ON THE CONCEPT OF ELASTICITY USED IN SOME FAST REACTOR ACCIDENT ANALYSIS CODES

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SUMMARY

The analysis to be presented will restrict attention to the elastic part of the elastic-plastic constitutive equation used in several Fast Reactor Accident Analysis Codes and originally applied by M.L. Wilkins: Calculation of Elastic-Plastic Flow, UCRL-7322, Rev. 1, Jan. 1969.

It is shown that the used elasticity concept is within the frame of hypo-elasticity. On the basis of a test found by Bernstein it is proven that the state of stress is generally depending on the path of deformation. Therefore this concept of elasticity is not compatible with finite elasticity.

For several simple deformation processes this special hypo-elastic constitutive equation is integrated to give a stress-strain relation. The path-dependence of this relation is demonstrated. Further the phenomenon of hypo-elastic yield under shear deformation is pointed out.

The relevance to modelling material behaviour in primary containment analysis is discussed.

1. Introduction

For the analysis of the integrity of the primary containment system in case of a Hypothetical Core Disruptive Accident in a Fast Reactor several accident analysis codes are now in use. These computer programs solve the basic equations of continuum mechanics in two dimensions by finite difference techniques assuming appropriate constitutive equations for the fluid and solid materials. In the study presented here the attention is restricted to the constitutive equations of the solid materials as they are proposed for use in some of the accident analysis codes [1-4]. The original work of Wilkins [5,6] has been an important source for the formulation of these elastic - plastic constitutive equations. Since "incremental" plasticity is involved the "elastic law" is formulated in an incremental or rate type equation taking account of large deformation gradients.

The equations given by Wilkins [5,6] and others [1-4] are formulated for axisymmetry; there correction termes δ_{rr}, δ_{zz}, δ_{rz} appear which take care of the rotation of the particle. These equations have the following form in cartesian coordinates. The rate of deformation tensor

$$d_{kl} = \tfrac{1}{2}\left(v_{k,l} + v_{l,k} \right). \tag{1}$$

is assumed to be separable in an elastic and a plastic part in an additive manner

$$d_{kl} = \underset{e}{d}_{kl} + \underset{p}{d}_{kl} \tag{2}$$

Here $v_k(x_i,t)$ are the velocity components and a comma $(\;)_{,k}$ and $(\;)_{,K}$ denotes partial differentation with respect to the spatial coordinates x_k $(k=1,2,3)$ and material (Lagrangian) coordinates X_K (K=1,2,3) respectively. The elastic deformation rate $\underset{e}{d}_{kl}$ is assumed to be given by

$$\overset{\circ}{t}_{kl} = \lambda \, \underset{e}{d}_{mm} \delta_{kl} + 2\mu \, \underset{e}{d}_{kl} \tag{3}$$

where the constants λ and μ correspond to Lame's constants of the infinitestimal theory of elasticity and δ_{kl} is the unit tensor; the usual summation convention applies. Here $\overset{\circ}{t}_{kl}$ is the co-rotational stress rate according to Jaumann

$$\overset{\circ}{t}_{kl} \equiv \frac{Dt_{kl}}{Dt} - t_{km}\,\omega_{lm} - t_{ml}\,\omega_{km} \tag{4}$$

where

$$\frac{Dt_{kl}}{Dt} = \frac{\partial t_{kl}}{\partial t} + \frac{\partial t_{kl}}{\partial x_m} v_m \tag{5}$$

is the material rate of the Cauchy stress t_{kl} and

$$\omega_{kl} = \tfrac{1}{2}\left(v_{k,l} - v_{l,k} \right) \tag{6}$$

is the spin, which describes the angular velocity of the rigid rotation. The plastic deformation not described here is certainly the most important aspect in the constitutive equation. However, we will restrict our attention strictly to the elastic law eq. (3), since it seems worthwhile to spend some effort for a rigorous understanding of the physical concepts behind eq. (3); thus for the following discussion we put simply $\underset{e}{d}_{kl} = d_{kl}$.

Elasticity concepts identical or similar to eq. (3) have been used by different authors e.g. Cameron and Scorgie [7], Hartzmann [8], Hibbit, Marcal and Rice [9] and Osias and Swedlow [10]. Comparing this concept with available constitutive models in nonlinear continuum mechanics one may easily find that this concept is a special case of hypo-elasticity [11-13]. Hypo-elastic material behavior is defined by the constitutive equation

$$\overset{\circ}{t}_{kl} = A_{klmn}(t_{rs})\, d_{mn} \qquad (7)$$

$$A_{klmn} = A_{lkmn} = A_{lknm}$$

The right hand side of eq. (7) is linear in the deformation rate d_{mn} and necessarily isotropic in t_{rs} and d_{mn} [11, 12]. All variables in eq. (7)are quantities defined in the instantaneous configuration (Eulerian description) and the concept of finite strain is not used. Since the corotational stress rate and the deformation rate are linearly related a change of time-scale does not affect the state of stress in a given configuration; thus the stress at time t depends only on the order of past configurations but not on the time-rate at which these past configurations are passed.

The hypo-elastic equation (7) represents a system of differential equations for the stress at a fixed particle if the deformation history and initial conditions for the stress are prescribed. Thus a stress-strain relation has to be obtained by integration.

The usual concept of elasticity is based on two physical assumptions:
(i) The stress at time t depends only on the strain at time t but not on the history of deformation. This condition is realized in the formulation (Chauchy-elasticity)

$$t_{kl} = f_{kl}(x_{i,M}) = f_{lk}\,, \qquad x_{i,M} = \frac{\partial x_i(X_L,t)}{\partial X_M} \qquad (8)$$

where $x_i = x_i(X_L,t),\ i=1,2,3$ describes the motion of a particle with material coordinates X_L and $x_{i,M}$ is the deformation gradient with respect to the stress free natural state of the body. Since this relation must be invariant under rigid body motions the functions f_{kl} are not arbitrary but have to be form invariant [11].
(ii) Usually an additional criterion is applied: The deformation energy per unit mass depends only on the initial and final configurations. This condition assures the existence of a strain energy function $\sigma(x_{i,L})$ such that eq. $(8)_1$ is of the form [12]

$$t_{kl} = \rho \frac{\partial \sigma}{\partial x_{l,M}}\, x_{k,M} \qquad (9)$$

where ρ is the density in the deformed configuration. Eq. (9) defines the Green or hyper-elastic materials.

In analysing hypo-elasticity it is of primary interest to know the conditions under which a hypo-elastic material is elastic in the sense of Cauchy or Green. In the following the relation of the hypo-elastic material defined by eq. (3) to elasticity (in the sense of Cauchy) is discussed based on the work of Bernstein and Ericksen [15-17].

2. Path-dependence of stress

We will briefly present the conditions under which the hypo-elastic eq. (7) will be integrabel to give a path-independent relation between stress and deformation gradient such as eq. (8) . Assuming that the velocity $v_i = Dx_i/Dt$ is a function of the material coordinates the spatial velocity gradient $v_{i,j}$ is given by (cartesian coordinates)

$$v_{i,j} = v_{i,M} X_{M,j} = \frac{D x_{i,M}}{Dt} X_{M,j} \tag{10}$$

where $X_{M,j}$ is the inverse of the deformation gradient: $x_{i,M} X_{M,j} = \delta_{ij}$

Applying eq. (1), (4-6), and (10) to eq. (7) yields

$$\frac{Dt_{kl}}{Dt} = B_{klqp} X_{M,q} \frac{D x_{p,M}}{Dt} \tag{11}$$

with

$$B_{klqp} = A_{klqp}(t_{rs}) + \frac{1}{2}\left(t_{kq}\delta_{lp} + t_{ql}\delta_{kp} - t_{kp}\delta_{lq} - t_{pl}\delta_{kq}\right). \tag{12}$$

Material differentiation of eq. (8) gives

$$\frac{Dt_{kl}}{Dt} = \frac{\partial f_{kl}}{\partial x_{p,M}} \frac{D x_{p,M}}{Dt} \tag{13}$$

If we assume that the hypo-elastic material is elastic in the sense of Cauchy (eq. $(8)_1$) then the right hand sides of eq. (11) and (13) are to be equal for all deformation processes $x_{p,M}(t)$. Since $x_{p,M}$ and $D(x_{p,M})/Dt$ are in general independent from each other the six functions f_{kl} must satisfy

$$\frac{\partial f_{kl}}{\partial x_{p,M}} = B_{klqp} X_{M,q} \quad ; \tag{14}$$

according to eq. (12) B_{klqp} is only depending on t_{rs} and thus f_{rs} . Eq. (14) represents an overdetermined system of partial differential equations of first order. If the above assumption is valid then this system has to admit a solution. A necessary and sufficient condition for the integrability of the system can be developed using standard methods of the theory of partial differential equations [19]. Consider the overdetermined system of partial differential equations

$$\frac{\partial z_\alpha}{\partial y_\beta} = g_{\alpha\beta}(y_1, \ldots, y_{\bar\beta}, z_1, \ldots, z_{\bar\alpha}) \qquad \begin{cases} \alpha = 1, 2, \ldots, \bar\alpha \\ \beta = 1, 2, \ldots, \bar\beta \quad , \ \bar\beta > \bar\alpha \end{cases}$$

where the functions $g_{\alpha\beta}$ are continuously differentiable with respect to their arguments y_β and z_α . A unique solution of this system is assured if the necessary and sufficient integrability condition

$$\frac{\partial g_{\alpha\beta}}{\partial y_\gamma} + \frac{\partial g_{\alpha\beta}}{\partial z_s} g_{s\gamma} - \frac{\partial g_{\alpha\gamma}}{\partial y_\beta} - \frac{\partial g_{\alpha\gamma}}{\partial z_s} g_{s\beta} = 0 \tag{15}$$

is identically satisfied in the arguments y_β and z_α .

Let

$$C_{klpM} = B_{klqp} X_{M,q} \tag{16}$$

then the application of the condition eq. (15) to the system eq. (14) taking account of the different indicial notation we obtain

$$\frac{\partial C_{klpM}}{\partial x_{m,L}} + \frac{\partial C_{klpM}}{\partial t_{rs}} C_{rsmL} - \frac{\partial C_{klmL}}{\partial x_{p,M}} - \frac{\partial C_{klmL}}{\partial t_{rs}} C_{rspM} = 0 . \tag{17}$$

With eq. (17) and

$$\frac{\partial X_{N,q}}{\partial x_{p,K}} = - X_{N,p} X_{K,q}$$

and taking the contracted product of eq. (17) with $x_{i,M} x_{j,L}$ finally yields the test condition derived first by Bernstein [17]:

$$\frac{\partial B_{klip}}{\partial t_{rs}} B_{rsjm} - \frac{\partial B_{kljm}}{\partial t_{rs}} B_{rsip} + \delta_{im} B_{kljp} - \delta_{jp} B_{klim} = 0. \qquad (18)$$

This equation represents a restriction on the stress t_{kl}. The satisfaction of this condition is necessary and sufficient for a hypo-elastic material to be Cauchy-elastic. If the application of the test shows that $A_{klmn}(t_{rs})$ does not satisfy eq. (19) identically in t_{rs} then for arbitrary deformations the stress cannot be represented by a function of the deformation gradients independent of the path of deformation. However, for restricted states of stress (e.g. hydro-static pressure) or deformations (e.g. simple extension) eq. (7) or (12) may be integrable to give a relation between stress and strain independent of the path of deformation.

Applying this test to the hypo-elastic equation (3) with

$$A_{klmn} = \lambda \delta_{mn} \delta_{kl} + \mu (\delta_{kn} \delta_{ml} + \delta_{km} \delta_{nl}) \qquad (19)$$

and obeying eq. (12) gives the following condition on stress

$$t_{kl} = 0 \quad , \quad k \neq l . \qquad (20)$$

Consequently the stress strain relation obtained from eq. (3) by integration for arbitrary deformation processes $x_{i,k}(t)$ will be path-dependent if shear stresses and thus shear deformation is involved; this result will be quantitatively verified in chapter (4).

The question now arises wether the hypo-elastic material eq. (3) shows dissipative effects in the sense that the deformation energy is positive in any closed deformation process with the same initial and final state of stress.

3. Path-dependence of deformation energy

The deformation energy generated in the time interval $0 \le \tau \le t$ is defined by

$$\int_0^t \int_V t_{kl} d_{kl} \, dV \, d\tau = \int_{V_0} w(X_M, t) \, dV_0$$

where V is the volume at time τ and V_0 is the volume in a reference configuration (reference density ϱ_0) and $w(X_M, t)$ is the deformation energy per unit initial volume:

$$w(X_M, t) = \int_0^t t_{kl} d_{kl} \frac{\varrho_0}{\varrho} d\tau \; ; \qquad (21)$$

the integration has to be performed along the particle path. Inversion of eq. (3) gives

$$d_{kl} = D_{klmn} \overset{\circ}{t}_{mn} \qquad (22)$$

where D_{klmn} is found to be

$$D_{klmn} = \frac{1}{2\mu} \left\{ \frac{1}{2} (\delta_{km} \delta_{ln} + \delta_{kn} \delta_{lm}) - \frac{\lambda}{3\lambda + 2\mu} \delta_{mn} \delta_{kl} \right\}. \qquad (23)$$

Further

$$t_{kl} d_{kl} = F_{mn} \overset{\circ}{t}_{mn} = F_{mn} \frac{\mathcal{D} t_{mn}}{\mathcal{D} t} \; ; \qquad F_{mn} = t_{kl} D_{klmn} . \qquad (24)$$

Note that all terms containing the spin ω_{kl} drop out in eq. $(24)_2$; this result is valid also for the general hypo-elastic equation (7) [15]. Thus

$$w(X_M, t) = \int_0^t \frac{\varrho_0}{\varrho} F_{mn} \frac{\mathcal{D} t_{mn}}{\mathcal{D} \tau} \mathcal{D}\tau = \frac{1}{2\mu} \int_0^t \frac{\varrho_0}{\varrho} \frac{\mathcal{D}}{\mathcal{D}\tau} \left(\frac{1}{2} t_{mn} t_{mn} - \frac{\lambda}{3\lambda + 2\mu} (t_{kk})^2 \right) \mathcal{D}\tau . \qquad (25)$$

If one ignores the density change $(\varrho \approx \varrho_0)$ it follows immediately that w does depend in this special case only on the initial and final state of stress.

Integration of the equation of mass conservation ($D\varrho/Dt = -\varrho d_{kk}$) following the motion of the particle (X_M = const.) and applying

$$d_{kk} = D_{kkmn}\,\overset{\circ}{t}_{mn} = G_{mn}\,\frac{\mathcal{D}t_{mn}}{\mathcal{D}t}\,, \qquad G_{mn} = D_{kkmn} \tag{26}$$

yields

$$\frac{\varrho_0}{\varrho} = \exp\left\{\int_0^t G_{mn}\,\frac{\mathcal{D}t_{mn}}{\mathcal{D}\tau}\,\mathcal{D}\tau\right\} \tag{27}$$

With eq. (23) it then follows from eq. (27) that the density ϱ does depends only on the initial and final hydrostatic pressure p

$$\frac{\varrho_0}{\varrho} = \exp\left\{\frac{3}{3\lambda+2\mu}\,(p_0 - p)\right\}\,, \qquad p = -\tfrac{1}{3}t_{kk}\,. \tag{28}$$

The line integral eq. (25) in the stress space is independent of the the path of integration if a total differential exists. Then a hypo-elastic potential φ exists satisfying the overdetermined system of differential equations

$$\frac{\partial\varphi}{\partial t_{mn}} = \frac{\varrho_0}{\varrho}F_{mn}\,.$$

This system has a unique solution if the integrability conditions (see eq. (15))

$$\frac{\partial\,\varrho_0/\varrho\,F_{mn}}{\partial t_{kl}} - \frac{\partial\,\varrho_0/\varrho\,F_{kl}}{\partial t_{mn}} = 0$$

are identically satisfied in t_{rs}. With eq. (27) this gives

$$F_{mn}G_{kl} - F_{kl}G_{mn} + \frac{\partial F_{mn}}{\partial t_{kl}} - \frac{\partial F_{kl}}{\partial t_{mn}} = 0 \tag{29}$$

Applying this condition to the hypo-elastic material defined by (eq. 23) yields the following reduced condition

$$t_{mn}\delta_{kl} - t_{kl}\delta_{mn} = 0 \tag{30}$$

The contracted product with δ_{kl} gives

$$t_{mn} = \tfrac{1}{3}t_{kk}\delta_{mn}\,.$$

Thus the integrability conditon is identically satisfied only for the special situation of a purely hydrostatic state of stress. This result shows that the deformation energy w is generally depending on the history of stress. Consequently the specific work will be non-zero in a closed deformation process with the same initial and final state of stress.

In answering the question raised at the end of chapter (2) we follow Bernstein and Ericksen (16) and consider two histories of stress in the time interval $0 \le \tau \le t$

(a) $t_{kl}(\tau) = r_{kl}(\tau)$, (b) $t_{kl}(\tau) = s_{kl}(\tau) = r_{kl}(t-\tau)$

Where $t_{kl} = r_{kl}$ are arbitrary prescribed functions. Case (a) and (b) correspond to the two directions an open path in stress-space may be passed. Let the point $r_{kl}(0)$ in stress space be denoted by number 1 and the point $r_{kl}(t)$ by number 2. Then the deformation energies per unit mass in case (a) and (b) are given by w_{12}/ϱ_1 and w_{21}/ϱ_2 respectively where ϱ_1 and ϱ_2 are the densities corresponding to state 1 and 2. Since here eq. (25) is simply a line integral in stress space it follows then easily from this property that

$$w_{12}/\varrho_1 = -\,w_{21}/\varrho_2 \tag{31}$$

i.e. the deformation energy per unit mass changes its sign if the stress path is traversed in the opposite direction.

We consider now a stress history where the initial and final state of stress are the same

$$t_{kl}(\tau) = r_{kl}(\tau) \qquad , \qquad r_{kl}(0) = r_{kl}(t) \ . \tag{32}$$

It has been proved above that for a closed stress path the deformation energy is generally non-zero. Assume that w is positive for the stress history given by eq. (32). Then it follows immediately from eq. (31) that any reversal of the stress history such that in the time interval $0 \le \tau \le t$

$$t_{kl}(\tau) = s_{kl}(\tau) \quad , \quad s_{kl}(\tau) = r_{kl}(t-\tau) \quad , \quad s_{kl}(0) = s_{kl}(t)$$

will simply reverse the sign of w; thus the deformation energy per unit initial volume will be negative. This result should be compared to standard constitutive models showing dissipative effects like Newtonian fluids and plastic deformation of metals: In these cases the constitutive equations are structured in such a way that $t_{kl}d_{kl}$ and thus w is positive or non-negative for any closed stress history. How the behavior of the hypo-elastic material is to be interpreted under due consideration of thermodynamic principles needs further investigation.

4. Examples

Consider an infinitesimal material element. During the motion of this particle we assume that the deformation gradient $x_{k,K}(X_M,t)$ at this particle is given by

$$(x_{k,K}) = \begin{bmatrix} f(t) & h(t) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

where $f(t)$ and $h(t)$ are piecewise smooth functions with initial conditions $f(0) = 1$, $h(0) = 0$. The appropriate choices of $f(t)$ and $h(t)$ define different deformation histories. The deformation rate and spin are given by

$$\begin{aligned} d_{11} &= \dot{f}/f \ , \qquad d_{12} = d_{21} = \tfrac{1}{2}(\dot{h} - \dot{f}/f\, h) \\ \omega_{12} &= -\omega_{21} = \tfrac{1}{2}(\dot{h} - \dot{f}/f\, h) \ , \end{aligned} \tag{34}$$

the other components vanish. The restricted deformation eq. (33) corresponds to a simultaneous stretching in x_1-direction and shearing in the $x_1 - x_2$ plane. The relation between stress and deformation gradient can now be obtained by integration of eq. (3) following the motion of the particle i.e. keeping Lagrangian coordinates constant. The hypo-elastic eq. (3) then reduces to a system of ordinary differential equations for the four non-vanishing stress components

$$\begin{aligned} \frac{Dt_{11}}{Dt} &= (2\mu + \lambda)\,\dot{f}/f + t_{12}(\dot{h} - \dot{f}/f\, h) \\ \frac{Dt_{22}}{Dt} &= \lambda\,\dot{f}/f - t_{12}(\dot{h} - \dot{f}/f\, h) \\ \frac{Dt_{33}}{Dt} &= \lambda\,\dot{f}/f \\ \frac{Dt_{12}}{Dt} &= \tfrac{1}{2}[2\mu - (t_{11} - t_{22})](\dot{h} - \dot{f}/f\, h) \ . \end{aligned} \tag{35}$$

Three different deformation histories are considered; the corresponding paths of deformation are illustrated in Fig. 1:

Case I: $0 \le \tau \le t^*$: Simple stretching in x_1-direction
$f = f(\tau)$, $k \equiv 0$, followed by
$t^* \le \tau \le t$: Shear deformation at constant stretch
$f = f^* = const.$, $k = k(\tau)$, $k(t^*) = 0$, $k(t) = k^*$

Case II: $0 \le \tau \le t^*$: Simple shear, $f \equiv 1$, $k = k(\tau)$, $k(0) = 0$, $k(t^*) = k^*$, followed by
$t^* \le \tau \le t$: stretching in x_1-direction
$f = f(\tau)$, $f(t^*) = 1$, $f(t) = f^*$, $k = k^* = const.$

Case III: Simultaneous stretching and shearing with f and k linear in time.

In case I and II the integration has to be performed in a piecewise manner. At $\tau = 0$ vanishing stresses are assumed. The following result showing the two stress components t_{11} and t_{12} at time t demonstrate very clearly the path dependence of the stress:

$$t_{11}/\mu = \begin{cases} 1 + (1+\frac{\lambda}{\mu})\ln f^* - (1-\ln f^*)\cos k^* & \text{Case I:} \\ 1 + (1+\frac{\lambda}{\mu})\ln f^* - \cos(k^*(1-\ln f^*)) + \frac{1}{k^*}\sin(k^*\ln f^*) & \text{Case II:} \\ 1 + (1+\frac{\lambda}{\mu})\ln f^* - \cos(k^*\frac{\ln f^*}{f^*-1}) + \frac{f^*-1}{k^*}\sin(k^*\frac{\ln f^*}{f^*-1}) & \text{Case III:} \end{cases}$$

$$t_{12}/\mu = \begin{cases} (1-\ln f^*)\sin k^* & \text{Case I:} \\ \sin(k^*(1-\ln f^*)) - \frac{1}{k^*}(1-\cos(k^*\ln f^*)) & \text{Case II:} \\ \sin(k^*\frac{\ln f^*}{f^*-1}) - \frac{f^*-1}{k^*}(1-\cos(k^*\frac{\ln f^*}{f^*-1})) & \text{Case III:} \end{cases} \quad (36)$$

A close evaluation of case I and II (see eq. (36))reveales that in each of the two phases - stretching and shearing - the specific form of the function $f = f(\tau)$ and $k = k(\tau)$ in the appropriate time interval has no influence on the final state of stress. Thus for simple stretching we have

$$k \equiv 0 \quad : \quad t_{11}/\mu = (2+\frac{\lambda}{\mu})\ln f \quad , \quad t_{12} = 0$$

and in case of shear we find

$$f \equiv 1 \quad : \quad t_{12}/\mu = \sin k \quad , \quad t_{11}/\mu = 1 - \cos k .$$

At first sight the functions eq. (36) look very different. For f^* approaching 1 and k^* approaching 0 the three cases will become indistinguishable. In Table 1 and 2 a numerical evaluation of the formulars eq. (36) is given. It is seen that the relative difference between the three values of t_{11}/μ is significant only in a region where the logarithmic strain $\varepsilon = \ln f$ as well as the shear angle $\Theta = tg^{-1}k$ is fairly large; the relative difference between the values of t_{12}/μ are considerable at large strains ε over the whole range of shear angles considered. The calculation of the elastic range of coldworked stainless steel Type 316 at room temperature shows that only the upper left corner of the tables is at yield. Thus the region where path dependence is becoming significant for the examples presented is clearly in a range where plastic deformations occur and will dominate the deformation behavior. Finally in Fig. 2 the shear stress is shown as a function of the shear angle for a shear deformation without stretch in the x_1-direction ($k \neq 0$, $f \equiv 1$). This figure demonstrates the phenomenon of hypo-elastic yield at a shear angle of Θ=57,5°; this phenomenon has been found by Truesdell [14] for a more complex hypo-elastic material. It should be noted that this instability does occur at a shear stress equal to the shear modulus μ, i.e. far beyond the load carrying capacity of steel.

5. Conclusions

The analysis presented has shown that the constitutive equation describing elastic material behavior as proposed for use in several accident analysis codes [1-4] actually defines a hypo-elastic material of grade zero. By use of Bernstein's test it has been proved that the stress-strain relation is generally path dependent except for simple extension or compression. A numerical evaluation of the stress-deformation relation for three different paths of deformation obtained by integration has been performed; these results indicate that significant path-dependence does occur only when reaching extremely high tensile and/or shear stresses.

The phenomenon of hypo-elastic yield has been shown to occur for this material; however this type of instability is far beyond the load carrying capacity of steel. Further it has been proved that generally the deformation energy per unit mass is path-dependent. Thus for a stress path with the same initial and final state of stress the specific energy will be generally non-zero; it will be either positive or negative according to the direction the stress path is run through. This result needs further analysis under due consideration of thermodynamic aspects.

It is expected that the deficiencies of this special hypo-elastic material as a model for elastic behavior is of minor importance in an elastic-plastic constitutive equation. However, presently it cannot be excluded that this hypo-elastic model may possibly lead to significant deviations from elasticity for special situations e.g. problems of stability or cyclic straining where small deviations may accumulate.

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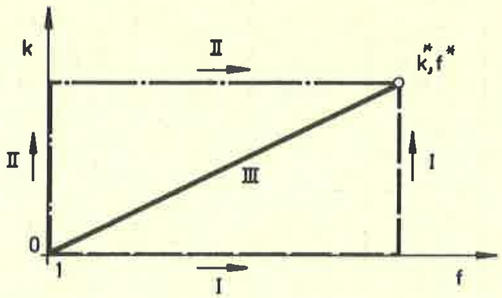

Fig. 1 Path of deformation according to case I, II, and III

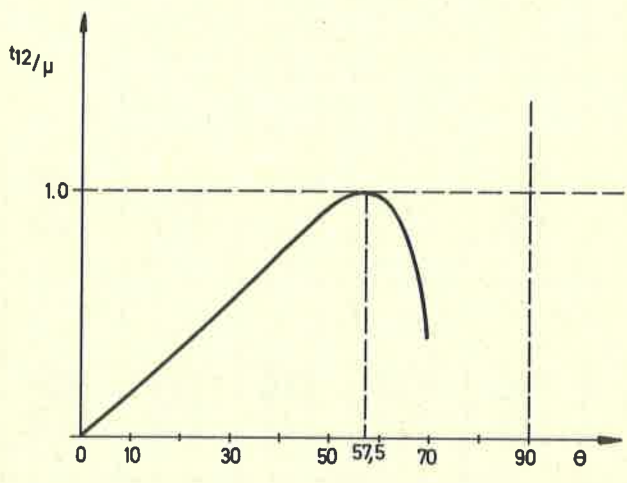

Fig. 2 Shear stress vs. shear angle $\Theta = tg^{-1} k$; simple shear deformation of a Jaumann hypo-elastic material of grade zero; hypo-elastic yield at $\Theta = 57{,}5°$

Table 1. Final tensile stress t_{11}/μ for three different paths of deformation and various stretches and shear angles $\Theta = tg^{-1}k$

ε=ln f / θ (°)	10^{-4}	10^{-3}	10^{-2}	5.0 10^{-2}	10^{-1}	5.0 10^{-1}
I II 0.1 III	3.515228 10^{-4} 3.515227 " 3.515228 "	3.501521 10^{-3} 3.501519 " 3.501520 "	3.500150 10^{-2} 3.500148 " 3.500149 "	1.750014 10^{-1} 1.750013 " 1.750014 "	3.500013 10^{-1} 3.500011 " 3.500012 "	1.750000 1.750000 1.750000
1.0	5.023204 10^{-4} 5.023052 " 5.023154 "	3.652182 10^{-3} 3.652030 " 3.652132 "	3.515080 10^{-2} 3.514929 " 3.515031 "	1.751447 10^{-1} 1.751374 " 1.751424 "	3.501370 10^{-1} 3.501232 " 3.501330 "	1.750076 1.750031 1.750075
15.0	3.603055 10^{-2} 3.602702 " 3.602938 "	3.914843 10^{-2} 3.911321 " 3.913675 "	7.032727 10^{-2} 6.997815 " 7.021281 "	2.088999 10^{-1} 2.072222 " 2.083795 "	3.821156 10^{-1} 3.789250 " 3.812015 "	1.767842 1.757466 1.767703
30.0	1.624220 10^{-1} 1.624067 " 1.624170 "	1.654261 10^{-1} 1.652732 " 1.653761 "	1.954673 10^{-1} 1.939509 " 1.949776 "	3.289837 10^{-1} 3.216764 " 3.267617 "	4.958793 10^{-1} 4.819341 " 4.919867 "	1.831044 1.784462 1.831046
45.0	4.600017 10^{-1} 4.599635 " 4.599898 "	4.627380 10^{-1} 4.623565 " 4.626187 "	4.901007 10^{-1} 4.863100 " 4.889339 "	6.117128 10^{-1} 5.932960 " 6.064475 "	7.637278 10^{-1} 7.282234 " 7.545933 "	1.979848 1.851842 1.984555
60.0	1.160790 1.160736 1.160778	1.162896 1.162347 1.162772	1.183951 1.178437 1.182752	1.277529 1.249520 1.272433	1.394501 1.337550 1.386566	2.330278 2.041943 2.380529

Table 2. Final shear stress t_{12}/μ for three different paths of deformation and various stretches and shear angles $\theta = tg^{-1}k$

$\varepsilon = \ln f$ / $\theta(^{o})$	10^{-4}	10^{-3}	10^{-2}	$5.0\ 10^{-2}$	10^{-1}	$5.0\ 10^{-1}$
I II 0.1 III	1.745156 10^{-3} 1.745156 " 1.745156 "	1.743585 10^{-3} 1.743584 " 1.743585 "	1.727877 10^{-3} 1.727790 " 1.727935 "	1.658064 10^{-3} 1.655882 " 1.659509 "	1.570797 10^{-3} 1.562071 " 1.576542 "	8.726651 10^{-3} 6.544990 10^{-4} 1.008906 10^{-3}
1.0	1.745243 10^{-2} 1.745243 " 1.745243 "	1.743672 10^{-2} 1.743672 " 1.743673 "	1.727964 10^{-2} 1.727878 " 1.728022 "	1.658147 10^{-2} 1.655973 " 1.659595 "	1.570876 10^{-2} 1.562164 " 1.576627 "	8.727089 10^{-3} 6.545552 " 1.008972 10^{-2}
15.0	2.647279 10^{-1} 2.647285 " 2.647281 "	2.644896 10^{-1} 2.644958 " 2.644921 "	2.621068 10^{-1} 2.621561 " 2.621392 "	2.515167 10^{-1} 2.514767 " 2.518471 "	2.382789 10^{-1} 2.374840 " 2.393577 "	1.323772 10^{-1} 1.001306 " 1.536091 "
30.0	5.457510 10^{-1} 5.457572 " 5.457534 "	5.452598 10^{-1} 5.453215 " 5.452831 "	5.403476 10^{-1} 5.409300 " 5.403943 "	5.185153 10^{-1} 5.206715 " 5.200487 "	4.912251 10^{-1} 4.936603 " 4.950378 "	2.729028 10^{-1} 2.130134 " 3.210257 "
45.0	8.413868 10^{-1} 8.414169 " 8.413980 "	8.406295 10^{-1} 8.409298 " 8.407413 "	8.330563 10^{-1} 8.359760 " 8.341856 "	7.993974 10^{-1} 8.121658 " 8.053105 "	7.573239 10^{-1} 7.783311 " 7.698338 "	4.207355 10^{-1} 3.570081 " 5.133365 "
60.0	9.869279 10^{-1} 9.870544 " 9.869735 "	9.860396 10^{-1} 9.873024 " 9.864954 "	9.771564 10^{-1} 9.895728 " 9.816933 "	9.376753 10^{-1} 9.950511 " 9.599136 "	8.883240 10^{-1} 9.912900 " 9.317696 "	4.935133 10^{-1} 5.584515 " 6.852916 "