A FRICTION STRESS METHOD FOR THE CYCLIC INELASTIC BEHAVIOR OF METALS

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SUMMARY

Inelastic deformation and fatigue analyses require that computational models of inelastic material behavior be capable of simulating the various plastic stress-strain phenomena such as the memory of prior history and cycle dependent transient hardening, softening, relaxation and creep associated with cyclic loads. This paper presents such a formulation in which the transient phenomena are uniquely described in terms of a friction stress parameter and the memory phenomenon is simulated by the characteristics of a mechanical model comprising of "Hookean Spring—Friction Slider" elements connected in series, the spring and slider within each element being connected in parallel. The formulation is ideally suited for programming on a digital computer.

A recent study of several structural steels and aluminum alloys showed that transient changes in the stress-strain curve during constant axial strain cycling were primarily due to changes in the lengths of initial "elastic" parts while the non-linear portions remained virtually unchanged in shape. This "elastic" part termed the "Yield Range Increment" (YRI), represents a change in the yield strength of the material. Physically it is equivalent to an intrinsic friction stress that must be exceeded for the onset of macroscopic plastic deformation in each half cycle. Such a friction stress level is a function of the substructural changes that occur during the transient behavior.

Cyclic hardening or softening is denoted by a cycle dependent change in YRI, whereas cyclic relaxation or creep is denoted by a relative difference in YRIs between two consecutive half-cycles. Under constant strain cycling, the YRI approaches an approximately unique saturation value independent of the prior history. This is consistent with the observations of dislocation structures. In structural aluminums studied, the saturation level of YRI is virtually independent of the strain amplitude whereas, in the case of steels it is dependent on the strain amplitude. These facts also correctly predict the observed difference in transient behavior between steels and aluminums. The rate at which YRI approaches saturation seems to be directly proportional to the difference between the current and saturation values of YRI and inversely proportional to the number of half-cycles. In general the observations seem to have a reasonable physical basis.

The fact that the non-linear portions of the stress-strain curve remain unchanged in shape during cycling implies that the plastic strain hardening rate ($d\sigma/d\epsilon_p$) is a unique function of the plastic strain excursion in each half-cycle and independent of prior history. In terms of the rheological model this means that the relative difference between the friction stress levels of the sliders in various elements remains unchanged under cyclic loads and is an intrinsic material property. At present, the formulation is limited to uniaxial state of stress and room temperature conditions (viscous effects are neglected). Possibilities of extending the formulation to elevated temperature conditions and multiaxial stress-states are being investigated.

1.0 INTRODUCTION

The stress-strain behavior of structural metals under cyclic loading conditions is complicated by cycle dependent plastic phenomena such as cyclic hardening or softening, cyclic creep or relaxation and memory of prior cyclic history. A large number of articles dealing with these phenomena have appeared in the literature over the last decade. These include phenomenological descriptions, microstructural studies and quantitative modelling techniques. This paper describes a unified modelling technique based on some recent observations of the hysteretic characteristics of several steels and aluminum alloys. The discussion is limited to room temperature condition (time dependent behavior is negelected), uniaxial stress state and small plastic strains (up to ~ 0.02) typically in the realm of low cycle fatigue.

2.0 MEMORY OF PRIOR CYCLIC HISTORY

For simplicity, this feature will be initially considered independent of the various cycle dependent transient phenomena. Figure 1 illustrates a simple example of "Memory". The stress-strain curve has been represented by piecewise linear segments only in order to use the same figure for describing an operating model later. If the material is loaded through OA, then unloaded through AB and finally loaded in the tensile direction, it apparently "remembers" the previous point of unloading A, and follows the path AC which is the continuation of the original path OA, thus exhibiting a discontinuity at A. This phenomenon is a consequence of the internal stress distribution caused by the previous loading history.

2.1 A Memory Model

A rheological model consisting of Hookean springs and friction sliders such as the one shown in fig. 2 is commonly used to represent the memory behavior. This model is physically motivated and it adequately represents the stress-strain behavior of structural metals as influenced by their polycrystalline and multiphase nature. The postulation (often named after Masing [1]) that the unloading hysteresis curve is geometrically similar to the monotonic stress-strain curve but magnified by a factor of two and the so called Bauschinger effect are also exhibited by this model. Based on this rheological model several formulations for simulating the memory behavior have been proposed [2-4]. The one which uses a simple set of rules ideally suited for digital computation will be adopted [4]. It is best to illustrate this with an example.

2.2 Memory Rules

Initially the stress-strain curves are assumed to be identical in tension and compression. Also the stress-strain curve is represented by a set of piecewise linear segments whose lengths, slopes and number are suitably chosen. For the present illustration a five segment representation is used as shown in fig. 1.

The rules are stated as follows:

1. The availability of a segment during the current loading (or reversal*) which is either tensile or compressive, depends on the prior loading history and is denoted by an "availability coefficient".
2. The absolute sum of the availability coefficients for <u>each</u> segment in the tensile and the compressive directions is always equal to <u>TWO</u>. Initially the availability coefficient in tension or in compression for <u>each</u> segment is <u>ONE</u>.

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*A reversal is defined as the hysteresis path between one peak to the next valley or vice versa.

3. During a current loading (or reversal) the stress-strain path is defined by segments starting from the first, in the consecutive order and to the extent they are available in the direction of loading, until the desired stress or strain limit is reached.
4. The availability of an element in a given direction (tension or compression) decreases to the extent the segment is used in that direction but increases by an equal amount in the opposite direction.

2.3 An Example

In brief, the procedure amounts to a simple "bookkeeping" operation of the availability coefficients as illustrated in Table 1 for the example of fig. 1. At the start of the tensile loading sequence OA, one unit of each segment is available in tension and hence the stress-strain path OA consists of unit length of segments (1) to (3) and 0.8 unit of (4) at which stage point A is reached. The availability coefficients of these segments in tension are reduced by these amounts, but at the same time their compressive availability coefficients are correspondingly increased. The availability coefficient of (5) is unaltered as it was not used. The next sequence viz. reversal AB which is compressive, uses two available units of (1) and 1.3 units of (2) at which stage the desired limit B is reached. The next sequence BC which is tensile, uses two units of (1), 1.3 units of (2), none of (3), 0.2 of (4) and one unit of (5).

As mentioned earlier, this formulation so far has not incorporated cycle dependent phenomena. If in the example of fig. 1, cycling is continued between fixed strain limits of A and B, this model exhibits fixed stress limits, i.e., it produces identically shaped hysteresis paths between A and B. However, a real material exhibits a variation in the stress limits with such cycles. These variations could be considerable depending on the material in question and may be especially significant in cumulative fatigue damage analyses [5]. A simple modification of the above 'memory' formulation to include cycle dependent transient phenomena is considered next.

3.0 CYCLE DEPENDENT TRANSIENT PHENOMENA

These phenomena are usually observed under constant strain or stress amplitude cycling conditions and are classified as cyclic hardening, cyclic softening, cyclic creep and cyclic mean stress relaxation. In general these are transient and a stable state is approached rapidly, except in the case of cyclic creep which may lead to tensile failure or buckling due to the excessive accumulation of tensile or compressive mean strain. It should be noted that these phenomena are not due to time dependent viscous effects but rather a consequence of substructural changes associated with reversed plastic straining and therefore are characterized as being cycle dependent. Therefore they should be distinguished from conventional relaxation and creep behavior observed under static conditions. Excellent descriptions of the observed cycle dependent deformation behavior of various structural metals are available [6-9] and hence is not discussed here.

3.1 Modelling Transient Phenomena

A recent study [10] of the hysteretic behavior of several metals including steels and aluminum alloys showed that transient changes in stress-strain curves during constant amplitude cycling were primarily due to changes in the initial "linear" parts while the non-linear portions remained virtually unchanged in shape. This change in the "elastic" part is defined as the Yield Range Increment (YRI) as it denotes a change in the yield strength. A cycle dependent increase or decrease in YRI denotes cyclic hardening or cyclic softening respectively, whereas a relative difference in YRI between two consecutive reversals describes cyclic relaxation or cyclic creep depending on the controlled limits (strain or stress). This is illustrated for the cases of cyclic hardening, relaxation and creep in figs. 3a, b and c. The YRI's AB, CD and EF account for the transient phenomena whereas the non-linear portions BC, DE and FG remain identical in shape. As saturation is approached the difference in YRI's between consecutive reversals decreases. Thus during the transient period the hysteresis loops are not fully closed but at saturation they are identical as the YRI's in the tensile and compressive hysteresis curves have reached a common saturation level.

3.2 Cycle Dependence of YRI

It is relatively easier to measure changes in YRI between two reversals rather than its variation within a given reversal. However this knowledge is necessary for predicting cyclic relaxation or cyclic creep. Physically the YRI is equivalent to a change in the internal friction stress of the material which is a function of the substructural changes associated with cyclic plastic loading. Consistent with dislocation theories [11], and observations of changes in stored energy of cold work [12, 13] the initial part of a reversal should be associated with some recovery (or softening) whereas work hardening processes dominate the latter portion of the reversal. A sudden release of stored energy of cold work associated with unloading has been observed by several investigators [12-14]. With this knowledge, it is tentatively assumed that the YRI remains unchanged during the initial unloading "elastic" part of the reversal, followed by a certain amount of sudden reduction in YRI due to recovery, which then gradually builds up towards a value at the end of the reversal. This terminal value is dictated by whether it is a hardening, softening or saturated situation. This is schematically illustrated in fig. 4. Adopting a simple view it is further assumed that this increase in YRI during the latter portion of the reversal is proportional to the absolute flow stress level. During transient hardening or softening, the YRI approaches an approximately unique saturation level independent of prior cyclic history [15]. This is consistent with observations of dislocation structures [16]. In structural aluminums studied the saturation level of YRI appears to be independent of the strain amplitude, whereas in steels it is dependent on the strain amplitude. This is illustrated for SAE 1045 normalized steel and 2024-T4 aluminum in fig. 5. This fact also explains the observed difference in relaxation behavior of steel and aluminum which is shown in fig. 6a, b. The steel exhibits softening whereas the aluminum exhibits hardening. The rate per reversal at which the YRI approaches saturation appears to be proportional to the difference between the saturation and current values of YRI and inversely proportional to the number of reversals [15].

3.3 Combination of Memory and Transient Behavior Models

The foregoing observations provide a complete basis for describing the cycle dependent transient phenomena in terms of the history dependent friction stress parameter, namely YRI. In terms of the memory model described earlier, the transient behavior is described by changes in the length of the first (elastic) segment as governed by the variation in YRI, while its slope E is assumed to remain constant. The fact that the non-linear portions of the hysteresis curves remain unchanged during transient conditions means, that the lengths and slopes of the other segments remain unaltered. This is equivalent to stating that the plastic strain hardening rate ($\frac{d\sigma}{d\epsilon_p}$) is a unique function of the plastic strain excursion in the reversal and is independent of prior cyclic history. An illustrative simulation of the behavior of 2024-T4 aluminum is described next.

4.0 A COMPUTER SIMULATION

A computer program to simulate the combined transient and memory behavior of 2024-T4 aluminum was written in "Fortran" language. Table 2 lists material data used in the program. For simplicity the quantities C and $\delta\sigma_R$ have been arbitrarily chosen as constants although they are functions of strain amplitude. Table 3 lists expressions used to characterize the cycle dependence of YRI for describing the transient behavior. A typical simulation of the hysteresis loops depicting cyclic hardening in first five cycles and cyclic relaxation in the last five cycles is shown in fig. 7a and b. The qualitative similarity between the real relaxation behavior of 2024-T4 aluminum in fig. 6b and that of simulation in fig. 7b may be noted.

5.0 CONCLUSIONS

A digital computer oriented modelling technique for the inelastic stress-strain behavior of structural metals has been presented. The model uses a single friction stress parameter to describe the cycle dependent transient phenomena and a set of rules to characterize memory of prior cyclic history.

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NOMENCLATURE

- C - coefficient of cyclic hardening/softening
- E - Young's modulus
- m_1 - availability coefficient of the first segment
- σ - stress
- ϵ - strain
- $\Delta\sigma$ - stress range
- $\Delta\bar{\sigma}_i$ - stress increment of i^{th} segment
- $\Delta\epsilon$ - strain range
- $\Delta\bar{\epsilon}_i$ - strain increment of i^{th} segment
- $\delta\sigma$ - Yield Range Increment (YRI)
- $\delta\sigma_R$ - Recovery portion of YRI
- $\delta\sigma_s$ - Saturation level of YRI
- $\delta\sigma_i$ - YRI at the end of initial static load
- r - subscript denotes beginning of r^{th} reversal

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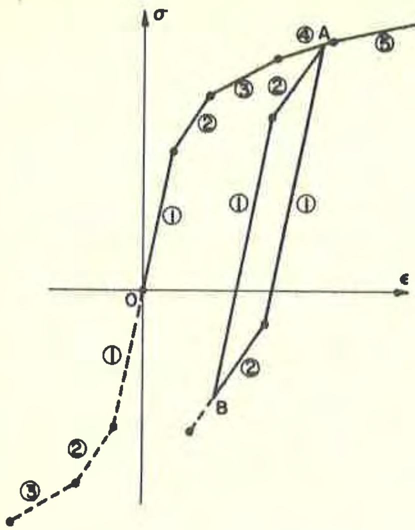

Fig. 1. An example of load history

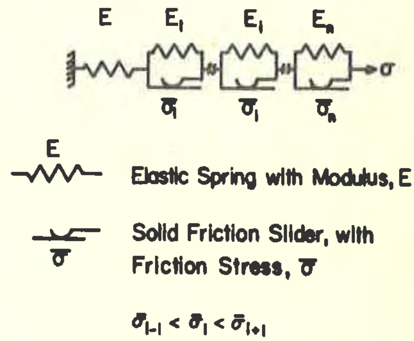

Fig. 2. A rheological model

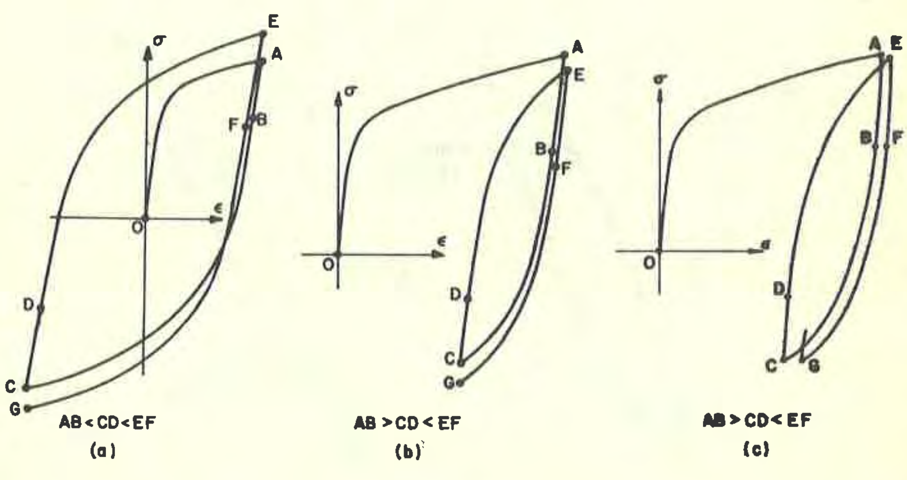

Fig. 3. Modelling cycle dependent transient phenomena, (a) Hardening, (b) Relaxation and (c) creep. AB, CD and EF are YRI s and the non-linear parts BC, DE, and FG are identical in shape.

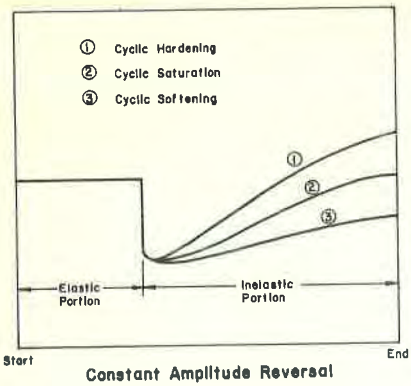

Fig. 4. Variation of YRI within a given reversal

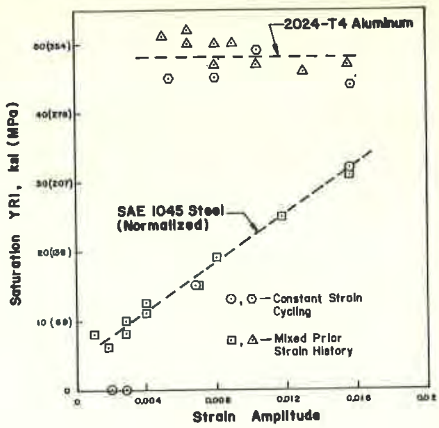

Fig. 5. Saturation values of YRI in SAE 1045 normalized steel and 2024-T4 aluminum.

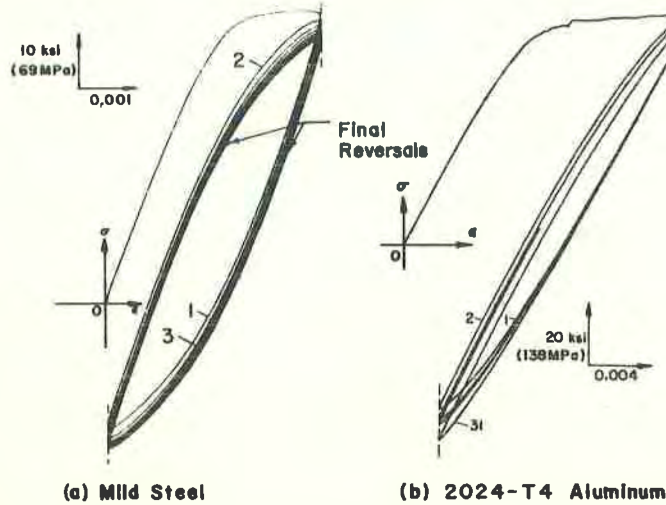

Fig. 6. Cyclic relaxation behavior of (a) SAE 1018 steel and (b) 2024-T4 aluminum.

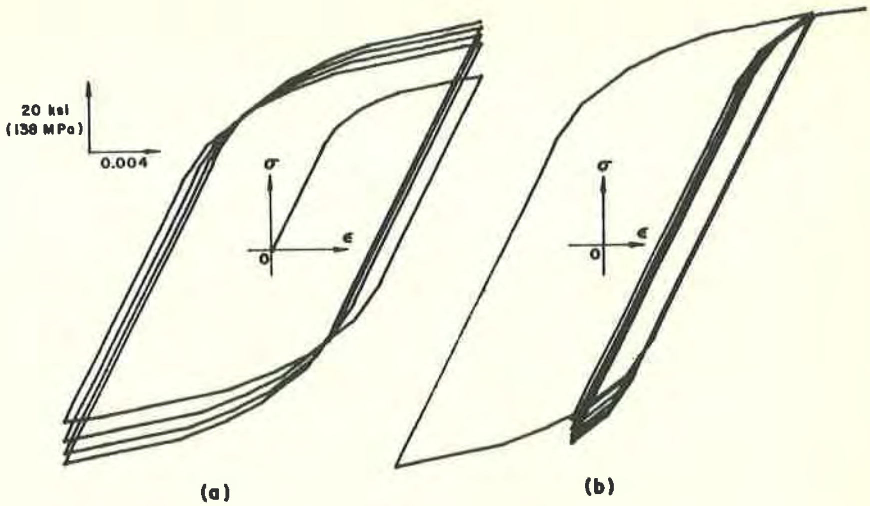

Fig. 7. A computer simulation of typical cyclic hardening, relaxation and memory behavior of 2024-T4 aluminum; (a) 1-8 reversals at strain limits of ± 0.012, (b) 9-10 reversals at ± 0.012, 1-9 reversals at ± 0.012 to - 0.002 and 1 reversal at - 0.002 to + 0.015.

TABLE 1 - AN EXAMPLE OF MEMORY (SEE FIG. 1)

Loading Path			Availability Coefficient of Segments									
			①		②		③		④		⑤	
			T	C	T	C	T	C	T	C	T	C
OA	T	S	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
		E	0.0	2.0	0.0	2.0	0.0	2.0	0.2	1.8	<u>1.0</u>	<u>1.0</u>
AB	C	S	0.0	2.0	0.0	2.0	0.0	2.0	0.2	1.8	1.0	1.0
		E	2.0	0.0	1.3	0.7	<u>0.0</u>	<u>2.0</u>	<u>0.2</u>	<u>1.8</u>	<u>1.0</u>	<u>1.0</u>
BC	T	S	2.0	0.0	1.3	0.7	0.0	2.0	0.2	1.8	1.0	1.0
		E	0.0	2.0	0.0	2.0	<u>0.0</u>	<u>2.0</u>	0.0	2.0	0.0	2.0

(S - Start, E - End, T - Tension, C - Compression, Unused Segments Underlined)

TABLE 2 - MATERIAL (2024-T4 ALUMINUM) DATA

LINEAR SEGMENTS

No. (i)	$\Delta\bar{\sigma}_i$ KSI	$\Delta\bar{\epsilon}_i$
1	30.00	0.003
2	5.00	0.0007
3	5.00	0.0013
4	2.50	0.0010
5	2.50	0.0013
6	2.50	0.0025
7	1.25	0.0012
8	1.25	0.0021
9	1.20	0.0022

$E = 10 \times 10^3$ KSI

$C = 0.1$

$\delta\sigma_i = 0.0$ KSI

$\delta\sigma_s = 50.0$ KSI

$\delta\sigma_R = 5.0$ KSI

TABLE 3 - CYCLE DEPENDENCE OF YRI

a) Hardening/softening per reversal:

$$\delta\sigma_{r+1} - \delta\sigma_r = C(\delta\sigma_s - \delta\sigma_r)$$

b) Variation of YRI within a reversal:

1. $\Delta\sigma_o = m_1(\Delta\bar{\sigma}_1 + \delta\sigma_r/2)$

2. $\delta\sigma = \delta\sigma_r$ .... $\Delta\sigma < \Delta\sigma_o$

3. $\delta\sigma_o = \delta\sigma_r - \delta\sigma_R$; If $\delta\sigma_r < \delta\sigma_R \longrightarrow \delta\sigma_o = 0$... $\Delta\sigma = \Delta\sigma_o$

4. $\delta\sigma = \delta\sigma_o + [C(\delta\sigma_s - \delta\sigma_r) + \delta\sigma_R] \dfrac{|\sigma|}{|\sigma_r|}$ ..... $\Delta\sigma > \Delta\sigma_o$