Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

On modeling and quantification of dependent fragilities in NPP structures & equipment

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ABSTRACT: A method for the analysis and quantification of dependent failures in nuclear power plants is proposed, making use of partial beta factors and fragility curves.

1 INTRODUCTION

Dependent failure in nuclear systems (structures and equipment) has been recognized as a major source to the seismic risk in NPPs during the last decade. Several concepts and models have been developed for taking into account the dependence among basic/component failure events in the PSA (Probabilistic Safety Assessment) of NPP structures and equipment (Apostolakis 1989), (Hirschberg 1989). Among them the CCF (Common Cause Failure) analysis has got a special attention (NUREG/CR-4780 1988), (Hirschberg 1989), (Ballard 1989). A model developed within CCF is based on the so-called β-factor (Mosleh 1989), (Ballard 1989). On another hand, the seismic risk analysis of NPP structures and equipment has led to the concept of fragility and to models based on it (Kennedy et al 1980), (Hwang 1989), (Ravindra & Johnson 1991). The analysis and evaluation of dependence between the component fragilities has been less taken into acccount. Some approaches to this problem are due to M.K.Ravindra & J.J.Johnson (1991) and to A.Yamaguchi (1991). We have also proposed a couple of ways to consider and analyze the dependence relationship among component fragilities (Cărăuşu & Vulpe 1991), (Vulpe & Cărăuşu 1993).

In this paper we first extend some representations for compound failures in complex systems, consisting of series and/or parallel subsystems, together with the corresponding formulas for evaluating failure probabilities / fragilities of (sub)systems. Our research attempts to meet a requirement [stated in (Apostolakis 1989)] that 'a need exists for analytical approaches for treating seismically induced common cause failures in seismic PSAs of nuclear power plants'. In the next section, the 'β-factor method' is extended so as to replace a common β factor by a set of partial β factors β_i, each of them associated to a certain environment [in the terminology of P.Dörre (1989)] or to a certain failure mode/mechanism. Then we propose a method to quantify the partial β factors in terms of fragility models (under the "double lognormal format") for (sub)systems. A couple of numerical examples are also included in Sections 2 and 3.

2 EVALUATION OF FAILURE PROBABILITIES FOR SYSTEMS AND SUBSYSTEMS

Let us consider a structural / equipment system in an NPP - denoted by S - which consists of m distinct components. In fact, this assumption needs not be understood in a strictly physical sense ; it would rather mean that the failure of the entire system can be expressed in terms of m component failure events. Let us denote the latter ones by $A_1, A_2, \ldots, A_m$. Then the event E_k – meaning that k out of m components fail – can be expressed as

$$(1) \qquad E_k = \bigcup_{j=1}^{\binom{k}{m}} \bigcap_{i=1}^{m} B_i \quad \text{where} \quad B_i = \begin{cases} A_i & \text{for} \quad i \in \varphi(j) \\ \bar{A}_i & \text{for} \quad i \in \overline{1,m} \setminus \varphi(j) \end{cases}$$

where $\bar{A}_i$ is the complementary of event A_i, $\overline{1,m}$ denotes the set $\{1, 2, \ldots, m\}$ and $\varphi : \{1, 2, \ldots, \binom{m}{k}\} \longrightarrow C^k_{\overline{1,m}} \subset \mathbb{N}^k$ is a function which selects combinations of k indices from the set $\overline{1,m}$. In other words, $\varphi(j) = (i_1, i_2, \ldots, i_k) \subset (\overline{1,m})^k_*$, where M^k_* denotes the set of ordered k-tuples of distinct members of the set M ; here, the order of indices $i_1, i_2, \ldots, i_k$ is not essential but we may anyway take $i_1 < i_2 < \ldots < i_k$ and therefore the set $\{i_1, i_2, \ldots, i_k\}$ can be assimilated to an ordered k-tuple with distinct components. The order of indices would be relevant only when the failure within a (sub)system is assumed to develop sequentially, along the so-called failure paths (Murotsu 1983). Corresponding to expression (1) of the event E_k, its probability under the assumption of independence between the component failure events A_i will be given by

$$(2) \qquad P(E_k) = \sum_{j=1}^{\binom{m}{k}} \prod_{i=1}^{m} P(B_i) = \sum_{j=1}^{\binom{m}{k}} \prod_{i=1}^{m} q_i$$

where

$$(3) \qquad q_i = \begin{cases} p_i = P(A_i) & \text{for} \quad i \in \varphi(j) \\ 1 - p_i & \text{for} \quad i \in \overline{1,m} \setminus \varphi(j) \end{cases}$$

Three types of multiple failure events are considered in (Yamaguchi 1991), namely E_{OR}, E_{AND} and $E_{k/m}$, but they may be expressed in terms of either A_i or E_k as follows :

$$(4) \qquad E_{\mathrm{OR}} = \bigcup_{i=1}^{m} A_i = \bigcup_{k=1}^{m} E_k,$$

$$(5) \qquad E_{\mathrm{AND}} = \bigcap_{i=1}^{m} A_i = E_m,$$

$$(6) \qquad E_{k/m} = \bigcup_{\ell=k}^{m} E_\ell .$$

In Eq.(6), $E_{k/m}$ means the failure of at least k components among the m possible ones, and such events are relevant in the risk analysis of redundant nuclear systems.

A failure mode could.be defined as consisting of k specific component failures in system S. Therefore, several failure modes could occur with the same number k of failing components. The difference between two such k-modes would be not relevant when the "population" of component failures is homogeneous (from the probabilistic/statistical point of view). However, such two modes are distinct in the other case. This remark shows the utility of considering the function $\varphi : \{1, 2, \ldots, \binom{m}{k}\} \longrightarrow C^k_{\overline{1,m}}$ which allows to "sort out" from all the $\binom{m}{k}$ possible combinations those ones corresponding to failure modes which are impossible to occur, as deduced from engineering judgement (experts' opinion) and/or statistical evidence.

Corresponding to expressions (4), (5) and (6) of the three types of composite events, algebraic (analytic) expressions may be derived for corresponding failure probabilities. Such expressions may be found in (Yamaguchi 1991) in the particular case $m = 3$. However, an important question arises as regards the mutual relationship among the component events A_i. This problem has been extensively studied in the PRA (Probabilistic Risk Assessment) of nuclear power plants, mainly in the framework of the so-called CCF (Common Cause Failure) analysis ; a synthesis of the procedures, including the theoretical background, may be found in (NUREG/CR-4780 1988, 1989). The CCF concepts regard the dependence relations among the component failures within the 'basic events' corresponding to specific accident sequences in NPPs. A more general concept seems to be the DFA (Dependent Failure Analysis) as stated in (Ballard 1989).

The dependent failure is taken into account in terms of the Beta Factor Model (βFM), in (Yamaguchi 1991) - Eqs.(17) to (19), that give the probabilities of the events E_{OR}, E_{AND} and $E_{k/m}$ for a system consisting of three components with partial correlation expressed by means of the β factor as a measure of the fraction of CCF. We give a first extension of these formulas for the case of m components in a (sub)system subjected to common cause failure :

$$(7) \qquad P(E_{\mathrm{AND}}) = (1-\beta)^m p^m + \beta p,$$

$$(8) \qquad P(E_{\mathrm{OR}}) = \binom{m}{1}(1-\beta)p - \binom{m}{2}(1-\beta)^2 p^2 + \ldots + (-1)^{m-1}\binom{m}{m}(1-\beta)^m p^m + \beta p = \\ = \sum_{\ell=1}^{m} (-1)^{\ell-1}\binom{m}{\ell}(1-\beta)^{\ell} p^{\ell} + \beta p,$$

$$(9) \qquad P(E_{k/m}) = \sum_{\ell=k}^{m} P(E_{\ell}) = \\ = \sum_{\ell=k}^{m-1}\left[\binom{m}{\ell}(1-\beta)^{\ell} p^{\ell} - \binom{m}{\ell}(1-\beta)^{\ell+1} p^{\ell+1}\right] + \binom{m}{m}(1-\beta)^m p^m + \beta p .$$

In all these three equations $p = P(A_i)$ for $i = 1, 2, \ldots, m$ is the common probability of independent failure for all the components, while the last term βp in the r.h.s. of each equation quantifies the failure probability due to the common cause.

Another extension of this model would consist in considering the 'partial beta factors', as mentioned in (Ballard 1989). Unfortunately, this partial βFM is not enough explicitly presented in this reference. The idea supporting the use of partial beta factors is the necessity of defense against the CC failures by means of diversity. As it is well known, the components in a 'clump' (e.g., a parallel subsystem) have to be redundant, that is, stochastically identical/similar. This would imply that the failure of one component under specific conditions would involve the failure of the whole system. A defense against this risk would just consist in assigning a certain diversity in the behaviour of similar (or 'probabilistically identical') components.

We suggest the use the concept of 'environment' in a differrent sense that the one of (Dörre 1989), namely in the sense of placing stochastically identical components (members of a CCF group) in different environments resulting in different behaviours when subjected to a specific set of external/functional actions.

Let us consider a (sub)system consisting of m similar components, whose 'success' (that is, state of non-failure) is characterized by 's-out-of-m' surviving components and let us denote this event by $[s/m]$ with the meaning that the system survives if at least s components do not fail $(1 \leq s \leq m)$. Then the system will fail when at least $m-s+1$ will have failed. It follows that a failure event of type 'k-out-of-m' as the one denoted above by $E_{k/m}$ is relevant for $k = m - s + 1$. Therefore we have to limit ourselves to the quantification of such failure events. The other events may be expressed in terms of $E_{k/m}$: $E_{\text{AND}} = E_{m/m}$ and $E_{\text{OR}} = E_{1/m}$.

The m components of the (sub)system S are assumed to behave in n distinct environments with a diversity factor γ_i associated to each of them ; of course, the environments have to be considered as mutually exclusive and the associated diversity factors should be subjected to condition

$$(10) \qquad (\forall i \in \{1, \ldots, n\})\ \gamma_i \in [0, 1] \quad \text{and} \quad \sum_{i=1}^{n} \gamma_i = 1.$$

Let us define the partial β factors by $\beta_i = \beta \gamma_i$; consequently, $\sum_{i=1}^{n} \beta_i = \beta$. For illustration, let us consider a parallel system of 4 components as in Fig.1 a), with the 'success event' $[3/4]$. Hence the failure event for this system will be $E_{2/4}$. A beta factor of $\beta = 0.4$ is assumed. Next, this system is split up into two subsystems with 2 components each assigned to two different environments (each subsystem in one environment). The common failure rate for each of the four components is taken as $Q_t = 0.02$ (this Q_t is termed the 'total unavailability' of a component in (NUREG/CR-4780 1988) and it has to be assimilated with p that occurs in Eqs.(7) to (9).

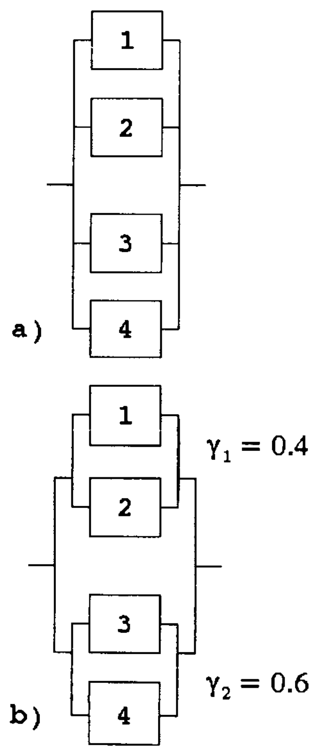

Fig.1 A redundant [3/4] parallel system

The system unavailability $Q(S)$ can be calculated using Eq.(9) for $P(E_{2/4})$, with the numerical data above mentioned in the case of a single beta factor (see Fig.1 a)). To see the difference when the four components are grouped in two pairs, we assumed two diversity factors $\gamma_1 = 0.4$, $\gamma_2 = 0.6$ for the two pairs of components, as in Fig.1 b). Of course, in the latter case formula (9) for $P(E_{2/4})$ had to be adapted but we cannot give it here (as it would take too much space). The numerical values obtained for $P(E_{2/4})$ in the two cases are:

a) $P(E_{2/4}) = 0.00883565$ without γ_i's
b) $P(E_{2/4}) = 0.00826719$ with γ_i's

It follows that the system unavailability in the case when subsystems with specified diversity factors were considered amounts to 93.5663% of $Q(S)$ obtained with the "classical" βFM, as in case a).

Possibilities to make use of diversity factors for reducing the CCF effect will also be discussed in connection with the fragility models, just in the next section.

3. DEPENDENT FRAGILITIES IN NPP SYSTEMS

As already mentioned in the Introduction, the problem of dependent failure under the impact of seismic actions has been less studied in the references available to us. We have proposed a way to take into account the dependence between the component fragilities in (Vulpe & Cărăuşu 1993). In this section we propose another method to employ the fragility models in order to reduce the CCF effect. Anyway, it is obvious that the CCF models cannot be directly used with the fragility concept by simply replacing 'unavailability' or 'probability of failure' with 'fragility', in view of the functional nature of the latter. Fragilities are conditional probabilities on the values taken by a random seismic input parameter like PGA (peak ground acceleration) or PGV (peak ground velocity). Hence the fragility is not a numerical value but a function/curve, although this does not appear explicitly in Eqs. (15), (16), (17) of (Yamaguchi 1991).

We now give some extensions of these formulas to the case of an arbitrary number m of components in a (sub)system subjected to CC failure. In fact - according to an earlier remark - it suffices to deal with Eq.(17) for the fragility of $E_{k/m}$ event since E_{AND} and E_{OR} can be expressed in terms of the former. If the random PGA parameter A takes the current value a, β_R^* and β_R' denote the common (dependent) and independent parts of the variability due to randomness as considered, e.g., by Ravindra & Johnson (1991), and $\breve{A}_\ell$ is the median PGA capacity of component ℓ, then the fragility corresponding to the event $E_{k/m}$ will be given by

$$P(E_{k/m} \mid A=a) =$$

$$= \int_0^\infty \Big[\sum_{j=1}^{\binom{m}{k}} \prod_{\ell \in \psi(j)} \Big[1 - \Phi\Big[\frac{1}{\beta_R'} \ln \Big(\frac{a}{x\breve{A}_\ell}\Big)\Big]\Big] \cdot \prod_{\ell \in \varphi(j)} \Phi\Big[\frac{1}{\beta_R'} \ln \Big(\frac{a}{x\breve{A}_\ell}\Big)\Big] + \tag{11}$$

$$+ \prod_{k=1}^{m} \Phi\Big[\frac{1}{\beta_R'} \ln \Big(\frac{a}{x\breve{A}_k}\Big)\Big] \cdot \phi\Big[\frac{\ln x}{\beta_R^*}\Big]\Big] \frac{dx}{x\beta_R^*} ,$$

where Φ is the standard normal cdf, ϕ is the standard normal pdf, φ is the function defined in Section 2 and $\psi(j) = \overline{1, m} \setminus \varphi(j)$. According to (Yamaguchi 1991), the β factor corresponding to the event $E_{k/m}$ can be evaluated by equating the r.h.s. of Eq.(9) to the r.h.s. of Eq.(11). Obviously, this would not be a simple task for larger values of m. The problem could be reduced to the evaluation of the β factors for the events $E_k, E_{k+1}, \ldots, E_m$.

The problem of reducing the fragility associated to a failure mode (of a structural subsystem) or to an accident sequence (involving equipment components) could be stated in terms of the diversity factors γ_i associated to different "environments" in a broader sense. For instance, three identical pump trains in the feedwater system placed in a star-like setup (at 120° between each other) would be subjected to different values of ground motion intensity, resulting in different fragilities. This would decrease the common part of the variability β_R^* for such a redundant subsystem, involving a greater value for β_R'. Let us see the numerical example presented in Table 1.

Table 1

$\breve{A}_\ell$ (g)	β_R	β_R^* / β_R'	β_U	β_U^* / β_U'	β_c	β_c^* / β_c'	HCLPF (g)
		0.25		0.30		0.390	
2.40	0.31	0.183	0.53	0.437	0.614	0.474	0.6027

Some of the numerical values of fragility parameters $\beta_R^*, \beta_U^*, \beta_R', \beta_U', \breve{A}_\ell$ for SWS pumps were found in (Ravindra & Johnson 1991); the other were computed by us, including the composite variabilities β_c and the HCLPF value. It is also useful to compute the ratios $\delta_R = \beta_R' / \beta_R^* = 0.733$ and $\delta_U = \beta_U' / \beta_U^* = 1.456$. They give measures of the independent vs common variabilities due to randomness / uncertainty and they can also be used for simplifying the expression of integrals as the one in Eq. (11), together with the function $\alpha(a) = [\ln(a/\breve{A}_\ell)]/\beta_R^*$. Next we computed the coefficients of p^2 for a parallel subsystem consisting of three SWS pumps, taking into account the terms except βp in the r.h.s. of Eq.(9) and in the generalized version of this equation with diversity factors included : $\gamma_1 = 0.5$, $\gamma_2 = 0.2$, $\gamma_3 = 0.3$. In (Ravindra & Johnson 1991) the values of the two parameters for this type of equipment are set to $\beta = 0.25$, $p = 2.01E{-}5$. In the case of a single β factor this coefficient amounts to $1.6875 - 0.7621\,p$, and for the partial β factors $\beta_i = \gamma_i \beta$ it amounts to $2.5194 - 4.7618\,p$. Equating the r.h. sides of Eqs.(9) and (11) in both cases, the resulting equation is expected to return a lower value of the system fragility for the partial factor model.

SUMMARY AND CONCLUSIONS

The dependence among the failure probabilities and fragilities of structural and equipment components in NPPs has been investigated. As a difference to our former approaches, we have not considered the assumption of sequential failure here, what made possible to use existing models for CCF as the β factor but generalized to partial factors expressed in terms of diversity coefficients on β. A couple of measures for the fraction of CC failure in the component fragility have been proposed. It still remains to test other CCF models (as MGL, for instance) in order to select the bes approach to quantifying the dependent fragilities for NPP structures and equipment.

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