Quantifying of Ductile Fracture Damage Evolution by Homogenization Approach

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Abstract

The purpose of this paper is to outline a procedure of the formulation of damage evolution equation as coupled with inelastic deformation and accounting for some microstructural features of a metallic material. The deterioration mechanism by void growth from spherical inclusions is studied and novel numerical results are given in terms of second order damage-tensor representation.

The damage tensor is introduced as an entity pertaining to damage zone presence within a representative sample subject to homogeneous finite deformation of its outer faces. In the paper emphasis is made on the onset of deterioration process. It is being represented in terms of "small damage" tensor associated through the micro-macro transition with the infinitesimal macrostrain measure.

1. Introduction

An adequate modelling of damage evolution is essential for the Plasticity-Damage Coupled Theory that may provide reasonable assessments of ductility limitations in deformed solid. The objective of this paper is to give a quantitative representation of the damage evolution based on micromechanical data (sect. 3). This representation is articulated in terms of a tensor variable to account for oriented character of damage. We consider an elastic-plastic metal having periodic structure resulting from some regular inclusion spacing. A representative sample ("unit cell") may be defined in its initial configuration Γ_0 as an element comprising a single, hard inclusion centered within. Voids are supposed to grow from hard inclusions in the course of advanced straining. In a current Γ_t- configuration a representative element is thus viewed as a sort of a composite made up of quasi-homogeneous ductile matrix and of a damage zone in a form of an oriented cavity or a crack-like defect around the particle. The attribute "macroscopic" refers to quantities applied to unit-cell faces and representing the porous aggregate behaviour. They may constitute, according to the postulate by HILL |1|, unweighted volume averages of their microscopic counterparts. The latter are pointwise quantities within a representative sample. Denoting by x_i and X_J the current and reference positions of a material point, the macroscopic deformation gradient $x_{i,K} = F_{i,K}$ can be introduced as a basic kinematical macrovariable ; as such it will be designated with a bar over : ${}_{(mac)}F_{iK} = \bar{F}_{iK}$.

For a continuous x_i field within a sample (no damage zone) we have

$$\bar{F}_{iK} = \{F_{iK}\} \ , \quad \{F_{iK}\} = \frac{1}{V}\int_V F_{iK}\, dV = \frac{1}{V}\int_{S_{ext}} x_i N_K\, dS \qquad (1)$$

where the symbol { } designates a volume average. The expression on the extreme right is the surface expression of $\{\underset{\sim}{F}\}$ and thus of $\underset{\sim}{\bar{F}}$, obtained through the divergence theorem. $N_K\, dS = dS_K$ represents vector area of the external surface of the element. The volume of the element is V in Γ_o and v in Γ_t ; $dv = J\, dV$. It is assumed in (1) that the macroelement is subject to homogeneous boundary conditions and that the macrohomogeneity hypothesis holds

$$\bar{x}_i\Big|_{S_{ext}} = \bar{F}_{iK} X_K \quad , \quad \oint_{S_{ext}} (\bar{x}_i - x_i)\, dS_K = 0 \qquad (2)$$

2. Damage variables

The macro-micro decomposition for $\underset{\sim}{F}$ can be completed in the case of existence of damage zone within the sample. It is done by means of regular extension of the continuously derivable x_i field to the interior of a cavity (in the same manner as it was done by GURSON |2| for the microscopic velocity field v_i in a similar context). The extended field is equal to the matrix x_i field at the cavity surface. By transforming void volumetric integral into a surface one, we obtain

$$\bar{F}_{iJ} = \frac{1}{V}\left|\int_{V_m} F_{iJ}\, dV + \int_{S'} \overset{*}{x}_i\, dS'_J\right| = \{F_{iJ}\}_{V_m} + f_{\Gamma_o}\,\delta_{iJ} + \Psi_{iJ} \qquad (3)$$

with

$$\Psi_{iJ} = \frac{1}{V}\int_{S'} b_i\, dS'_J \ , \quad {}_{(cav)}x_k = \overset{*}{x}_k\Big|_{S'} = X_I\,\delta_{Ik} + b_k \ , \text{ and } f_{\Gamma_o} = \frac{V_{incl}}{V} \qquad (4)$$

where b_k is the discontinuity displacement vector from S' (inclusion surface) and δ_{Ik} represent shiffer's components. The subscript "m" accompanying V (and later v) indicates the matrix volume in a sample in Γ_o (and in Γ_t).

Based on typical boundary conditions (2) in terms of actual velocity v_i with reference to initial Γ_o or actual Γ_t- configuration we obtain analogously to (3), (4)

$$\dot{\bar{F}}_{iJ} = \{\dot{F}_{iJ}\}_{V_m} + \dot{\Psi}_{iJ} \quad ; \quad \dot{\Psi}_{iJ} = \frac{1}{V}\int_{S'} \dot{b}_i\, dS'_J \qquad dS'_J \text{ in } \Gamma_o \qquad (5)$$

$$\bar{l}_{ij} = \bar{v}_{i,j} = \{l_{ij}\}_{v_m} + \dot{\psi}_{ij} \quad ; \quad \dot{\psi}_{ij} = \frac{1}{v}\int_{s_{cav}} \dot{b}_i\, ds'_j \ ; \ ds'_j \text{ in } \Gamma_t \qquad (6)$$

$$ds'_j = ds_j^{(cav)}$$

In the key macro-micro transition formulae (3), (5), (6) the tensors Ψ_{iJ} and ψ_{ij} represent respectively the two-point (mixed) and Eulerian quantities relative to damage zone existence. The two may be chosen as basic "finite damage" characteristics and eventually as damage internal variables in plasticity-damage coupled theory, DRAGON |3|, DRAGON and CHIHAB |4|. A particular small-damage approximation discussed in |4| indicates infinitesimal microdeterioration linked to small deformation on a macroscale. This situation of common convergence of the deformation term and the damage one is certainly no generally valid. It might be however a useful approximation for the case of incipient damage by void growth while the matrix itself is plastically incompressible, i.e. for most of metallic materials at relatively early stage of plastic deterioration. The following decomposition equivalent to (5) and (6) is then applied

$$\dot{\varepsilon}_{ij} = \{\dot{\varepsilon}_{ij}\}_{V_m} + \dot{\phi}_{ij} \qquad \text{if} \qquad \dot{\underset{\sim}{F}} \overset{\sim}{-} \underset{\sim}{\bar{1}} = \dot{\underset{\sim}{\bar{\varepsilon}}}\,,\ \underset{\sim}{F} = \underset{\sim}{1}$$

$$\dot{\phi}_{ij} = \frac{1}{V}\int_{S'} \dot{b}_i N_j \, dS' \qquad \text{and } \dot{\underset{\sim}{\Psi}} \overset{\sim}{-} \dot{\underset{\sim}{\psi}} = \dot{\underset{\sim}{\phi}}\,,\ \underset{\sim}{\Psi} = \underset{\sim}{\psi} = \underset{\sim}{\phi} \tag{7}$$

Note that the integral in $(7)_2$ is taken over the initial cavity surface (i.e. that of an inclusion) as in (5) and not on à priori unknown cavity surface as in (6). In fact (7) is a reasonable approximation for both as long as incipient damage evolution is considered and the difference between the curent cavity form and its initial configuration can be neglected. From this point of view analogies with employement of small strain theory on undeformed body geometry are evident. The decomposition (7) makes appear the damage variable $\underset{\sim}{\phi}$ introduced in its integral form by VALULENKO and KACHANOV jr |5| and employed by DRAGON |6| and DRAGON and MRÓZ |7| to the microcracking-kinetics description in plastic/brittle solids. In fact in (7) we have a symmetric part of $\underset{\sim}{\phi}$, i.e. $\phi_{ij} = \phi_{(ij)}$. In the following we omit the brackets and we will be dealing with a symmetric part only.

3. Damage Evolution Equation Based on Micromechanical Data

The procedure that we propose in order to settle up the damage evolution equation is based on micromechanical solution providing the local velocity field v_i within an elementary sample. The latter contains an inner cavity and is subject to all around displacement boundary conditions compatible with homogeneous deformation of its outer faces. Suppose that such a solution can be found in the form

$$v_j = v_j\,(\dot{\underset{\sim}{\bar{\varepsilon}}}, \underset{\sim}{x}, k, \ldots) \tag{8}$$

where k is a parameter characterizing the sample. The relation with damage tensor rate is immediately set as $\dot{b}_i$ itself represents a local velocity vector on the cavity surface :

$$\dot{b}_i = v_i\big|_{S'} = \overset{*}{v}_i = \dot{\overset{*}{x}}_i \quad \Rightarrow \quad \dot{b}_i = \overset{*}{v}_i\,(\dot{\underset{\sim}{\bar{\varepsilon}}}, \overset{*}{\underset{\sim}{x}}, k, \ldots) \tag{9}$$

We deal with incipient damage kinetics, so we will employ $\underset{\sim}{\phi}$ from eqns. (7) as a damage variable. The representative sample being large compared with a cavity, we state

$$0 < (b_i\, b_i)^{1/2} \ll c - a \tag{10}$$

with 2c being the inclusion spacing distance and so the one relative to characteristic dimension of the sample (c is half-length of the edge of a cubic cell) ; a is the radius of a spherical inclusion. The vector-position $\overset{*}{x}_i$ on the cavity surface can be expressed by the following

$$\overset{*}{x}_i = x^\circ_i + b_i \tag{11}$$

as in the second eq. (4). In eq. (11) we discard the X_I and put x°_i as the initial position on discontinuity (inclusion) surface to stress infinitesimal damage context. Finally, we will integrate the expression of the type

$$\dot{b}_i = \overset{*}{v}_i\,(\dot{\underset{\sim}{\bar{\varepsilon}}}, x^\circ_i + b_i, k, \ldots) \tag{12}$$

over the inclusion surface according to eq. (7). The procedure makes emerge integrals of dyadic expressions $b_i N_j$, so the components of ϕ_{ij} itself. We obtain the kinetics equation in the form

$$\dot{\phi}_{ij} = \dot{\phi}_{ij}\,(\dot{\underset{\sim}{\bar{\varepsilon}}}, \underset{\sim}{\phi}, k) \tag{13}$$

A linearisation procedure discussed in DRAGON, CHIHAB |4| is to be carried out during integration of (12) in the infinitesimal damage context.

4. Results

We consider two forms of velocity field v_i for an isotropic unit sample ; they are as follows :

$$v_i = \dot{\bar{\varepsilon}}'_{ij} x_j + \frac{\dot{\bar{\varepsilon}}_{kk}}{3} \frac{c^3 x_i}{(x_m x_m)^{3/2}} \tag{14}$$

$$\hat{v}_i = \dot{\bar{\varepsilon}}'_{ij} x_j + \frac{\dot{\bar{\varepsilon}}_{kk}}{3} \frac{c^3 x_i}{(x_m x_m)^{3/2}} + 3\,\dot{\bar{\varepsilon}}'_{ij} \left(\frac{a}{r}\right)^6 \left[\frac{3}{2} - 2\left(\frac{a}{r}\right)^3\right] x_j + 9\left(\frac{a}{r}\right)^6 \left[\left(\frac{a}{r}\right)^3 - 1\right] \frac{x_i}{r^2} \dot{\bar{\varepsilon}}'_{kl} x_k x_l \tag{15}$$

where $r = (x_j x_j)^{1/2}$ and $\dot{\bar{\varepsilon}}'_{ij}$ represent the components of macrostrain deviator. The form (14) is based on RICE-TRACEY's |8| and GURSON's |2| analyses where only the second, spherically symmetric void-expansion term perturbs the trivial homogeneous solution $\dot{\bar{\varepsilon}}_{ij} x_j$. The more involved solution (15) yields the following kinetics equation

$$\bigcirc \quad \dot{\phi}_{ij} = \frac{\phi_{ij} + k\,\delta_{ij}}{\phi_{ll} + k} \frac{\dot{\bar{\varepsilon}}_{pp}}{3} + (\dot{\bar{\varepsilon}}'_{ik} \phi_{kj} + k\,\dot{\bar{\varepsilon}}'_{ij}) \left[1 - \frac{3}{2} \frac{k - 3\phi_{kk}}{\phi_{ll} + k} \left(\frac{k}{\phi_{rr} + k}\right)^2\right] \; ; \qquad i, j = 1, 2, 3 \tag{16}$$

In (16) a particular, hexagonal-like distribution of spherical inclusions is implied (the symbol ◯) ; $k = \frac{a^3}{c^3}$ represents a factor linked to inclusion size and spacing. Relatively simple step-by-step calculations based on (16) for prescribed macrostrain-path permit to trace cavity profile for a given strain/damage configuration. At the beginning, initial ϕ°_{ij} components are to be given in the zoroth step. They are small compared with a typical increment in the step-by-step process and are chosen in the manner to simulate physically reasonable void formation (decohesion) mechanism. In Fig. 1, three-dimensional proportional straining is being considered associated with damage represented by principal components of $\underset{\sim}{\phi}$. The formula (16) was employed with the void formation mechanism ϕ°_{ij} assumed coaxial with $\bar{\varepsilon}_{ij}$. The dotted-line "primitive theory" curves represent damage tensor components relevant to the kinetics equation based on (14) and discussed in DRAGON, |3|.

The two-dimensional versions of (13) concerning damage by cavity growth from cylindrical inclusion (in a plane perpendicular to a cylinder) are given below. They concern the hexagonal-like (○) and the cubic-like (□) inclusion arrays :

$$\bigcirc \quad \dot{\phi}_{ij} = \frac{\phi_{ij} + k\,\delta_{ij}}{\phi_{rr} + k} \frac{\dot{\bar{\varepsilon}}_{pp}}{2} + (\dot{\bar{\varepsilon}}'_{ik} \phi_{kj} + k\,\dot{\bar{\varepsilon}}'_{ij}) \left[1 - 2\left(\frac{k}{\phi_{rr} + k}\right)^3\right] \; ; \; i, j = 1, 2 \qquad k = \frac{a^2}{c^2} \tag{17}$$

$$\square \quad \dot{\phi}_{ij} = \frac{\phi_{ij} + \frac{\pi}{4} k\,\delta_{ij}}{\frac{4}{\pi}\phi_{rr} + k} \frac{\dot{\bar{\varepsilon}}_{pp}}{2} + (\dot{\bar{\varepsilon}}'_{ik} \phi_{kj} + \frac{\pi}{4} k\,\dot{\bar{\varepsilon}}'_{ij}) \left[1 - 2\left(\frac{k}{\frac{4}{\pi}\phi_{rr} + k}\right)^3\right] \tag{18}$$

The results obtained from (18) for uniaxial strain path are given in fig. 2 (solid line). They show a tendency to slightly oblate growth of a cavity with respect to $\bar{\varepsilon}_{(1)}$ axis initially. For similar results obtained from (17), see DRAGON and CHIHAB, |4|. The ϕ_2- values based on a simple velocity field i.e. on the two-dimensional version of (14) (the dotted line) are now inferior to these from (18) and do not indicate tendency to oblate growth. The influence of inclusion size and spacing ratio a/c is showed in Fig. 3 by employing eq. (17) for a/c = 0.1 and a/c = 0.2 for the strain path α = 0.

5. Conclusion

Affinity of damage tensorial variables to macroscopic finite deformation tensors is emphasized. The procedure leading to damage evolution equation based on micromechanical data has served here as guidance for obtaining quantitative damage growth relations (16), (17), (18). Assuming that $\dot{\underset{\sim}{\varepsilon}}$ is total elastoplastic strain rate supposes that a damage internal variable $\underset{\sim}{\phi}$ is coupled with elastic as well as plastic behaviour. The results presented here on damage evolution modelling constitute a step in view of formulation of the Plasticity-Damage Coupled Theory.

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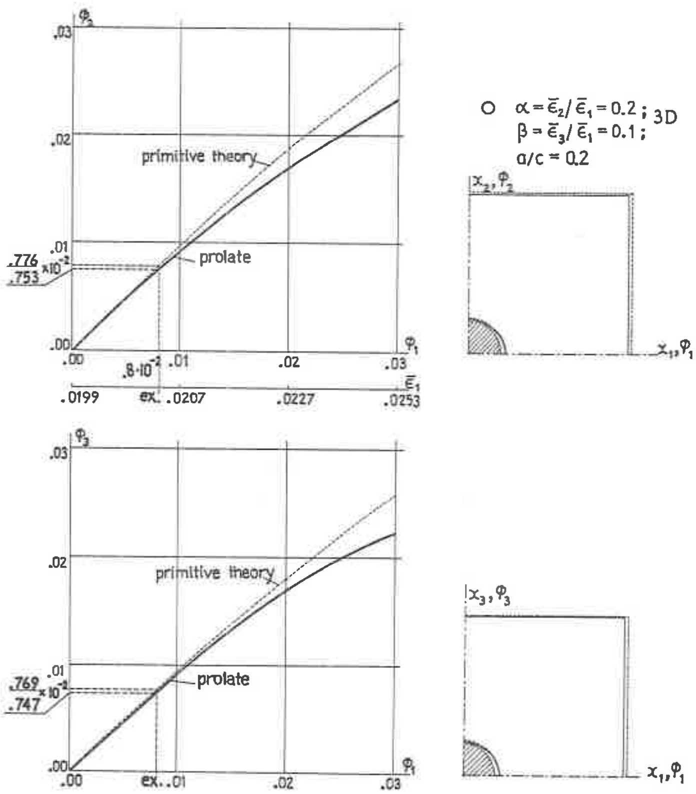

Fig. 1 Damage by void growth from spherical inclusion ; representation in terms of damage tensor φ.

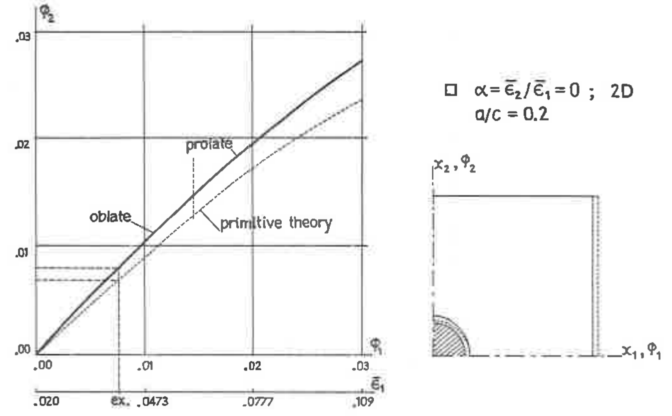

Fig. 2 Damage by void growth from cylindrical inclusion ; representation in terms of damage tensor φ. Transitory effect of oblate growth of a cavity is shown for the $\tilde{v}$-based theory (solid line).

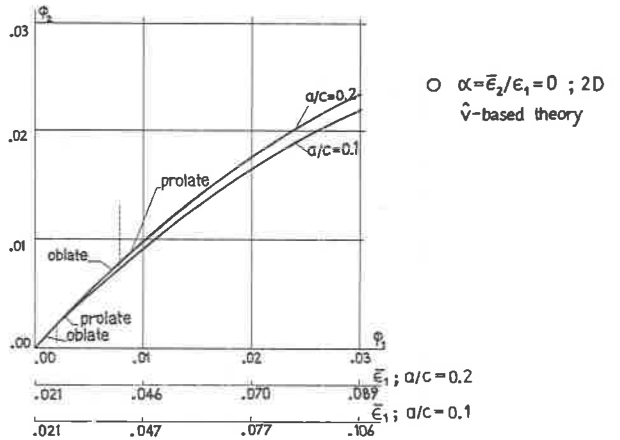

Fig. 3 Damage by void growth from cylindrical inclusion ; representation in terms of damage tensor φ. Influence of inclusion size and spacing ratio a/c.