

LIMIT MOMENTS FOR NON CIRCULAR CROSS-SECTION (ELLIPTICAL) PIPE BENDS

J. SPENCE

*Department of Mechanics of Materials, University of Strathclyde,
75 Montrose Street, Glasgow G1 1XJ, Scotland, United Kingdom*

G. E. FINDLAY

*Department of Mechanical and Production Engineering, Paisley College of Technology,
High Street, Paisley PA1 2BE, Scotland, United Kingdom*

SUMMARY

It is well known that the curved pipe or pipe bend is a key component in the design of pipework systems. This importance is even greater when the system is subject to combinations of loadings and temperature which cause inelastic effects such as plasticity or creep. Computational methods which include inelastic effects are proving complex and expensive resulting in an increasing interest in simplified approximate methods suitable for design purposes.

A number of experiment studies have been reported or are underway which investigate limit moments applied to pipe bends. Some theoretical work is also available. However, most of the work has been confined to nominally circular cross-section bends and little account has been taken of the practical problem of manufacturing tolerances. Many methods of manufacture result in bends which are not circular in cross-section but have an oval or elliptical shape. The present paper extends previous analyses on circular bends to cater for initially elliptical cross-sections. The loading is primarily in plane bending but out of plane is also considered and several independent methods are presented. No previous information is known to the authors. Upper and lower bound limit moments are derived first of all from existing linear elastic analyses and secondly upper bound moments are derived via a plastic analogy from existing stationary creep results. It is also shown that the creep information on design factors for bends can be used to obtain a reasonable estimate of the complete moment/strain behaviour of a bend or indeed a system.

The results are given in graphical form over the whole range of pipe bend geometries for several levels of ellipticity and should be of direct use to designers in simplified calculations for the integrity of pipework. They should also be of interest to experimenters for comparison with or planning of test work.

1. Introduction

In recent years, economic considerations coupled with new processes in the power, oil-refinery and chemical industries have produced trends towards larger and more complex pipework installations operating at higher temperatures and pressures. These installations must be designed so that they are capable of absorbing thermal expansions without excessive terminal reactions being exerted on adjacent structures. Because of their favourable flexibility characteristics, smooth pipe bends on welding elbows are generally recognised as being one of the more important features of any pipework system subject to temperature fluctuations.

Most of the analytical and experimental studies on pipe bends have been concerned with bends of circular cross-sections subject to external bending moments. In the present context of elliptic bends, it is of historical interest to note that the Parisian instrument-maker Bourdon patented his celebrated "tube" in 1850 although the principle of the pressurised-elliptic, curved tube had apparently been reported several years earlier by a German designer named Schinz.

However, most of the effort since has undoubtedly been directed towards understanding the behaviour of circular cross-section smooth bends and only recently have aspects other than linear elastic behaviour been considered. In the closely related field of pressure vessel engineering, current design procedures frequently permit some measure of controlled plastic deformation via criteria associated with "shakedown" and "limit analysis". An earlier paper by Spence and Findlay [1] considered the limit analysis of smooth bends having circular cross-sections and uniform thickness, under externally-applied in-plane bending moments and summarised other work prior to that time. Another theoretical limit analysis investigation has been reported by Calladine [2] which gives a result which is broadly in line with [1]. There has been renewed interest in the complete elastic/plastic response of pipe bends - see for example [3, 4, 5]. Simple methods of obtaining the complete moment/curvature relationship for bends are briefly mentioned in another paper in this conference [6] and the techniques are simply extensions to that used later herein in obtaining limit loads from creep information.

However to the authors' knowledge, no similar data has been published on bends with non-circular cross sections. While smooth bends can be successfully forged or fabricated with more-or-less uniformly circular cross-sections, very often in the interests of economy these components are manufactured by bending from straight pipes which invariably lead to non-circular or elliptic cross sections. The present paper will briefly examine the limit analysis behaviour (or plastic collapse of smooth bends with elliptic cross-sections and constant thickness under the application of externally applied, in-plane bending. Theoretical limit loads or moments are evaluated using an existing linear elastic analysis in conjunction with the relevant yield-interaction diagrams, the equations for which are detailed by Marcal and Turner [7] leading to upper and lower bound solutions. The same elastic result is also used with a simplified yield surface to estimate lower bound limit loads in the manner employed in [2] for circular cross-sections. Lastly as a check, the results of a creep analysis [8] are manipulated to give approximate upper bound limit moments.

Where possible, the results are presented and discussed in non-dimensional form and cover most practical pipe bend geometries over a range of initially elliptic cross sections.

The information contained herein should provide useful information for design situations.

2. Summary of the Limit Theorems

The limit theorems of plasticity are well known. They were briefly summarised in [1]. For convenience the two most important ones are repeated here.

- (a) The Lower Bound Theorem : If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances certain external loads, and at the same time does not violate the yield condition, those loads will be carried safely by the structure. The loads will be either low or correct.
- (b) The Upper Bound Theorem : If an estimate of the plastic collapse load of a body is made by equating internal rate of dissipation of energy to the rate at which external forces do work in any postulated mechanism of deformation of the body the estimate will be either high or correct.

3. Approximate Limit Moments from Linear Elastic Analysis

3.1 Suitable Elastic Solution

In the present context, it is convenient to utilise the asymptotic solution of Clark and Reissner [9] which was based on thin-shell theory applied to a toroidal shell having an elliptic cross-section and subject to in-plane bending. The maximum stress levels occur at the neutral axis of the cross section and can be expressed by

$$\frac{\sigma_{\phi B} I}{M b} = \pm \frac{1.892}{\lambda^{2/3}} \frac{B_1(\phi)}{\pi (b/a)^{1/3}} \quad \text{at } \phi = 0$$

$$\sigma_{\theta B} = \pm \nu \sigma_{\phi B}$$

$$\sigma_{\theta D} = 0, \quad \sigma_{\phi D} \neq 0$$

$$\text{where } B_1(\phi) = \int_0^{2\pi} \frac{(b/a)^4 \sin^2 \phi}{(1 - e^2 \sin^2 \phi)^{5/2}} d\phi, \quad e^2 = 1 - (b/a)^2. \quad (1)$$

The limits of applicability of the above equations were specified to be $\lambda < 0.5$ and $b > a$ with (b/a) of the order of unity. However, a comparison with an analysis based on minimum potential energy was made by the authors in ref [10] and it was found that the above eqns were satisfactory for a wider range of "b/a" values (i.e. $0.5 \leq b/a \leq 1$).

The actual asymptotic solution does yield a finite, non-zero direct meridional stress ($\sigma_{\phi D}$) but this is small compared with the bending component and is neglected in the present assessment. It was shown in [1] that its influence varies with R/r and it lowers the limit moments slightly but the effect was relatively small.

3.2 Limit Moments from Interaction Surfaces

The method follows the work in [1]. It is convenient to employ the equations corresponding to the interaction surfaces which were established generally by Marcal and Turner [7]. The stress equations (1) are expressed in terms of non-dimensional stress resultants:

$$\begin{aligned}
 m_\phi &= \frac{M_\phi}{M_o} = \frac{2}{3} \frac{1.892}{\lambda^{2/3}} \frac{Mb}{I \sigma_y} \frac{B_1(\phi)}{\pi(b/a)^{1/3}} \\
 m_\theta &= \nu m_\phi \\
 n_\phi &= \frac{N_\phi}{N_o} = 0 \\
 n_\theta &= \frac{N_\theta}{N_o} = 0
 \end{aligned} \tag{2}$$

the sign convention being such that positive 'M' produces tension on the outside surface corresponding to an externally applied bending moment tending to close the bend. The terms $N_o = 2h \sigma_y$ and $M_o = h^2 \sigma_y$ are the usual rigid-plastic resultants, when applied separately.

Equations (2) can now be substituted into the governing equations defining the interaction surfaces, from which it emerges directly that lower and upper bound limit moment factors (m_L) are 1.5 and 2.31 respectively, (if $\nu = 0.3$), where:

$$m_L = \frac{M_L}{M_Y} \tag{3}$$

While these factors are identical to the case of circular cross-sections, (neglecting $\sigma_{\phi o}$ term), the actual values of the limit moments M_L are, of course, related to the first yield moment for the elliptical bend which depend on λ and b/a .

As explained in [1] the effect of a finite N_θ tends to reduce the factors but the effect is relatively small and has been ignored here.

3.3 Limit Moments by the approach in [2]

Essentially the method is the same as in 3.2 above except that one works directly from the appropriate yield surface. In [2] Calladine examined the asymptotic elastic distributions for M_ϕ and N_θ taken from [9] in a general form (for $\lambda < 0.5$) around a circular cross-section. Since the asymptotic solution assumes that $N_\theta \gg N_\phi$ and $k_\theta = 0$ Calladine suggested that a yield surface relating M_ϕ and N_θ would be appropriate. The one chosen is the well established ellipse for a Mises material and corresponds to a symmetrically loaded cylindrical shell with zero longitudinal force

$$\left(\frac{N_\theta}{N_o}\right)^2 + \frac{3}{4} \left(\frac{M_\phi}{M}\right)^2 = 1 \tag{4}$$

Observations from the elastic distributions indicated that if a constant self-equilibrating M_ϕ^* , equal to $-0.228 M_\phi^{\max}$ ($\phi = 0^\circ$), was introduced then " $(1 - .228)M_\phi^{\max}$ " and N_θ^{\max} would simultaneously reach their maximum possible values (i.e. $\frac{2}{3} M_o$ and N_o respectively). It should be noted that in the text of [2], 0.228 was misprinted as 0.288. The interesting single result emerges from the Calladine analysis for circular bends that for $\lambda < 0.5$, the limit moment factor $m_L = M_L/M_Y$ is constant and equal to 2.25. Again it should be noted that 2.0 was quoted at the end of [2]. Comparisons with the other results in [1] show that the value of 2.25 is surprisingly high for a lower bound. Despite this restriction the same approach will be applied to the elliptic case since it yields additional information. It should be noted that M_θ has been neglected although $k_\theta = 0$.

Examination of the linear elastic analysis in [9] for elliptic cross-sections reveals that the M_ϕ , N_θ distributions are similar to those for circular cross-sections except that they are multiplied by a factor $B_1(\phi)/(\pi b/a)$. In following through the analysis a moment factor m_L results which is exactly the same as before, namely 2.25 and independent of the

pipe bend factor or b/a . This is an interesting result and in line with that of section 3.2 above. The result still seems rather high but its independence of b/a is important.

Having the benefit of the above technique of using an assumed residual constant moment, it is a relatively easy matter to extend the method of section 3.2 to include the effect. Indeed, one simply needs to divide by $(1 - 0.228)$ but again the values would be rather high.

4. Approximate Limit Moments from Creep Analysis

Where a simple power (n) law of creep is used there is a direct analogy with linear elasticity when $n = 1$ and a similar analogy with rigid plasticity when $n = \infty$. Where creep analyses have been conducted for high n values, limit loads can be obtained directly by putting the highest stress equal to the yield stress (or criteria). The process is simplified by the approximate linearity of the redistributed maximum stresses with the reciprocal of the stress index [11]. The process is explained in some detail in [1] and it is shown that bounds on the limit loads result. In [8] a total potential energy type analysis was used and this via the above technique results in upper bounds. As before

$$m_L = \frac{\text{max stress for } n = 1}{\text{max stress for } n = \infty}$$

The minimisation within the creep analysis means that the bound is optimised to some extent. Once more the effect of N_ϕ has been neglected. Its inclusion would only be noticeable for small R/a and would tend to reduce the factors.

5. The Straight Elliptical Pipe

The straight elliptical pipe may also be of interest. It is relatively simple to derive the limit moment directly as

$$M_{Le} = \sigma_y 2h a b (b/a)^3 4 \int_0^{\pi/2} (\sin \phi) / (1 - e^2 \sin^2 \phi)^2 d\phi \tag{5}$$

where $e^2 = 1 - (b/a)^2$

and the corresponding moment for first yield as

$$M_{ye} = \sigma_y 2h a b B_1(\phi) \tag{6}$$

whereas the limit and first yield moments for a circular cross-section pipe are

$$M_{Ls} = \sigma_y 8h r^2 \tag{7}$$

$$M_{ys} = \sigma_y 2h \pi r^2 \tag{8}$$

The evaluation of the integrals can be avoided by an alternative derivation taking moments about the neutral axis of the forces due to the fully plastic distribution acting at the centroid of the semi-ellipses. This of course gives the same values. For convenience we define

$$m_e = M_{Le} / M_{ye} \quad , \quad m_s = M_{Ls} / M_{ys}$$

Of course the values of m_e for example can also be obtained from the creep results for straight elliptical pipes in [8]. They are identical to those obtained from the above formulation. Where non dimensionalisation is made with respect to a circular cross section the approximation is made that $r = (a + b)/2$ which is considered to be a reasonable

assumption [12].

6 Discussion of Results

The results for the straight pipe are shown in the two alternative presentations m_e and m_s in Fig 1. These are useful in that they give the asymptotic values for pipe bends of various ellipticity.

Fig 2 shows some results of m_L from the various analyses over a range of the pipe bend factor. The results which are independent of the pipe factor are at best valid for $\lambda \leq 0.5$. The two solid lines refer to the method of section 3.2 and the dotted line to the method of section 3.3. It will be seen that the upper bound results for the creep analysis are close together which is not surprising considering the pattern of the other results. The $b/a = 1.2$ results had a slight scatter about the $b/a = 1.0$ values and thus could not be easily distinguished. The $b/a = 1.4$ had a little more scatter tending to be lower at low λ . These b/a values greater than unity are of less interest in any case. It should be noted in passing that the circular case in the region of $\lambda = 1.0$ has been improved slightly compared with [1].

The final diagram, Fig 3 shows the limit moments for bends expressed in terms of the limit moment for a circular straight M_L/M_{LS} . This presentation separates the results slightly for different b/a . A slightly different scale is also shown $(1 + b/a)^2 M_L/M_{LS}$ which has the merit of further separation if one wishes it. The point is made again that the collapse load of a bend can be much less than that of a straight pipe. At this point it is worth referring again to remarks made in [1] and reiterated in [2] about reference stresses for creep deformation; they apply equally well here. Reference stress values were given in [13] and Fig 3 here has the same form. The relationship between them using the approximation for the reference stress $\sigma_R = (M/M_L)\sigma_y$ is

$$\left(\frac{M}{h a^2 \sigma_R} \right) = \xi_0 = 2 (1 + b/a)^2 M_L / M_{LS}$$

where ξ_0 is the parameter used in [13]. Of course the values are slightly different since in [13] a representative range of the stress index was used.

The limitations and general reservations concerning the type of approach used here have already been mentioned in [1] and will not be repeated here. It must be emphasised that all the analyses are approximate to a lesser or greater extent except those for the straight pipe which are exact.

It can be seen that moderate variations from the circular form do not have a large effect on the load carrying capacity of these bends. Nevertheless, it is apparent that there is sufficient difference that tests performed on bends of unknown cross-sectional form could give misleading results. The presentation of Fig 2 may then be useful. The analogy between a single term non linear creep constitutive relationship and a plasticity relationship for monotonic loading is obvious. A simple extension of the analogy used here, employing (creep) flexibility factors, allows approximations to the complete elastic/plastic response of a bend under bending loading. This is briefly mentioned in [6]. The difficulty of extending the present limit factors to out of plane bending is substantial but some creep results have been established^[14] and initial limit loads estimated in this fashion are encouraging [6].

7 Conclusions

Approximate plastic collapse moment factors for pipe bends having non-circular cross-sections have been derived and presented in various ways. If factored in terms of the first yield moment for the same geometry the results are (virtually) independent of b/a over the range considered herein. In terms of the collapse loads for the equivalent circular straight pipe the bends results are significantly lower and the smaller b/a the lower the collapse moment.

8 Nomenclature

- a semi axis of cross section at 90° to plane of the bend
- b semi axis of cross section in plane of the bend
- $B_1(\phi)$ elliptic function
- h semi thickness
- I second moment of area of cross-section
- M applied in plane bending moment
- M_Y moment to cause first yield in elliptical pipe bend
- M_{Ye} moment to cause first yield in elliptical straight pipe
- M_{Ys} moment to cause first yield in circular straight pipe
- M_L limit moment for elliptical pipe bend
- M_{Le} limit moment for elliptical straight pipe
- M_{Ls} limit moment for circular straight pipe
- $M_o = h^2 \sigma_y$
- $M\phi, m\phi$ etc. moment stress resultants
- $m_L = M_L / M_Y$
- $m_e = M_{Le} / M_{Ye}$
- $m_s = M_{Ls} / M_{Ys}$
- $N_o = 2h\sigma_y$
- $N\phi, n\phi$ etc. stress resultant
- n stress index in creep law
- R radius of curvature of pipe bend centreline
- r = $(a + b)/2$
- $\lambda = 2hR/a^2$
- v Poisson's ratio
- ξ_o reference stress parameter $M/(ha^2\sigma_R)$
- σ_D, σ_B stress, direct or bending
- σ_R reference stress
- σ_Y yield stress

References

- 1 SPENCE J and FINDLAY G E "Limit loads for pipe bends under in-plane bending"
2nd Intl Conf on Pressure Vessel Technology San Antonio Texas
vol 1 paper 1 - 28 pp 393-399 1973
- 2 CALLADINE C R "Limit Analysis of curved tubes"
Jn of Mechanical Engineering Science vol 16 No.2 pp 85-87 1974
- 3 MELLO R M, GRIFFIN D S "Plastic collapse loads for pipe elbows using inelastic
analysis" Trans ASME J Press Vess Techn 96 p 177 (1974)
- 4 MELLO R M, SCHELLER J D "Simplified inelastic (plastic and creep) analysis of
pipe elbows subjected to in plane and out of plane bending" ASME 75-PVP-62 (1975)
- 5 MELLO R M, GRIFFIN D S, SCHELLER J D "Simplified inelastic (plastic and creep)
analysis of pipe elbows in support of the MLTF" WARD-HT-3045-10 (June 1975)
- 6 BOYLE J T, SPENCE J "The analysis of piping system for creep"
3rd SMIRT Conference San Francisco August 1977 L2/h
- 7 MARCAL PV, TURNER C E "Elastic solution in the limit analysis of shells of
revolution with special reference to expansion bellows" Jn of Mechanical Engineering
Science vol 3 No.3 pp 252-257 1961
- 8 SPENCE J "Stationary creep stresses for elliptical cross-section pipe bends
subject to in-plane bending" Pressure Vessels & Piping Conf ASME Miami Beach
1974 74 PVP 1 Jn of Pressure Vessel Technology ASME
- 9 CLARK R A, REISSNER E "Bending of curved tubes" Advances in Applied Mechanics
vol II 1951
- 10 FINDLAY G E, SPENCE J "Bending of pipe bends with elliptic cross-sections"
Welding Research Council Bulletin No.164 pp 8 - 16 1971
- 11 CALLADINE C R "A rapid method for estimating the greatest stress in a structure
subject to creep" Conf on Thermal Loading and Creep, Proc Institution of
Mechanical Engineers vol 178 (Part 3L) pp 5-15 to 5-23 1964
- 12 SPENCE J, FINDLAY G E "Manufacturing tolerances for oval pipe bends"
Process Engineering pp 62-67 December 1972
- 13 SPENCE J "The creep deformation of elliptical cross section pipe bends under in
plane bending" Intl Conf Creep and Fatigue ASME/IMEchE Philadelphia/Sheffield
1974
- 14 SPENCE J, BOYLE J T "An analysis for out of plane bending on curved pipes in
creep" IWGFR Specialists Meeting on High Temperature Structural Design Technology
Champion 1976

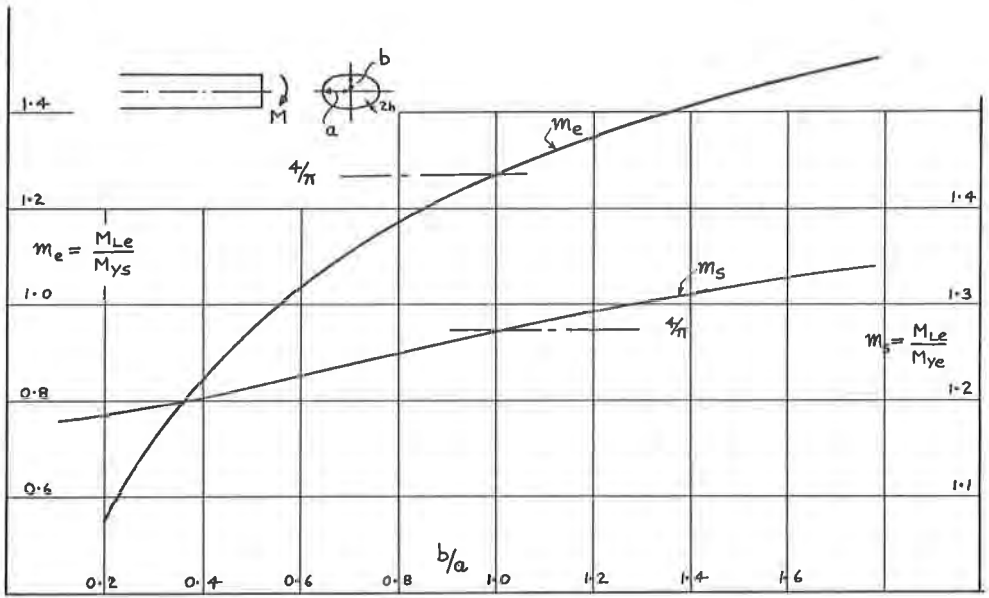


Fig 1 Limit moment factors for straight elliptical pipes

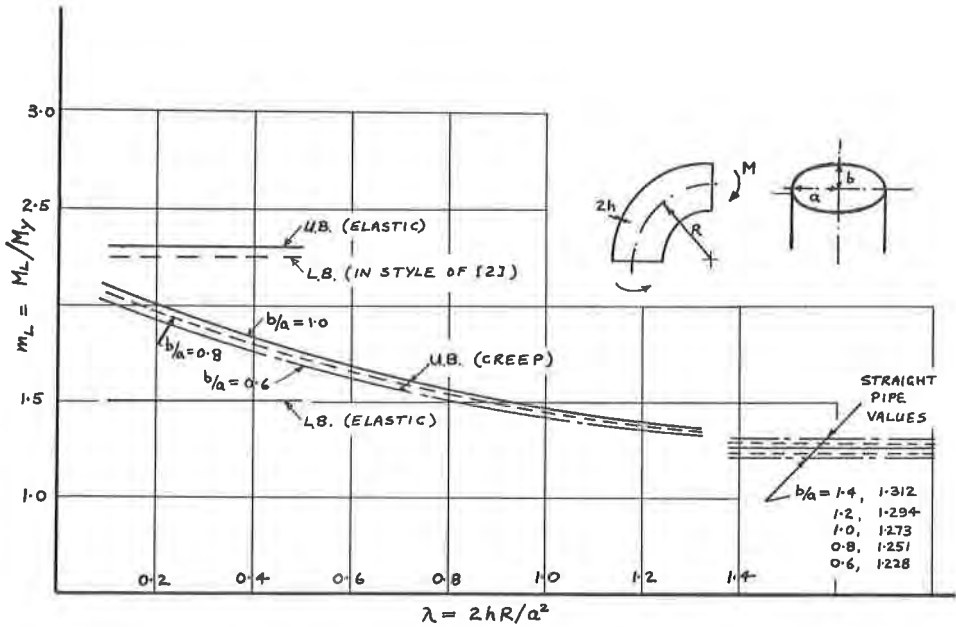


Fig 2 Limit moment factors for elliptical pipe bends

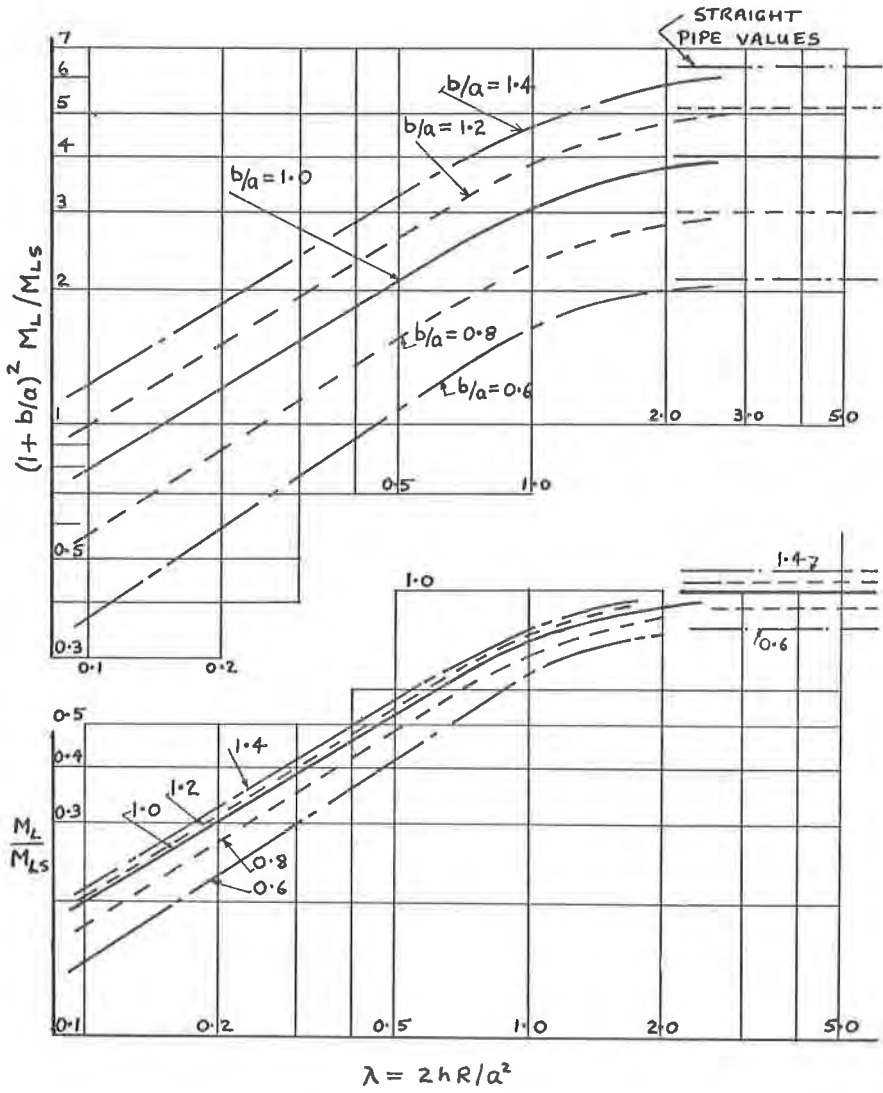


Fig 3 Limit moments relative to circular straight pipes