Seismic Response Analysis for a Containment of Nuclear Power Plant in Consideration of Soil Structure Interaction

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1. INTRODUCTION

In this paper the substructure method is applied. The super-structure of system is analyzed by a semi analytical curved ring shell FEM[1]. The displacements in shell around the circumference direction are expanded in Fourier series. Based on the theory of thin shells, the computational formulas for FEM have been derived. This method has some advantages, such as simplicity of one dimensional division, high precision of results and stress field in whole structure may be shown in detail. An approximate analytical method[2] is applied to the coupling vibration of translation and rocking of an embedded foundation. The seismic response of a containment has been obtained in time domain in a numerical example and SSI effects have been discussed.

2. SEMI ANALYTICAL FEM FOR A REVOLUTION SHELL

The displacements on the middle surface of a revolution shell coordinate and a general cylindrical coordinate system are expressed by u, v, w and u_r, v, u_z, respectively (Fig. 1). Under any load, the displacements in the circumference can be expanded in Fourier series as follows:

$$u_r = \sum_{n=0}^{\infty} u_{rn}(s)\cos n\theta \qquad u_z = \sum_{n=0}^{\infty} u_{zn}(s)\cos n\theta \qquad v = \sum_{n=0}^{\infty} v_n(s)\sin n\theta \tag{1}$$

The deflection angle β and the strain ε_φ in meridian direction are follows

$$\beta = \sin\varphi \frac{\partial u_r}{\partial s} - \cos\varphi \frac{\partial u_z}{\partial s} = \sum_{n=0}^{\infty} (\sin\varphi \frac{du_{rn}}{ds} - \cos\varphi \frac{du_{zn}}{ds})\cos n\theta = \sum_{n=0}^{\infty} \beta_n \cos n\theta$$

$$\varepsilon_\varphi = \cos\varphi \frac{\partial u_r}{\partial s} + \sin\varphi \frac{\partial u_z}{\partial s} = \sum_{n=0}^{\infty} (\cos\varphi \frac{du_{rn}}{ds} + \sin\varphi \frac{du_{zn}}{ds})\cos n\theta = \sum_{n=0}^{\infty} \varepsilon_{\varphi n} \cos n\theta \tag{2}$$

In the above expansions the coefficients are functions of s only. Now,we adopted the interpolation functions of displacement as

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$$u_{rn}=\alpha_1+\alpha_2\xi+\alpha_3\xi^2+\alpha_4\xi^3 \qquad u_{zn}=\alpha_5+\alpha_6\xi+\alpha_7\xi^2+\alpha_8\xi^3 \qquad v_n=\alpha_9+\alpha_{10}\xi \tag{3}$$

in which ξ=s/L. Choosing u_{rn}, u_{zn}, v_n, β_n, $\varepsilon_{\varphi n}$ as nodal freedom degrees and substituting the nodal conditions where ξ=0, ξ=1 in eqs.(3), we have

$$\{\alpha\} = [\alpha_1\ \alpha_2\ \dots\ \alpha_{10}]^T = [A]_{10\times10}\{\delta^e\}_{10\times1} \tag{4}$$

in which, $\{\delta^e\}=\left[u_{rn_i}\ u_{zn_i}\ v_{n_i}\ \beta_{n_i}\ \varepsilon_{\varphi n_i}\ \middle|\ u_{rn_{i+1}}\ u_{zn_{i+1}}\ v_{n_{i+1}}\ \beta_{n_{i+1}}\ \varepsilon_{\varphi n_{i+1}}\right]^T$ is called element nodal displacement vector. The relation between element displacement and nodal freedom degrees is

$$\{d_n\} = [u_{rn}\ u_{zn}\ v_n]^T = [N]\{\delta^e\} \tag{5}$$

in which [N] is the displacement matrix and the components of it are given by

$$[N] = \begin{bmatrix} 1-3\xi^2+2\xi^3 & 0 & 0 & L\sin\varphi_i(\xi-2\xi^2+\xi^3) & L\cos\varphi_i(\xi-2\xi^2+\xi^3) \\ 0 & 1-3\xi^2+2\xi^3 & 0 & L\cos\varphi_i(-\xi+2\xi^2-\xi^3) & L\sin\varphi_i(\xi-2\xi^2+\xi^3) \\ 0 & 0 & 1-\xi & 0 & 0 \end{bmatrix.$$
$$\begin{bmatrix.} 3\xi^2-2\xi^3 & 0 & 0 & L\sin\varphi_{i+1}(-\xi^2+\xi^3) & L\cos\varphi_{i+1}(-\xi^2+\xi^3) \\ 0 & 3\xi^2-2\xi^3 & 0 & L\cos\varphi_{i+1}(\xi^2-\xi^3) & L\sin\varphi_{i+1}(-\xi^2+\xi^3) \\ 0 & 0 & \xi & 0 & 0 \end{bmatrix} \tag{6}$$

The relations between strain or stress of middle surface element and nodal freedom degrees are

$$\{\varepsilon\}=\left[\ \varepsilon_{\varphi n}\ \varepsilon_{\theta n}\ \gamma_{\varphi\theta n}\ \chi_{\varphi n}\ \chi_{\theta n}\ \tau_n\ \right]^T = [B]_{6\times10}\{\delta^e\} \tag{7}$$

$$\{\sigma\}=\left[\ N_{\varphi n}\ N_{\theta n}\ N_{\varphi\theta n}\ M_{\varphi n}\ M_{\theta n}\ M_{\varphi\theta n}\ \right]=[D]_{6\times6}[B]_{6\times10}\{\delta^e\}=[S]_{6\times10}\{\delta^e\} \tag{8}$$

in which [B], [D], [S] are the geometrical matrix, the elastic matrix and the stress matrix respectively.

Applying the principle of virtual work, the stiffness matrix and mass matrix of an element can be obtained

$$\left[K^e\right]_{10\times10}= \int_0^1 Lr[B]^T[D][B]d\xi \qquad \left[M^e\right]_{10\times10}= \int_0^1 \rho Lhr[N]^T[N]d\xi \tag{9}$$

Analogous to displacements the external loads might be expanded as

$$q_{Ur} = \sum_{n=0}^{\infty} q_{Urn}\cos n\theta \qquad q_{Uz} = \sum_{n=0}^{\infty} q_{Uzn}\cos n\theta \qquad q_V = \sum_{n=0}^{\infty} q_{Vn}\sin n\theta \tag{10}$$

When the nth component is considered, according to the principle of virtual work the equivalent nodal force vector is obtained

$$\left\{p^e\right\} = \left\{\begin{matrix} p_i^e \\ p_{i+1}^e \end{matrix}\right\} = \left\{\begin{matrix} \int_0^1 Lr[N]^T \left\{\begin{matrix} q_{Uzn}(s) \\ q_{Urn}(s) \\ q_{Vn}\ (s) \end{matrix}\right. & \text{where q is surface loads} \\ \int_0^1 Lrh[N]^T \left\{\begin{matrix} q_{Uzn}(s) \\ q_{Urn}(s) \\ q_{Vn}\ (s) \end{matrix}\right. & \text{where b is body loads} \end{matrix}\right. \tag{11}$$

The global stiffness matrix, the global mass matrix and the global force vector can be obtained in proper order to assemble the contributions of all element matrices.

3. IMPEDANCE OF THE FOUNDATION

The impedance of an embedded foundation consists of two parts, one of them is induced by under soil and the other by side soil of a foundation.

The relations of impeding horizontal force P, moment M, which the soil exerts on the embedded cylindrical foundation, and the displacements of the foundation u_0, φ_0 are presented from reference[2].

$$\left\{\begin{matrix} P \\ M \end{matrix}\right\} = \begin{bmatrix} K_H + i\omega C_H & K_{H\varphi} + i\omega C_{H\varphi} \\ K_{H\varphi} + i\omega C_{H\varphi} & K_\varphi + i\omega C_\varphi \end{bmatrix} \left\{\begin{matrix} u_0 \\ \varphi_0 \end{matrix}\right\} = \left\{ \begin{bmatrix} K_H & K_{H\varphi} \\ K_{H\varphi} & K_H \end{bmatrix} + i\omega \begin{bmatrix} C_H & C_{H\varphi} \\ C_{H\varphi} & C_H \end{bmatrix} \right\} \left\{\begin{matrix} u_0 \\ \varphi_0 \end{matrix}\right\} \tag{12}$$

4. SEISMIC SOIL STRUCTURE INTERACTION FOR CONTAINMENT

4.1 Establishment for Motion Equations

When seismic ground motion takes place on a site which the containment is situated in, the displacement of any point on the containment is composed of rigid horizontal displacement $\bar{u}_0 = u_g + u_0 + (H-Z)\varphi_0$ and the elastic displacement u_r, v, u_z relative to foundation (Fig. 2). Let us suppose that the containment is divided into NE elements, the quantities of the nodal freedom degrees are N=5NE+5. Besides, adding the translation u_0 and rocking angle φ_0 the foundation which are caused by interaction, the unknown quantity of the system are N+2. According to D'Alembert's principle the N+2 equations of motion can be correspondingly established as follows:

$$[M]\{\ddot{\delta}\} + \{M_u\}\ddot{u}_0 + \{M_\varphi\}\ddot{\varphi}_0 + [C]\{\dot{\delta}\} + [K]\{\delta\} = -\{M_u\}\ddot{u}_g \tag{13}$$

$$\{M_u\}^T\{\ddot{\delta}\} + \left(M_{ss} + \frac{Mb}{\pi}\right)\ddot{u}_0 + M_{sg}\ddot{\varphi}_0 + \frac{p(t)}{\pi} = -\left(M_{ss} + \frac{Mb}{\pi}\right)\ddot{u}_g \tag{14}$$

$$\left\{M_{\varphi}\right\}^T\left\{\ddot{\delta}\right\} + M_{sg}\ddot{u}_0 + \left(M_{gg} + \frac{I_{\varphi}^b + I_{\varphi}}{\pi}\right)\ddot{\varphi}_0 + \frac{M(t)}{\pi} = -M_{sg}\ddot{u}_g \tag{15}$$

In the above eqs, [M], [C] and [K] are N×N mass matrix, damping matrix and stiffness matrix of a superstructure, respectively. $\{\delta\}$ is N×I nodal relative displacement vector of a superstructure to its foundation. u_0, φ_0 and $\ddot{u}_g$ are translation, rocking angle of a foundation and acceleration of ground, respectively, P(t), M(t) are determined by eqs.(12). M_b I^b_{φ} are foundation mass and mass moment of inertia with respect to its own horizontal axis, respectively.

$I_{\varphi} = \sum_{i=1}^{NE} I_{\varphi}^{(i)}$, $I_{\varphi}^{(i)} = \pi\int_0^1 \rho hLr^3 d\xi$ is mass moment of inertia of the element with respect to its own horizontal axis.

As the space is forbidden, here the expressions for M_u, M_{φ}, M_{ss} ··· etc. are not given.

The seismic response motion equations in matrix form in consideration of SSI of a containment can be written as follows:

$$\begin{bmatrix} [M] & \{M_u\} & \{M_{\varphi}\} \\ \{M_u\}^T & M_{ss} + \frac{M_b}{\pi} & M_{sq} \\ \{M_{\varphi}\}^T & M_{sq} & M_{qq} + \frac{I_{\varphi}^b + I_{\varphi}}{\pi} \end{bmatrix} \begin{Bmatrix} \{\ddot{\delta}\} \\ \ddot{u}_0 \\ \ddot{\varphi}_0 \end{Bmatrix} + \begin{bmatrix} [C] & \{0\} & \{0\} \\ \{0\}^T & \frac{C_H}{\pi} & \frac{C_{H\varphi}}{\pi} \\ \{0\}^T & \frac{C_{\varphi H}}{\pi} & \frac{C_{\varphi}}{\pi} \end{bmatrix} \begin{Bmatrix} \{\dot{\delta}\} \\ \dot{u}_0 \\ \dot{\varphi}_0 \end{Bmatrix}$$

$$+ \begin{bmatrix} [K] & \{0\} & \{0\} \\ \{0\}^T & \frac{K_H}{\pi} & \frac{K_{H\varphi}}{\pi} \\ \{0\}^T & \frac{K_{H\varphi}}{\pi} & \frac{K_{\varphi}}{\pi} \end{bmatrix} \begin{Bmatrix} \{\delta\} \\ u \\ \varphi \end{Bmatrix} = -\begin{Bmatrix} \{M_u\} \\ M_{ss} + \frac{M_b}{\pi} \\ M_{sq} \end{Bmatrix} \ddot{u}_g \tag{16}$$

If SSI is not considered, the formula (16) will be reduced to equation (17), just as it is for a rigid foundation.

$$[M]\left\{\ddot{\delta}\right\} + [C]\left\{\dot{\delta}\right\} + [K]\left\{\delta\right\} = -\left\{M_u\right\}\ddot{u}_g \tag{17}$$

4.2 Solution of the motion equations

The equations (16) should be to deal with, when boundary conditions are introduced in them. As functions of impedance are dependent on frequency of excitation, equation (16) have to be solved first in frequency domain and then by inversion to time domain. In terms of reference[3], however, the dynamic response of a structure will be with enough accuracy if impedance functions for any frequency might be replaced by those of fundamental frequency of a system. Therefore equations(16) can be solved direct in time domain, once fundamental frequency of a system has been obtained.

5. NUMERICAL EXAMPLE AND ANALYSIS

Let us consider a containment which is a combination shell to be composed of a cylindrical shell with a height=50m and a semi spherical shell top with a radius=18m. It is made of reinforced concrete with material density =2500Kg/m^3, elasticity modulus=2.7489x10^{10}pa, Poison's ration=0.1667, thickness=1m. It is built on an embedded cylindrical foundation with radius=20m, validly embedded depth=10m. The material density and the Poison's ratio in soil 1875Kg/m^3,and 1/3, respectively. The shear moduli in soil beside and under foundation are 0.6×10^{10}pa and 1.5×10^{8}pa, respectively.

The first five natural frequencies, of horizontal excitation are given in Table 1. It is found that SSI to be considered the natural frequencies of a system are reduced and its effect is most significant for fundamental frequency of a system and it will decrease rapidly with increment of the mode number.

Table 1. Natural Frequencies (rad/s)

Mode number	I	II	III	IV	V
Frequencies ω for rigid soil	23.635	75.113	174.610	281.060	358.880
Frequencies $\bar{\omega}$ for soft soil	12.860	67.053	164.830	286.690	346.560
$\lvert\bar{\omega} - \omega\rvert/\omega\times100\%$	45.59%	10.73%	5.60%	4.40%	3.43%

The NS component of El-Centro earthquake record is selected as an input in this paper. Deformation and stress in containment to a horizontal excitation are shown in Fig. 2, in which u_r, u_z, N_φ, N_θ, M_φ, M_θ, vary as $\cos\theta$ form around the circumference, v, $N_{\varphi\theta}$, $M_{\varphi\theta}$ as $\sin\theta$ form, respectively. The large displacements are produced in semi spherical shell and the maxima stresses are located near bottom of containment and attenuate gradually with height increment. The membrane stresses are the main stresses, bending moments exist in a small area near containment bottom which is caused by edge effects.

Some of quantities versus time are shown from Fig. 3. It can be found that SSI will reduce deformation and stress in a containment, but there are additional rigid displacements generated by a translation and a rocking angle of the foundation, they make total displacements of containment in a increment.

6. CONCLUSIONS

6.1 This analytical and numerical combination method used in this paper may be applicable to analyze dynamic response of various revolution shells, such as hyperbolic cooling towers. It can not only give detailed deformation and stress distribution all over the superstructure to avoid drawback caused by a simplified lumped mass modeling to be selected, but also can show the character of a shell curved surface element more evident than other ring one.

6.2 With depth increment of a foundation embedment SSI effect could be reduced and damping impedance increases evidently, so that the seismic response of a structure will be reduced. The harder the soil, especially the foundation under soil, the less the SSI effect.

6.3 SSI effect displays mainly in reduction of natural frequencies, deformation and stress of a system. The membrane stresses are main stresses and bending moments exist only near the bottom of a containment.

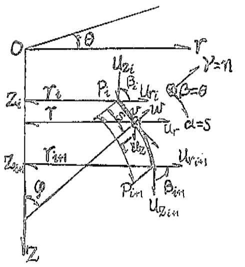
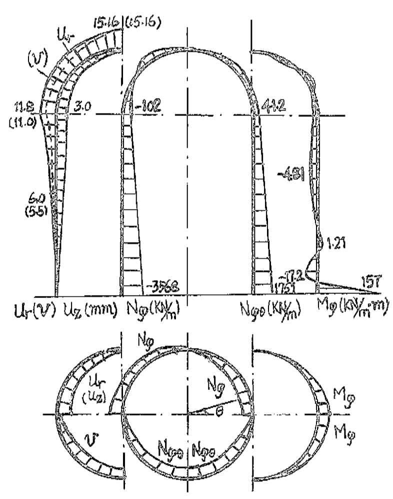


Fig.1 Displacement on the middle surface of a revolution shell element



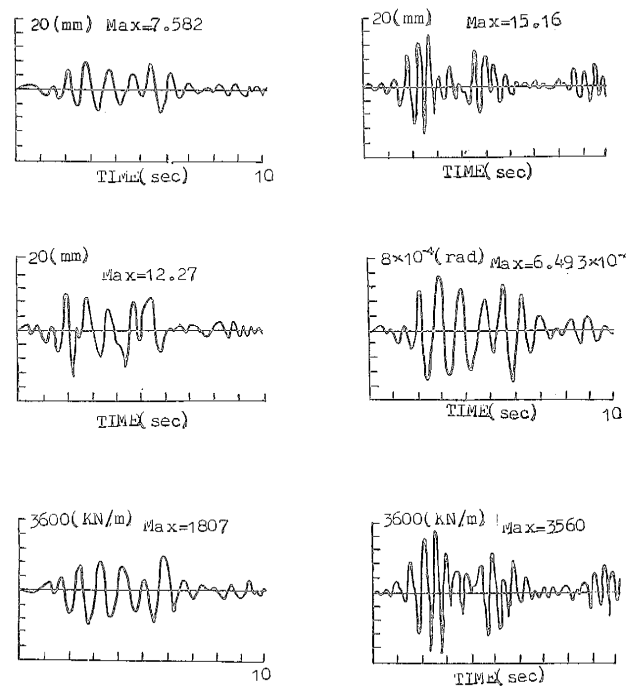


Fig.3 some of quantities versus time

Fig.2 Deformation and stress in containment to a horizontal excitation

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