spectrum:

$$S(f,\xi) = \Gamma\ s(f,\xi),$$

where Γ is the ZPA and $s(f,\xi)$ a non dimensional response spectrum. For frequencies below and in the vicinity of the natural frequency of the oscillator, typically between f_0 and $f_0/2$, it is assumed that $s(f,\xi)$ can be written in the form of:

$$s(f,\xi) = s(f_0,\xi_0)\ (f/f_0)^p / (\xi/\xi_0)^q$$

In a conventional log-log scheme, and for a given damping value, it means that the response spectrum takes the form of a straight line with a p slope. As a matter of example, a non-dimensional response spectrum is presented in Fig. 2. With this spectrum, for the case of an f_0=6 Hz oscillator, p can be estimated at p=0.54. Conventional design spectra are generally in the form of smooth curves, so that estimating p value is very easy.

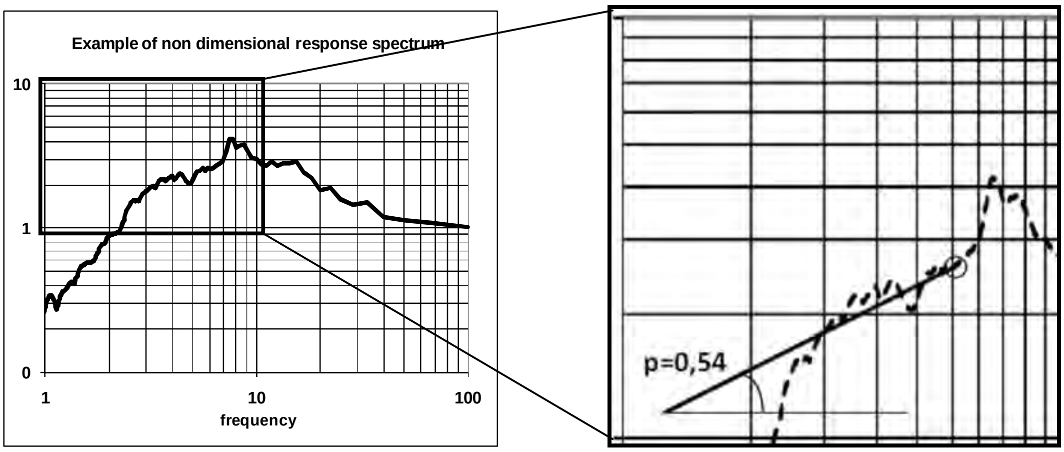

Fig. 2: Example of estimate of p value for an f_0=6Hz oscillator

Dependence on damping is governed by the q factor. Random vibration theory leads to a 0.5 q value, which is retained in the present paper for numerical applications. Some authors and some standards recommend a slightly lower value.

Principle of the approach

A non dimensional response spectrum $s(f,\xi)$ is selected, characterized by its p value in the vicinity of f_0. For the $\{f_0,\xi_0\}$ oscillator, Γ_e is calculated so that under this input X is equal to X_e. It means that Γ_e is so that:

$$X_e.= \Gamma_e\ s(f_0,\xi_0) /(2\pi f_0)^2.$$

Then, taking into account the effective frequency and effective damping in plastic regime, Γ_u is calculated so that under this input X is equal to X_u:

$$X_u.= \Gamma_u\ s(f_u,\xi_u) /(2\pi f_u)^2,$$

where f_u, and ξ_u are the effective frequency and effective damping corresponding to $X=X_u$.

We introduce $\gamma=\Gamma_u/\Gamma_e$, which can be regarded as the margin on the seismic load (for a system designed so that $X=X_e$ under the considered seismic load). The core of the paper consists of discussing γ as a function of μ, τ, p and q.

Load categorization, primary part of a seismic stress

Two possible outputs of γ calculation process deserve a special interest:

- In case the output is $\gamma=1$, it means that the seismic load should be regarded as a force-controlled load (a primary load in the mechanical engineering terminology),
- while in case it is $\gamma=\mu$, it means that the seismic load should be regarded as a displacement-controlled load (a secondary load).

Of course γ values, different from 1 and μ, are anticipated, meaning that a seismic load cannot be regarded *a priori* as primary or secondary. In terms of mechanical engineering the primary part σ_P of an inertial stress σ is:

$$\sigma_P=\sigma/\gamma.$$

MARGIN CALCULATION AND LOAD CATEGORIZATION

Generic formula

A generic formula for γ is easily derived from above mentioned assumptions. First γ reads as:

$$\gamma=\mu\ s(f_0,\xi_0)/s(f_u,\xi_u)\ (f_u/f_0)^2.$$

Then, the selected form of the non-dimensional spectrum leads to:

$$\gamma=\mu\ (f_u/f_0)^{2-p}\ (\xi_u/\xi_0)^q.$$

Perfectly damaging system

This case is characterized by $\tau=0$ and $\xi_u=\xi_0$. A consequence of $\tau=0$ is that $k_u/k_0=1/\mu$ and consequently $f_u/f_0=(1/\mu)^{1/2}$, resulting in:

$$\gamma=\mu^{p/2}.$$

The formula drives to the conclusion that both the ductile capacity of the structure or component and the slope of the response spectrum in the vicinity of the natural frequency of the system should be considered concurrently when making decision about stress classification. The above very simple formula reminds that a pseudo-acceleration response spectrum with a slope p=2 is a constant displacement spectrum (a perfect displacement-controlled load), while with a slope p=0 it is a constant force spectrum (a perfect force-controlled load). The formula generalizes this result and enables to address any spectral shape.

Perfectly plastic system

This case is characterized by $\tau=1$corresponding to and $f_u=f_0$, resulting in

$$\gamma=\mu\ (\xi_u/\xi_0)^q,$$

Practically, with the above mentioned assumptions about damping, the formula reads:

$$\gamma=\mu\ (1+0.4\mu)^{1/2}.$$

If disregarding the additional damping effect, the formula takes the following very simple form, meaning that in this theoretical case, the seismic load should be regarded as secondary, regardless the shape of the input spectrum and any possible additional damping effect:

$$\gamma=\mu.$$

Intermediate cases

For other τ values, it should be first considered that $k_u=(1-\tau)k_u^*+\tau k_0$, resulting in $k_u/k_0= (1-\tau)/\mu+\tau$. Then f_u/f_0 is derived from k_u/k_0, leading to the sister equivalent following formulae, quoting respectively the perfectly damaging and the perfectly plastic cases:

$$\gamma = \mu^{p/2} (\mu\tau+1-\tau)^{1-p/2} (\xi_u/\xi_0)^q \text{ , or } \gamma = \mu (\tau+(1-\tau)/\mu)^{1-p/2} (\xi_u/\xi_0)^q \text{ .}$$

With the above mentioned assumptions relating to effective damping and q value, the formulae take the practical following forms:

$$\gamma = \mu^{p/2} (\mu\tau+1-\tau)^{1-p/2} (1+0.4\mu\tau)^{1/2} \text{ , or } \gamma = \mu (\tau+(1-\tau)/\mu)^{1-p/2} (1+0.4\mu\tau)^{1/2}.$$

PRACTICAL APPLICATIONS

Stress categorization abacus

Setting $\gamma=1$ or $\gamma=\mu$ in the above formula leads to a relationship between μ, τ and p, which can be plotted in the form presented in Fig 3(top) for $\mu=10$. The thick solid curve corresponds to $\gamma=\mu$, meaning that for every $\{\tau, p\}$ couple on this curve, the seismic load should be regarded as a displacement-controlled load (or seismic stresses should be regarded as secondary in mechanical engineering terminology). For any case above this solid curve, the seismic load is even less damaging than expected in case of a displacement-controlled load.

The thick dotted curve corresponds to $\gamma=1$, meaning that for every $\{\tau, p\}$ couple on this curve the seismic load should be considered as a force-controlled load (or seismic stresses should be regarded as primary in mechanical engineering terminology). For any case below this solid curve, the seismic load is even more damaging than expected in case of a force-controlled load.

For a $\{\tau, p\}$ lying between these two curves it should be considered that a seismically induced stress σ has a primary part σ_P so that $\sigma_P = \sigma/\gamma$. The thin curve corresponds to the average value between 1 and μ. For instance for $\mu=10$, the case $\{\tau=0.3, p=0\}$ is located on this thin curve. It results in $\gamma=5.5$, meaning that, in such conditions, the primary part of a seismic stress is $\sigma_P = \sigma/5.5$. Incidentally it should be observed that the primary part of a secondary stress is not nil but equal to σ/μ.

Similar schemes can be plotted for other μ value, for instance $\mu=5$ and $\mu=15$ are presented in Fig. 3. The global pattern is similar, leading to the conclusion that, presented in such a manner, the final output is not very sensitive to the μ value, at least in the range of reasonably expected values. Of course γ is strongly dependant on μ, the similar pattern comes from the fact that the line $\gamma=\mu$ is plotted on every scheme instead of γ= 5, 10 or 15.

Flexible versus stiff structures or components

As discussed for instance in [5], when addressing its response to a dynamic input, stiffness or flexibility of a structure or a component should be discussed by comparison of its first (or dominant) frequency to the central frequency of the input motion. In case the eigenfrequency is lower than the input central frequency the structure or component should be regarded as flexible, and regarded as stiff in the opposite case. It has already been observed that a seismic load trends to act as a secondary load on flexible structures or components, while it trends to act as a primary load on stiff systems [5].

This well known result is better quantified by the approach presented in this paper: A flexible structure or component being by definition so that its eigenfrequency is lower than the central frequency of the input motion, the slope of the response spectrum in the vicinity of its eigenfrequency is positive ($p>0$), while it is negative ($p<0$) in case of a stiff structure or component. However the proposed approach enables to quantify differences, and principally how different are available margins, between for instance p=1 and p=2.

Regarding flexible structures, their response is governed by the ascendant part of the input spectrum. For very low frequency the slope of the spectrum cannot be less than p=2, otherwise the input displacement would not be limited (this is a drawback of the NRC spectrum, the slope of which is even lower than 1 in the low frequency range), meaning that for such flexible structures, the seismic input is a secondary load. Practically, every flexible structure can potentially go towards this part of the spectrum, to the extent a large ductile capacity is available. This underlines the necessity of ductile capacity as a major feature for security under seismic load: the available margin is directly equal to the available ductility.

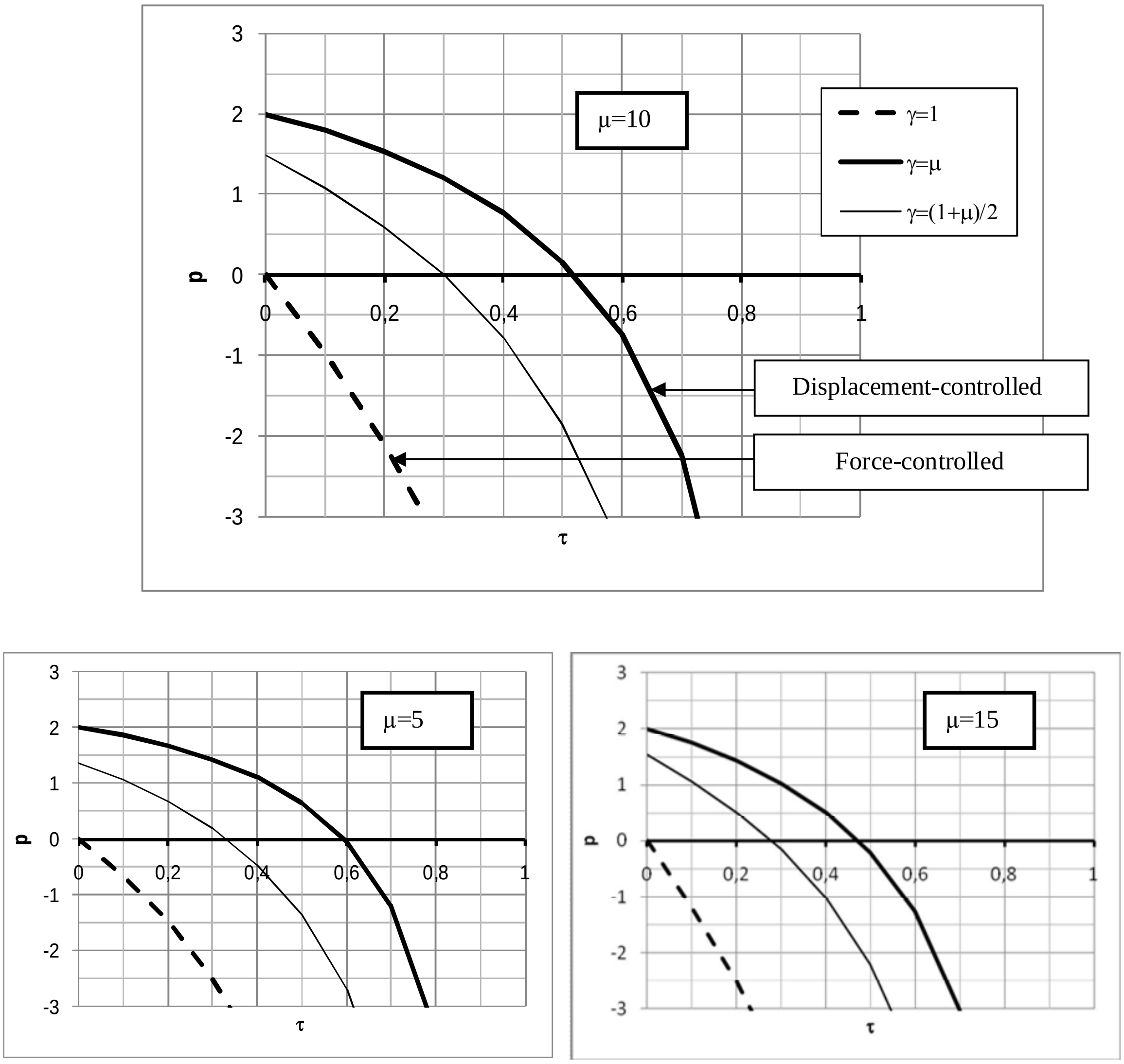

Fig. 3: Load/Stress categorization versus μ, p and τ.

Regarding stiff structures, their response is governed by the slope of the decreasing part of the input spectrum from the peak (the "peak" takes more the form of a plateau when considering the NRC spectrum) to the ZPA. In case this slope is abrupt (e.g. p=-3) a significant τ value should be available in order to avoid that the severity of the seismic load be even larger than a primary load. In this regard a gentle slope such as adopted by the NRC spectrum (p=-0.75 for 5% damping) is favourable because a very small τ value, lower than 0.1, is sufficient to guaranty that the seismic input can at the worst be regarded as a primary load.

Both facets of the problem (flexible and stiff structures) were addressed by Newmark when discussing the inelastic response spectrum [1]. Newmark concluded that for flexible structures the inelastic response spectrum should be derived from the elastic response spectrum by divided the later by μ (displacement controlled load), while for stiff structures it should not be modified (force controlled load). In the range of intermediate frequencies Newmark proposed that the elastic response spectrum is divided by a factor $(2\mu-1)^{1/2}$ corresponding to a velocity controlled load. In this paper a velocity controlled input corresponds to p=1. From the above presented formula, it is once again concluded that a τ value lower than 0.1 is sufficient to derive that Newmark's proposal is on the safe side.

Concrete structures

Interpretation of the above results for concrete structures should be considered with care. In the case of concrete structures, path from the elastic response to the plateau is more complicated than presented in Fig. 1 .There is first a concrete cracking effect that has a significant impact on structural effective stiffness. This phenomenon appears early before yielding of R-bars, which in first approximation can be regarded as the starting point of the plateau. Consequently the eignenfrequency of the virgin structure (before cracking) should not be regarded as its f_0 value; more reasonably this f_0 value corresponds to the first R-bar yielding. Useful considerations on the subject are available in reference [6].

Piping systems

In the frame of design criteria, only very limited plastic strains are permitted for piping systems. As observed experimentally and confirmed by theoretical approaches, it results in the fact that the effective frequency is practically equal to f_0, meaning that τ is very close to 1. It should therefore be concluded (see right part of schemes in Fig. 3) that, regardless the slope of the input spectrum in the vicinity of f_0, the seismic load should always be regarded as secondary in the design procedure. When discussing actual margins, corresponding to μ values larger than 10, a lower τ value should be considered leading to the conclusion that, depending on the spectrum slope, the available margin could be lower than μ, remaining however still comfortable.

CONCLUSION

Seismic response of a family of oscillators has been studied, a parameter τ ($0<\tau<1$) has been introduced in order to cover all possible cases from "perfectly damaging" (τ=0) to "perfectly plastic" (τ=1) systems. Concrete structures are in the range of low τ values, while mechanical components, in particular piping systems, are in the range of large τ values. It is concluded that the seismic input motion cannot be regarded a priori as of the displacement-controlled type (primary) or of the force-controlled type (secondary). Both the τ value of the structure or component under consideration and the slope, p, of the input response spectrum in the vicinity of its (dominant) eigenfrequency should be considered concurrently when making decision about load or stress categorization. Consequently the primary part σ_P of a seismically induced stress σ is given by

$$\sigma_P = \sigma/\gamma.$$

A formula has been established that provides γ as a function of τ and p, taking into accounts some assumptions on damping effects induced by the system non linear response. The formula depends also on μ, the ductile capacity of the system. However outputs have been presented in an abacus, in a form that is practically not sensitive to μ value.

REFERENCES

[1] US-Nuclear Regulatory Commission *Development of Criteria for Seismic Review of Selected Nuclear Power Plants*, NUREG/CR-0098, May 1978.

[2] Vanmarke, E.H., *Random Fields*, MIT Press, 2003.

[3] Labbé, P.B., "Ductility demand and design of piping systems", *Proccedings of the 10th European Conference of Earthquake Engineering (Vienna, 1994)*, Balkema, Rotterdam 1995, pp. 2689-2693.

[4] English, W.F. *Piping and fitting dynamic reliability program, first, second, third and forth semi-annual reports*, General Electric, San Jose, California, 1985-1988.

[5] International Atomic Energy Agency, *Seismic evaluation of existing nuclear power plants*, IAEA-Safety Reports series N° 28, Vienna, 2003.

[6] Brun, M., Reynouard, J.M., Jézéquel, L., "A simple shear wall model taking into account stiffness degradation", *Engeneering Structures*, 2003, vol. 25 pp. 1-9.