# The Prediction of Air Leakage Rate Through Cracks in Pressurized Reinforced Concrete Containment Vessels

J. Tinkler, R. Del Frate
*Mechanical Engineering Dept.*
S.H. Rizkalla
*Civil Engineering Dept.*
*University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada*

Abstract

Measurements of air leakage through cracks in specimens of reinforced concrete were analysed on the basis that the compressible flow occurred without heat transfer between the air and the concrete. The resultant friction coefficients were then correlated with the crack size and roughness parameters.

The empirical relations were rearranged so that leakage rates could be predicted for a gas with known properties passing through cracks of known average dimensions. Checking the predicted leakage against the measured leakage rates gave agreement within ± 35%, the scatter being attributable to the very irregular nature of cracks. The method presented for the prediction of leakage can be used with confidence for typical cracks in reinforced concrete containment vessels.

## 1. Introduction

In the unlikely event that a reactor's safety system fails, the pressure inside the containment building can become high enough to cause cracking of the structures' concrete wall and result in leakage of gases into the atmosphere. To obtain data on the leakage rate of air through cracks in reinforced concrete, two experimental programs were conducted at the Universities of Alberta and Manitoba, Canada. Based on the measured data from both programs, a mathematical expression was proposed to predict the air leakage through cracked concrete by Rizkalla et al. [1].

However, the above mathematical formulation assumed isothermal air flow from the high internal pressure to atmospheric pressure through the concrete cracks in additional to other approximations. The main objective of this paper is to re-analyse the same measured data of air leakage and to include the compressibility effects on the air flow by assuming Fanno flow through the cracks.

## 2. Experimental Program

Eight reinforced concrete specimens were tested. The main variables considered are the concrete thickness and the percentage of steel reinforcement in the direction of the applied load. The specimens were reinforced in two directions and subjected to axial tensile membrane forces in one direction. Two air chambers were provided, one on each face of the specimen. The upstream chamber was filled with pressurized air and the down stream chamber was used to collect air that had leaked through the cracks of the specimen. A sketch of a typical specimen is shown in Figure (1). A detailed description of the experimental program and test results are given in by Rizkalla et al. [1]. The test sequence is illustrated by the flow chart shown in Figure (2).

## 3. Idealized Model

The flow through a concrete crack may be idealized as shown in Figure (3). The properties of the air at the inlet and the outlet, as it leaves the crack, are denoted i and o respectively. The flow along the crack can be idealized as an adiabatic flow of a compressible gas along a constant area duct with surface friction (Fanno flow, analysed in Shapiro [2]). The analysis treats the air as an ideal gas steadily flowing through a constant cross-sectional area with a constant friction coefficient along the length of the crack. The thermodynamic process is similar to that of throttling since there is no heat transfer to the air as it expands. Thus the temperatures, T, in both the high and low pressure reservoirs will be the same. Since this type of structure is subjected mainly to membrane loads, the width of the cracks should be reasonably uniform. In this analysis, the average crack width, W is the mean of the individual crack widths at a given load increment.

## 4. Average Coefficient of Friction, F

The results of a dimensional analysis by del Frate [3] for the average coefficient of friction, F, revealed that F can be expressed as follows:

$$F = f\,(\mu/GL,\ W/L,\ \varepsilon/L)\ , \qquad (1)$$

where $\mu$ = viscosity of the air

$G$ = mass flow rate of air/unit area

$L$ = length of the specimen

$W$ = average crack width

$\varepsilon$ = height of roughness on the concrete which is assumed constant for all specimens

At any given axial load increment, the average crack width, W, is constant. In addition, for the same specimen, the length, L, is constant. Thus the two parameters W/L and $\varepsilon/L$ are constant for the measured air leakage under constant axial load for each specimen. Therefore, for this set of measured values, the friction can be expressed as follows:

$$F = f\,(\mu/GL) \qquad (2)$$

Based on the experimental results, the relation between F and $\frac{\mu}{GL}$ for a given average crack width was found to be linear:

$$F = a + b\,(\mu/GL) \qquad (3)$$

where a and b are parameters dependent on W/L and $\varepsilon/L$ values, and possibly on the amount of steel, although this latter dependence could not be detected in the results.

Based on the experimental measurements, it was found that the variation of the parameters a and b with W/L was insignificant for a given value of $\varepsilon/L$ and no appreciable inaccuracy would be introduced by taking them equal to their arithmetic means. To examine the relation between F and $\varepsilon/L$, the tested specimens were classified into three different categories for ε/L (using a mean sand grain diameter of 0.76 mm for ε), from which the following relationship was established.

$$a = 28.1 - 10.5 \times 10^3 \, (\varepsilon/L) + 0.987 \times 10^6 \, (\varepsilon/L)^2$$
$$b = 677{,}000 - 223{,}000 \times 10^3 \, (\varepsilon/L) + 20{,}000 \times 10^6 \, (\varepsilon/L)^2 \qquad (4)$$

Flow Rate Equations

The flow can be described by applying the following concepts:

a) Isentropic flow between the high pressure reservoir and the crack inlet

b) Fanno flow through the concrete

c) Isentropic flow between the crack exit and the low pressure reservoir.

Following the sequence of the above concepts, three equations for the mass flow per unit cross sectional area, G, and the Mach numbers at the crack inlet and outlet, $M_i$ and $M_o$ respectively, could be obtained as follows:

$$\frac{G}{P_1}\sqrt{\frac{RT}{k}} = M_i \left(1 + \frac{k-1}{2} M_i^2\right)^{\frac{k+1}{2(k-1)}}$$

$$F = \frac{W}{2Lk} \left| \frac{1}{M_i^2} - \frac{1}{M_o^2} + \frac{k+1}{2} \ln \; \{M_i^2(1 + \frac{k-1}{2} M_o^2)\}/\{M_o^2(1 + \frac{k-1}{2} M_i^2)\} \right|$$

$$\frac{G}{P_2}\sqrt{\frac{RT}{k}} = M_o \left(1 + \frac{k-1}{2} M_o^2\right)^{\frac{k+1}{2(k-1)}} \qquad (5\ (a,b,c))$$

where
$k$ = ratio of specific heats (= 1.4 for air)
$P_1$ = reservoir pressure at inlet
$P_2$ = reservoir pressure at exit
$T$ = isentropic stagnation temperature for both high and low pressure reservoirs
$R$ = gas constant
$F$ = average friction as proposed by eqs. (3) and (4).

Using the experimentally measured values, the quantities L, $P_1$, $P_2$, R, T, ε, and W can be specified, leaving three unknowns $M_i$, $M_o$ and G. Due to the non-linearity of the equations, the Newton-Raphson method for multiple equations was be used to obtain the solution as follows:

$$[A]\ [X] = [B]$$

where
$$[X] = \begin{vmatrix} \Delta M_i \\ \Delta M_o \\ \Delta G \end{vmatrix}$$

$$[A] = \begin{vmatrix} \frac{W}{2.8L}\left|\frac{2.4}{M_i} - \frac{2}{M_i^3} - \frac{.48\, M_i}{1 + .2M_i^2}\right|, & \frac{W}{2.8L}\left|\frac{2}{M_o^3} + \frac{.48\, M_o}{1 + 2.M_o^2} - \frac{2.4}{M_o}\right|, & \frac{b\mu}{LG^2} \\ (1 + .2M_i^2)^3 + 1.2M_i^2\,(1 + .2M_i^2), & 0, & \frac{-1}{P_1}\sqrt{\frac{RT}{1.4}} \\ 0, & (1 + .2M_o^2)^3 + 1.2M_o^2\,(1 + .2M_o^2)^2, & \frac{-1}{P_2}\sqrt{\frac{RT}{1.4}} \end{vmatrix}$$

$$[B] = \begin{vmatrix} \frac{W}{2.8L} \left| \frac{1}{M_i^2} - \frac{1}{M_o^2} + 1.2 \ln \{M_i^2(1 + .2M_o^2)\}/\{M_o^2(1 + .2M_i^2)\} \right| - (a + \frac{b\mu}{GL}) \\ M_i (1 + .2M_i^2)^3 - \frac{G}{P_1} \sqrt{\frac{RT}{1.4}} \\ M_o (1 + .2M_o^2)^3 - \frac{G}{P_2} \sqrt{\frac{RT}{1.4}} \end{vmatrix}$$

The initial values of $1 \times 10^{-4}$ for each of $M_i$, $M_o$ and G were found to be satisfactory for convergence. The calculated and measured mass flow rate per unit area, G, can be expressed in non-dimensional form and compared as is done in Figures (4) and (5). A typical relation for a typical specimen (L5) is given in Figure (4) whilst Figure (5) combines the results for all specimens onto a single graph. The comparison represents a noticeable improvement over the original analysis, especially at the higher internal pressure zones. As mentioned in the earlier paper [1], the scatter of the results at the low rates could be attributed also to the low sensitivity of the instrumentation used in measuring the flow rates and crack width and does not necessarily reflect the accuracy of the proposed method.

## Summary and Recommendation

The proposed method to predict air leakage through cracked concrete represents a noticeable improvement over most of the existing methods in the literature. The results were especially good at higher internal-pressure ranges where the air compressibility effect is more pronounced. In future tests, the roughness should be examined and related to the sand grain size.

## References


[1] Rizkalla, S.H., Lau, B.L., and Simmonds, S.H., "Air Leakage Characteristics in Reinforced Concrete", Journal of Structural Engineering, Vol. 110, No. 5, May 1984, pp. 1149-1162.

[2] Shapiro, Ascher H., "The Dynamics and Thermodynamics of Compressible Fluid Flow", Vol. I, John Wiley and Sons, 1953.

[3] Del Frate, Robert "Fanno Flow through Cracked Concrete", B.Sc. Thesis, Mechanical Engineering Department, University of Manitoba, Canada, May 1983.


## Notation

a coefficient in eq. (3)
b coefficient in eq. (3)
F friction coefficient
G mass flow per unit area of cross section

k ratio of specific heats

L length of crack in flow direction

$M_o$ Mach number at crack exit

$M_i$ Mach number at crack inlet

$P_1$ reservoir pressure at inlet

$P_2$ reservoir pressure at exit

R gas constant

T total temperature

W average crack width

$\varepsilon$ crack roughness

$\mu$ absolute coefficient of viscosity

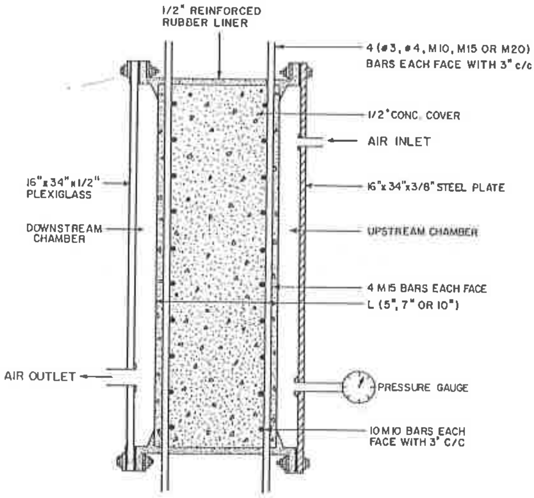


Fig. 1 Typical Leakage Specimen

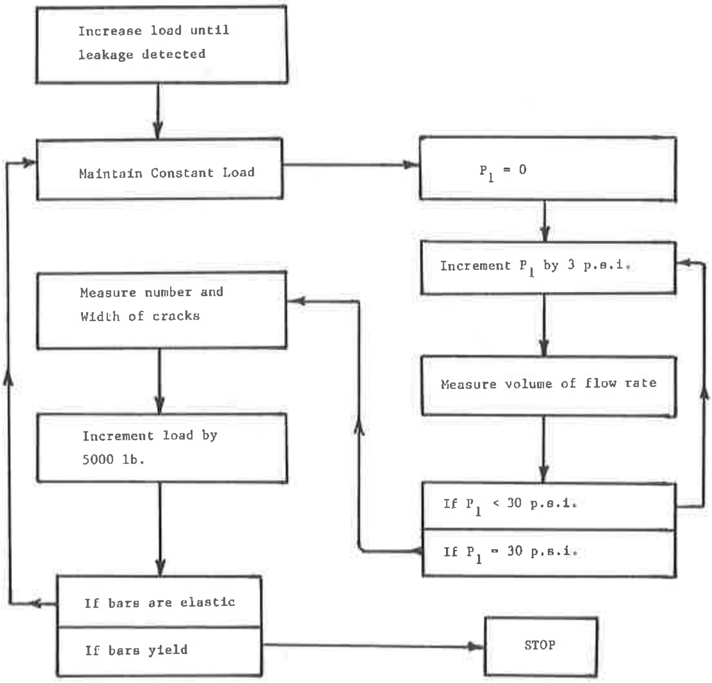


Fig. 2 Text Sequence

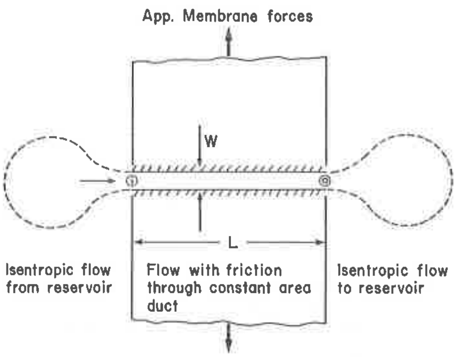


Fig. 3 Physical Model

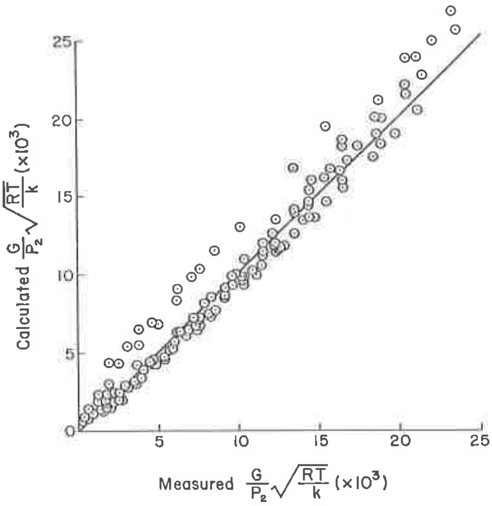


Fig. 4 Leakage Prediction Method Applied to Specimen L5

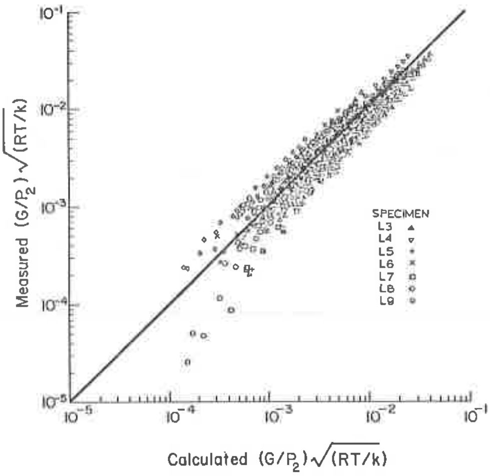


Fig. 5 Leakage Prediction Method Applied to the Experimental Model