Elastoplastic and Geometrically Nonlinear Thin Plates Analysis by the Secant Stiffness Diagonal Matrix

A. Mihanovic, A. Munjiza

University Split, Faculty of Civil Engineering, V. Maslese b.b., 58000 Split, Yugoslavia

Summary

The paper describes an iterative method called the method of the secant stiffness diagonal matrix used for the nonlinear analysis of the equilibrium of thin plates. The secant stiffness diagonal matrix is of heuristic origin. Since the stiffness matrix is diagonal the method can efficiently use the memory and the processor of a computer. The obtained results show that this method proves to be more efficient than the other methods when the load is applied at once. In incremental applications of load this method is more efficient in problems with geometrical non-linearity. This method has a special advantage when used on computers with a small memory and particularly on PC, when both the linear and nonlinear parts of a problem are solved by the same iterative procedure.

1. Introduction

A great number of papers related to the field of numerical analysis of nonlinear problems have been published during the last decade. This paper suggests the use of an iterative method for the non-linear analysis of this plates referred to as the method of the secant stiffness diagonal matrix.

Essentially the iterative procedure uses an approximative matrix of secant stiffness with only diagonal coefficients.

The method is compared with a method of initial tangent stiffness given in [1, 4] which is considered most efficient by the authors. The efficiency of the method is shown in Figures 5 and 6. The method was tested on thin plates with straight, curved and arbitrary boundary conditions. The plates were loaded in plane and lateral. The material nonlinearity, geometrical stiffness and large plate displacements were observed. The plate material is homogeneous, isotropic and elastoplastic.

2. Mechanical model of the plate

The deformations and stresses in the plate for the initial stiffness and the elastic material behaviour were adopted as in [2 Ch. 10.1]. The geometrical stiffness was defined as in [2 Ch. 19.3]. The Lagrange formulation of deformations caused by great displacements was used as in [2 Ch. 19.3]. In a non-elastic domain the material exhi-

bits ideal plastic behaviour as in [2 Ch. 18.4]. The criterion of the cross-section plastification was taken according to unit forces at the cross-section in view of [1]. The Robinson's function of the plastic potential F_1 was chosen according to [1].
Actual stresses and forces at a cross-section were determined using a simple heuristic expression

$$\underset{\sim}{\sigma}_{pl} = k \, \underset{\sim}{\sigma}_{el} \tag{1}$$

where k is a constant defined by the ratio

$$k^2 = \frac{F_{pl}}{F_{el}} \tag{2}$$

$F_{pl} < 1$ is a plastic potential, $F_{el} = 1$ is an elastic potential, and $k < 1$. Relations (1) and (2) were tested on examples 4.1 and 4.2, see Figures 2 and 4 showing very good accuracy.

3. Numerical model

The plates were modelled by eight-node semiloof elements with a reduced integration according to [1, 3]. The model equilibrium was established by the virtual work principles as in [2 Ch. 19].

$$\underset{\sim}{\Psi} = \underset{\sim}{p} - \underset{\sim}{f} = \underset{\sim}{0} \tag{3}$$

where

$$\underset{\sim}{p} = \int_V \underset{\sim}{\bar{B}}^T \underset{\sim}{\sigma} \, dv = (\underset{\sim}{K}_0(a) + \underset{\sim}{K}_\sigma(a) + \underset{\sim}{K}_1(a)) \, \underset{\sim}{a} \tag{4}$$

generalized internal forces in nodes dependent on displacement and including the initial stiffness, geometrical stiffness, large displacements and the elastoplastic relation between stresses and strains. The meaning of matrices in relation (4) is the same as in [2 Ch. 19].
The plate loading is applied in increments.

3.1 Iterative method with a diagonal matrix of the secant stiffness

3.1.1 The method for the first increment of loading

The following algorithm was used:

a) Linear displacements $\underset{\sim}{a}^{(1)}$ were computed by the standard frontal methods as in [4, 5]. Subsequently the stresses and nodal forces were calculated.

b) Displacements for the following steps were determined iteratively by means of the following relations

$$\underset{\sim}{a}^{(n+1)} = \underset{\sim}{a}^{(n)} + \Delta \underset{\sim}{a}^{(n)} \tag{5}$$

where

$$\Delta a_i^{(n)} = \frac{a_i^{(n)} - a_i^{(n-1)}}{p_i^{(n)} - p_i^{(n-1)}} \, (f_i - p_i^{(n)}) \tag{6}$$

Nodal and residual forces were determined after each iteration.

Relation (6) is subjected to constraint

$$\text{if} \quad \left|\Delta a_i^{(k)}\right| > \frac{\left|a_i^{(k)}\right|}{\Theta} \quad \text{then} \quad \Delta a_i^{(k)} = \frac{a_i^{(k)}}{\Theta} \tag{7}$$

In examples 4.1 and 4.2 we assumed $\Theta = 3$. The iteration is performed until the required accuracy is obtained. As accuracy criterion the displacement norm is taken

$$\text{DISPLACEMENT NORM} = \text{Max.} \left| \frac{a_i^{(k)} - a_i^{(k-1)}}{a_i^{(k)}} \right| < 0.001 \tag{8}$$

3.1.2 The procedure for other loading increments

For the first step $\underset{\sim}{a}^{(1)}$ the final values from the previous loading increment are taken. Further iteration is performed according to 3.1.1.b.

3.2 The mathematical aspect of the iterative procedure

Equation (6) is of a heuristic type for two or more dimensional vector space. It can be written as

$$\Delta a_i^{(n)} = c_i^{(n)} \Delta a_i^{(n-1)} = c_i^{(n-1)} \Delta a_i^{(n-2)} =$$

$$= c_i^{(1)} c_i^{(2)} \ldots c_i^{(n)} a_i^{(1)} = d_{ii}^{(n)} a_i^{(1)} \tag{9}$$

where

$$c_i^{(n)} = \frac{1}{\dfrac{R_i^{(n-1)}}{R_i^{(n)}} - 1} \quad , \quad R_i^{(n)} = f_i - p_i^{(n)} \tag{10}$$

Now relation (5) can be expressed as

$$\underset{\sim}{a}^{(n+1)} = [\underset{\sim}{I} + \underset{\sim}{D}_1 + \underset{\sim}{D}_2 + \ldots \underset{\sim}{D}_j + \ldots \underset{\sim}{D}_n]\ \underset{\sim}{a}^{(1)} \tag{11}$$

where $\underset{\sim}{I}$ is a unit and $\underset{\sim}{D}_j$ a diagonal matrix.
Relation (11) shows that the problem has been transferred from the m-dimensional vector space into m-onedimensional independent spaces. The condition of the solution convergence is

$$\lim_{n \to \infty} c_i^{(n)} = 0 \tag{12}$$

The results obtained on the practical examples of the plate analysis show that relation (12) has been satisfied, i.e. that the procedure converges. The constraint given by (7) speeds up the convergence.

4. Examples

4.1. Let us take a simply supported circular plate with the following characteristics:

modulus of elasticity $E = 1000$ kN/cm², Poisson's coefficient $\nu = 0.30$, yield stress $\sigma_o = 16$ kN/cm², plate thickness t=5 cm, plate diameter 2R=200 cm.

The plate is subjected to loading in various combinations. Results of the analysis are shown in Figures 1 and 2.

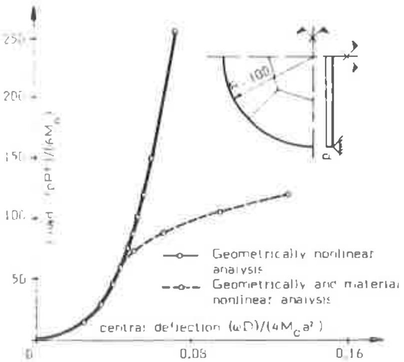

Figure 1. Simply supported circular plate. Geometrically and material nonlinear analysis

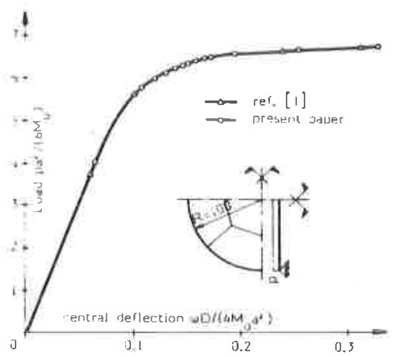

Figure 2. Simply supported circular plate. Material nonlinear analysis

4.2. A square simply supported plate with the following characteristics $E = 4000$ kN/cm², $\nu = 0.30$, $\sigma_o = 57{,}6$ kN/cm², t=2.5 cm, a=100 cm was analyzed.

One quarter of the plate was first modelled by 4 and subsequently by 16 semiloof elements. The results are practically the same and they are presented as one in Figures 3 and 4.

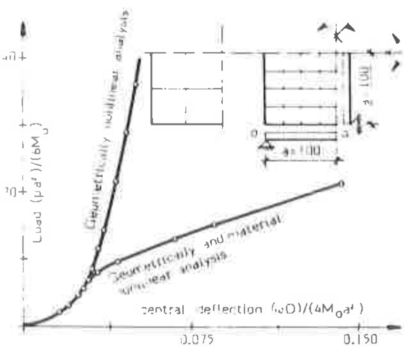

Figure 3. Simply supported square plate. Geometrically and material nonlinear analysis

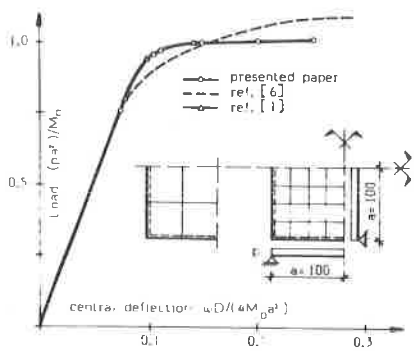

Figure 4. Simply suported square plate Material nonlinear analysis.

5. Comparison with other methods

The efficiency of the given iterative method was studied on example 4.2 modelled by 4 and 16 semiloof elements. The comparison was effected using the method of the initial stiffness matrix given in [4, 5] .
The results of the comparison are presented in Figures 5 and 6.

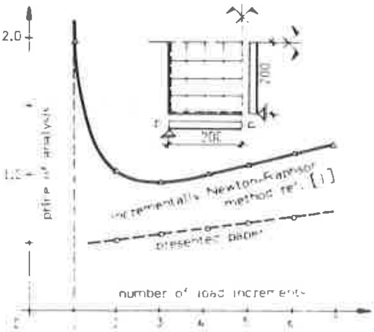

Figure 5. Efficiency of the presented method for geometrical nonlinearity

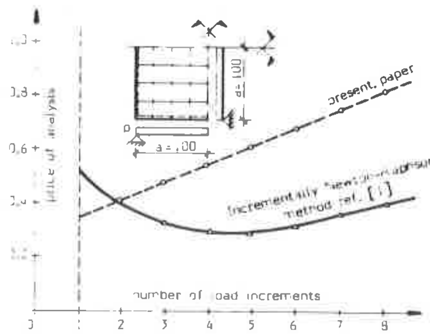

Figure 6. Efficiency of the presented method for the material nonlinearity

6. Conclusion

The described method of the secant stiffness diagonal matrix is used to solve the problems of nonlinear analysis of thin plates. In some cases this method uses the memory and processor of a computer more efficiently than the other methods.

According to the described examples it was shown that this method is significantly more efficient than the other methods for the cases of geometric nonlinearity without regard to the number of loading increments. In case of material nonlinearity the method is more efficient only for loading applied at once. In mixed problems the efficiency of the method will depend on whether the geometrical or material nonlinearity is more stressed.

On of the disadvantages of this method and of other iterative methods according to [7 Ch. 7] is that it is impossible to determine exactly the duration of the analysis.

This method can be also used to obtain a linear solution of the problem. The results obtained on actual examples show that the computation for the linear solution lasts twice as long as the same computation performed by a frontal method as in [4, 5] .
The suggested method is very efficient when we have a computer with a small memory, particularly a PC, as direct methods cannot be applied in such cases.

7. References

1 MARTINS, R.A.F., OWEN, D.R.J., "Elastoplastic and Nonlinear Thin Shell Analysis by Semiloof Element". Computer and Structures, Vol. 13., pp. 505-543, 1981.

2 ZIENKIEWICZ, O.C. - "The Finite Element Method", 3rd edition, McGrow Hill Book Company, London, 1977.

3. IRONS, B., AHMAD, S. "Techniques of Finite Elements" John Wiley & Sons, New York, 1980.

4. OWEN, D.R.J., FIGUEIRAS, J.A., DAMJANIĆ, F., "Finite Element Analysis of Reinforced and Prestressed Concrete Structures including Thermal Loading". Computer Methods in Applied Mechanics and Engineering 41, (1983), 323-366, North Holland

5. MIHANOVIĆ, A., DAMJANIĆ, F., GOTOVAC, B., MAROVIĆ, P., "Stability Analysis of Thin plates and Shells by Semiloof Elements", Proc. 7th Congres SDGK of Yugoslavia, Cavtat, April 1983, Book TE-2, Paper TE-55, 125-132

6 ARMEN, H., PIFKO, A., LEVINE, H.S., "Finite Element Analysis of Structures in the Plastic Range". N.A.S.A. Contractor Rep. CR. 1649, 1971.

7 BATHE, K.J., WILSON, L.W., "Numerical Methods in Finite Element Analysis". Prentice Hall, Inc. Englewood Cliffs, New Jersey, 1976.