Nonlinear free vibrations of shallow cylindrical shells at elevated temperature

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1 INTRODUCTION

Modern structures are often subjected to severe vibrations and high temperatures. In such cases vibrations and thermal stresses play an important and even a primary role. Linear and nonlinear analysis for plate stuctures have been examined by several authors but problems in shell analysis are very little compared to the present need in modern stuctures. Moreover, the nonlinear vibrations of shell stuctures at elevated temperature, perhaps, are yet to be examined.

The aim of the present paper is to investigate the behaviour of shallow cylindrical shells vibrating at large amplitudes including thermal effect. Though the present project of the author includes the nonlinear analysis under a general thermal boundary condition, the present paper concerns with a spesific temperature distribution. The analysis is based on Karman-Tsien field equations (1941) extended to thermal loading.

2 GOVERNING DIFFERENTIAL EQUATIONS

Consider a cylindrical shell of radius 'a', thickness 'h' and of length 'l'. The coordinates of any point P on the shell may be taken as (x,y) refered to an origin at an arbitrary point of one of the edges of the shell, x measuring along the axial and y that along the circumferential direction in the median surface of the undeformed shell. Besides a temperature distribution $\Theta(x,y,z)$, z measuring along the radial direction, the shell may be subjected to a constant load 'q'. Assume further that large vibrations take place principally perpendicular to the median surface and that the heat flux is not so high so as to change the material properties of the shell.

The dynamic Karman-Tsien field equations extended to the case including thermal effect can be expressed as

$$\Lambda' \equiv \nabla^4 F + (1/h)\,\nabla^2 N_T = E\left[w_{,xy}^2 - w_{,xx}\,w_{,yy} - (1/a)\,w_{,xx}\right] \qquad (1)$$

$$\Lambda'' \equiv D\,\nabla^4 w + \nabla^2 M_T/(1-\nu) = h\left[w_{,xx}F_{,yy} - 2F_{,xy}w_{,xy} + w_{,yy}F_{,xx} + (1/a)\,F_{,xx}\right] + q - \rho h\,(\partial^2 w/\partial t^2) \qquad (2)$$

where F is the stress function, is the mass density of the shell material, $D = Eh^3/12(1 - \nu^2)$, the flexural rigidity, E is the modulus of elasticity, w is the deflection along the transverse direction, is Poisson's ratio, t the time, $\nabla^2 \equiv (\ldots)_{,xx} + (\ldots)_{,yy}$ and

$$N_T = E\,\alpha_T \int_{-h/2}^{h/2} \Theta(x,y,z)\, dz$$

$$M_T = E\,\alpha_T \int_{-h/2}^{h/2} \Theta(x,y,z,)\, z\, dz \tag{3}$$

where α_T = the coefficient of linear thermal expansion of the material. Comma (,) notation denotes partial differentiation.

3 METHOD OF SOLUTION

As suggested by Nowinski (1963) the deflection function may be chosen in the separable form

$$w = f(t) \sin\alpha_i x \sin\beta_j y + f_o(t) \tag{4}$$

where $\alpha_i = i\pi/l$, $\beta_j = j/a$, and i,j representing the number of longitudinal half waves and the circumferential waves respectively. The boundary conditions imposed on w suggest that the local bending moments at the edges of the shell vanish identicall and conform to the conditions of the free support of the edges of the shell in an integral sense.

Consider a temperature distribution of the form

$$\Theta(x,y,z) = T \sin\alpha_i x \sin\beta_j y \tag{5}$$

where $\Theta = T_1 - T_2$ vanishes along the edges of the shell. Using equations (1), (4) and(5) a particular integral for the stress function may be obtained as

$$F^*(x,y,t) = (C_1 \cos 2\alpha_i x + C_2 \cos 2\beta_j y) f^2 + C_3 f \sin\alpha_i x \sin\beta_j y - (qa/h)\, x^2/2 \tag{6}$$

where

$$C_1 = (E\beta_j^2/32\alpha_i^2), \quad C_2 = E(\alpha_i^2/32\beta_j^2),$$

$$C_3 = E\left[(\alpha_i^2/a) + (\alpha_T T_o/f)(\alpha_i^2 + \beta_j^2)\right] / (\alpha_i^2 + \beta_j^2)^2 \tag{7}$$

f(t) anf $f_o(t)$ are unknown functions of time in equation (4) but $f_o(t)$ can be expressed in terms of f(t) by using the closure condition

$$\int_0^{2\pi a} (\partial v/\partial y)\, dy = 0 \tag{8}$$

where, v is the displacement along the y direction and

$$(\partial v/\partial y) = [F^*_{,yy} - \nu F^*_{,xx} - (E/2) w^2_{,y} + E\alpha_T(N_T/h)] /E \qquad (9)$$

From equations (5) and (8) it can be easily seen that

$$f_o(t) = (\beta_j^2 \; a/8) \; f^2(t) + (qa^2/Eh) \qquad (10)$$

Finally, the Galerkin procedure is applied to equation (2) and is represented in the integral form

$$\int_S \Lambda'' \, w \, dx \, dy = 0 \qquad (11)$$

where 'S' is the domain of integration over the median surface of the shell. A trivial but a lengthy calculation yields the following time differential equation

$$(d^2f/dt^2) + d_1 \; f + d_2f^3 + d_3 = 0 \qquad (12)$$

where

$$d_1 = \frac{E}{\rho a^2}\left[\frac{s^2}{12}\frac{(\lambda^2+j^2)^2}{(1-\nu^2)} + \frac{\lambda^4}{(\lambda^2+j^2)^2} + j^2\frac{qa}{Eh}\right]$$

$$d_2 = (E/\rho a^2) \; s^2 \; (\lambda^4+j^4)/16$$

$$d_3 = (E/\rho a^2) \; T^*h \quad \lambda^2/(\lambda^2+j^2)$$

$$= (i\pi \, a/l) \quad , \qquad s = (h/a)$$

$$T^*_o = (\alpha_T \, T_o \, a/h) \qquad (13)$$

4 FREE NONLINEAR VIBRATIONS

Since the primary object is to investigate the effect of temperature on the frequency one may restrict the proceedings by letting $q = 0$. Equation (12) then reduces to the well-known DUFFING's equation. For large vibrations with one degree of freedom one can introduce

$$f(t) = A \; g(t) \qquad (14)$$

which permits the use of normalised conditions

$$g'(0) = 0, \; g(0) = 1 \; . \qquad (15)$$

Further, when the steady state has been reached, that is, the frequency of vibration of the shell is equal to the frequency of the pulsation of the thermal load T^*_o one can put

$$T^* = T^{**} \cos\omega \, t \qquad (16)$$

The approximate solutions are then given by

$$g(t) = A \cos\omega t + \frac{d_2 \, (A/h)^3}{p} \cos 3 \; t$$

$$p = 32 \left[d_1 + \tfrac{3}{4} d_2 \, (A/h)^2 - d_3 /(A/h) \right] \qquad (17)$$

while the relative frequency ratio of the large and small vibrations is given by

$$(\omega / \omega_0) = \left[1 + \tfrac{3}{4} (d_2/d_1) (A/h)^2 - (d_3/d_1) / (A/h)\right]^{\frac{1}{2}}$$

5 OBSERVATION

From Figure 2 it can be seen that the effect of the temperature parameter T* is significant for lesser values of i and j for lower degree of temperature.

Unstable motion occurs in cases for higher temperature as well as for higher values of i and j.

More the number of longitudinal half waves and circumferential waves greater is the possibility of the jumping phenomenon.

For lesser values of i and j the ratio (ω / ω_0) practically remains constant however the large amplitude may be.

REFERENCES

von Karman, T. & Tsien, H.S. 1941. The buckling of thin cylindrical shells under axial compression. J. Aero/Space Sci.,8,303-312.

Nowinski, J.L. 1963. Nonlinear transverse vibrations of orthotropic cylindrical shells. AIAA Journal, 1, 617-620.

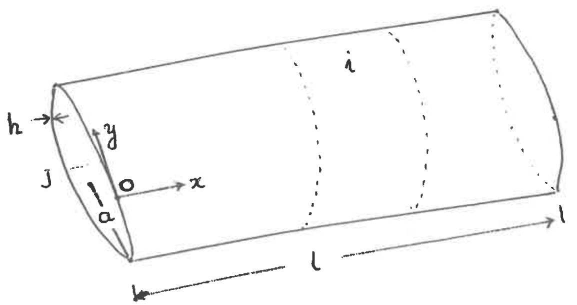

Figure 1. Dimensions of the cylindrical shell

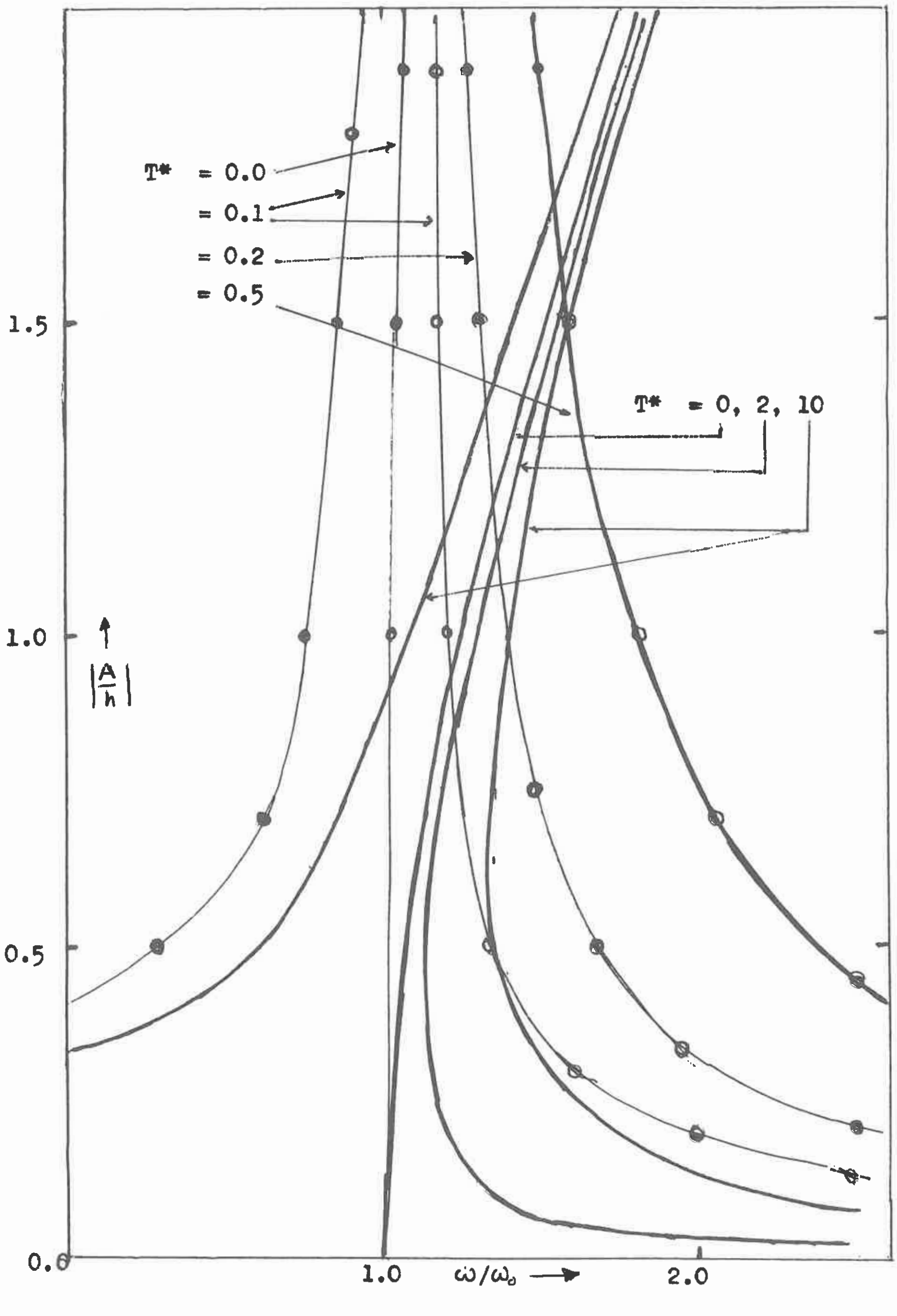

Figure 2. Frequency ratio (ω /ω_0) vs relative amplitude (A/h).
—⊖—⊖—⊖— (h =2, j = 4), ———————— (h = 40, j = 10).