SIMPLIFIED METHODS OF INELASTIC ANALYSIS FOR COMPONENTS OPERATING WITHIN THE CREEP RANGE

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SUMMARY

The paper examines simplified methods of inelastic analysis in the creep range with particular reference to variable temperature. For components that operate below the short-term shakedown limit, it is shown that design may be based on elastic, limit and shakedown analysis. The purpose of the shakedown calculation is to ensure that gross structural damage does not occur and that substantial regions of the structure do not distort significantly. The analysis is identical to a low temperature shakedown analysis but the stress is limited not only by the short-term yield stress but also by the time-dependent strength of the material. For steady loading, the shakedown method reduces to the conventional reference stress technique based on limit analysis.

Application of the shakedown method is illustrated by considering a simply supported circular plate subjected to a cyclic lateral pressure and a cyclic radial temperature distribution. The example demonstrates that the shakedown method is often simple to apply and that this is particularly true where the loading is primarily due to thermal effects. The results of the simplified analysis are in good agreement with those of a detailed finite-element solution.

Above the short-term shakedown limit, bounding theorems are presented for predicting the deformation of a component. The behaviour of a structure is described in terms of the behaviour of the same structure composed of material with the same elastic-plastic properties but which does not creep. The total deformation is bounded by the elastic-plastic deformation, the initial conditions and creep strains derived from the elastic-plastic stress history. The bounding theorem is applied to experiments reported in the literature on pressurised pipes subjected to repeated thermal shocks. The upper bound is in good agreement with the experimental results.

1. INTRODUCTION

In the primary design of components two structural modes of failure need to be considered: failure by gross distortion or ratchetting, and failure by creep rupture or fatigue. Recent work has shown that design procedures which ensure that such failures are avoided may be greatly simplified if the loadings are always within a modified shakedown limit. Above this limit, inelastic analysis is necessary but it is shown that some simplification is possible if bounding theorems are used.

2. DESIGN WITHIN A MODIFIED SHAKEDOWN LIMIT

The modified shakedown boundary is simply the shakedown boundary calculated using the short-term yield stress reduced by a factor $0.9n/(n+1)$ where n is the creep deformation index. This is based on the work of Leckie [1] and may be expressed formally as follows: if a time-independent residual stress field, $\underline{\rho}$, can be found such that

$$f\{\hat{\underline{\sigma}}(t) + \underline{\rho}, \theta\} \leq 0.9n\sigma_y(\theta)/(n+1) \text{ at all times } t, \qquad \text{eq. (1)}$$

the total inelastic strains can be bounded by reference stresses calculated without considering plasticity. In this expression $\hat{\underline{\sigma}}$ is the elastically calculated stress distribution, f is the yield function and $\sigma_y(\theta)$ is the yield stress at the temperature θ. The yield stress is not easily defined for a work-hardening material but comparison with experiments on simple structures [2,3] indicates that use of the 0.2% proof stress gives a good measure of the ratchetting/shakedown boundary.

Inequality (1) ensures that there is no significant interaction between short-term plastic deformation and creep deformation. It is then necessary to impose constraints to ensure that creep does not cause excessive deformation or rupture of components. It transpires [1,4,5] that a component of creep ductile material will be satisfactory if a residual stress distribution is found such that at all times,

$$f(\hat{\underline{\sigma}}(t) + \underline{\rho}) \leq \text{lower value of } \{\frac{n\sigma_y(\theta)}{n+1}, \bar{\sigma}(\theta)\} \text{ at time } t \qquad \text{eq. (2)}$$

where σ_y is probably set equal to the 0.2% proof stress

and $\bar{\sigma}$ is either the stress to reach the allowable strain of X% in the lifetime of the component, or the stress to rupture divided by a suitable factor.

Inequality (2) is a shakedown requirement in which the stress is limited by the yield stress σ_y and by the time-dependent stress $\bar{\sigma}$. Consider first the application to the creep rupture of structures of complex geometry subjected to steady load. Normal design factors against plastic collapse on initial loading ensure that inequality (1) is satisfied. Neglecting the safety factors on rupture stress, inequality (2) then predicts the loading P to give failure in time t as

$$P(t) = \sigma_r(t).\{P_{lim}/\sigma_y^*\} \qquad \text{eq. (3)}$$

where $\sigma_r(t)$ is the stress to give rupture in time t in the material of construction at the operating temperature,

and P_{lim} is the limit load of the structure calculated using a rigid plastic material of fictitious yield stress σ_y^*.

There is considerable experimental and theoretical evidence on a wide range of geometries for the use of eqn (3). This has been reviewed elsewhere and the factors of safety required on the rupture stress have been discussed [4,6].

The application of inequality (2) to cyclic loading is considered more fully here. The method is discussed in terms of the typical load cycle of Fig. 1a. The shakedown analysis (2) is shown schematically in Fig. 1b where the elastic stress change may arise from changes in temperature and/or mechanical load. The method is conceptually simple: the creep life is satisfactory if it is possible to demonstrate that shakedown will occur with a limiting stress of $\bar{\sigma}$ during the hot part of the cycle and a limiting stress of $n\sigma_y/(n+1)$ during the cold part of the cycle.

In order to illustrate the process of calculation for cyclic loading it is worth considering an example. The structure chosen is the simply supported circular plate of radius a and thickness h shown in Fig. 2a. This is subjected to a lateral pressure p and a radial temperature variation $\theta(r)$ which vary according to the loading sequence ABCA... shown in Fig. 2a. Creep strain rates are negligible at temperature θ_o so that significant creep only occurs during the hold period B.

The most severely loaded position is at the centre of the plate and the yield conditions are applied to the stresses at the upper and lower surfaces of the plate at this position. The allowable stress at temperature θ_o is denoted σ_y^c and that at the peak temperature θ^* as σ_y^h with $\beta = \sigma_y^h/\sigma_y^c$. For elastic behaviour, the pressure p induces equal hoop and radial stresses on the upper and lower surfaces of the plate of

$$\sigma = \pm 3(3+\nu)pa^2/8h^2 = \pm\ 1.9P$$

where stress has been normalised by σ_y^c, $P = 2pa^2/3h^2\sigma_y^c$ and Poisson's ratio ν has been taken as $\nu = 0.3$. The temperature field induces non-dimensional stresses at the centre of the plate equal to

$$\sigma = -\phi$$

on both upper and lower surfaces where $\phi = E\alpha\Delta T/3\sigma_y^c$. Here E is Young's modulus and α is the coefficient of thermal expansion.

For the shakedown solution a residual stress field σ_R proportional to the thermal stress field is assumed so that the stresses at the centre are $\sigma_R = \rho\phi$. This is superposed for loading B on the stress distribution for a rigid-plastic solution with uniform hoop stress $\sigma_\theta = \pm P$. Since hoop and radial stress components are the same at the centre of the plate the constraints to be applied are (inequality (2), Fig. 1b)

$$\text{at } \theta_o, \quad -1 \le \sigma \le 1 \;; \qquad \text{at } \theta^*, \quad -\beta \le \sigma \le \beta.$$

Applying these for the load sequence ABCA...

	Upper Surface	Lower Surface
B	$-\beta \le -P + \rho\phi \le \beta$	$-\beta \le P + \rho\phi \le \beta$
C	$-1 \le -P + (1+\rho)\phi \le 1$	$-1 \le P + (1+\rho)\phi \le 1$
A	$-1 \le 0.9P + (1+\rho)\phi \le 1$	$-1 \le -0.9P + (1+\rho)\phi \le 1$

For $P \geq 0$ and $\phi \geq 0$ these reduce to

$$P \leq \beta, \qquad 2P + \phi \leq 1 + \beta \qquad \text{eq. (4)}$$

The shakedown region defined by inequalities (4) is shown in Fig. 2b. The example demonstrates that the shakedown analysis can be simple to apply. This is particularly true for thermal loading where the thermal stress profile, necessarily self-equilibrating, provides a ready residual stress field which can be factored for use in the shakedown analysis. If σ_y^c is identified as $n\sigma_y/(n+1)$ at temperature θ_o and σ_y^h as the stress to X% creep strain in time t at θ^*, Fig. 2b defines a loading regime within which the membrane creep strain in the plate is less than X% in time t. Alternatively, for any given loading (P,ϕ) the shakedown analysis provides a reference stress which is that value of σ_y^h (i.e. that β) for which equality holds in one of (4); the creep strain prediction is then simply the creep strain at stress σ_y^h and temperature θ^* in time t.

In order to assess the creep strain predictions of the above analysis, a full elastic-plastic-creep analysis of the plate has been performed using the finite-element system BERSAFE [7]. The finite-element calculations have been obtained using a simple secondary creep law and a perfect plasticity model. Details of the finite-element results and comparison with the predictions of the shakedown method will be presented at the conference.

3. DESIGN ABOVE THE MODIFIED SHAKEDOWN LIMIT

In designing above the shakedown limit full inelastic computer solutions are normally required. This is the approach recommended by ASME Code Case N-47 which is the only design code that specifically addresses the problem of designing liquid metal breeder reactors. Recent work has shown that some simplification in design is possible above the shakedown limit but an elastic-plastic analysis is normally a minimum requirement. Ainsworth [8] has derived work and deformation bounds for the region beyond the short-term shakedown limit which apply to the steady cyclic state condition. These results may be extended to include the transitional region to the steady cyclic state and bound the total deformation [9]. A particular form of the bound used below is [9]

$$u(T) \leq u^*(T) - \tfrac{1}{2}u^*(o) + (1/nR)\int_o^T \int_V \dot{D}\,\{n\underline{\sigma}^*/(n+1)\}\,dV\,dt \qquad \text{eq. (5)}$$

where the actual displacement u(T) at time t in the direction of a constant additional dummy load R is bounded by the displacement u^* and stresses $\underline{\sigma}^*$ which occur in the structure subject to the actual load/temperature history plus the dummy load R. This fictitious structure has the same elastic-plastic properties as the material of the actual structure but is assumed not to creep. $\dot{D}(\underline{\sigma})$ is the product of stress tensor $\underline{\sigma}$ and the creep strain rate tensor at that stress.

Application of the bounding method is here illustrated by comparing the predictions with results reported by Corum et al. [10] of tests on straight sections of type 304 stainless steel pipe. The pipe was subjected to thermal shocks followed by dwell periods under internal pressure at 1100°F, as shown schematically in Fig. 3a, consisting of thirteen nominally identical cycles with a peak pressure of 700 psi and a hold time of 160 hr. The temperature profiles resulting from the thermal shocks are presented in Fig. 7 of [10] and show an approximately parabolic variation of temperature with radius. The peak temperature difference between inside and outside walls measured experimentally was 175°F. The pipe was thin-walled with a ratio of mean radius to thickness of 10.8. For the purposes of analysis, the pipe has been represented by an uniaxial model used by Bree [11] and others.

To calculate the upper bound a constant additional pressure R has been applied so that inequality [5] bounds the hoop strain by the calculated displacement u^* in the uniaxial model and by creep strains resulting from the calculated stress history σ^*. The value of additional pressure which minimized the bound after the first cycle was found to be about 20% of the applied pressure. However, the bound is not particularly sensitive to R and varies by less than 20% for R between 10% and 50% of the peak pressure. The elastic-plastic solution has been obtained using material properties taken from Corum [12] with, where necessary, mean values over the temperature range being used. Kinematic hardening behaviour has been assumed with the tenth cycle data of [12] used for all cycles except the first when first cycle data was used. Creep strains have been obtained directly from the data presented by Corum [12]. Further details of the elastic-plastic analysis are given in [9].

The upper bound (5) is compared with the experimental results [10] in Fig. 3. It can be seen that the upper bound is in very good agreement with experiment. It should be noted that the upper bound has no allowance for the effect on material properties of any creep-plastic interactions. However, a full analysis of the problem [13] using the same material data [12] has shown almost exact agreement with experiment. This suggests that any creep-plastic interactions have a negligible effect on the overall deformation in the present case and that use of the independently obtained plastic and creep data is justified. In general, however, the possibility of creep-plastic interactions should be considered when using the upper bound predictions as, indeed, they should be considered in a full analysis.

4. CONCLUSIONS

Below a modified shakedown limit reference stress techniques may be used to describe the deformation and failure of structures operating in the creep range. The approach requires a shakedown analysis similar to that used below the creep range. Beyond the modified shakedown limit, the approach is more complicated and some form of inelastic analysis is required. Some simplification is possible using bounding methods which still require inelastic analysis to determine the elastic-plastic response but which eliminate detailed analysis of the time-dependent creep deformation.

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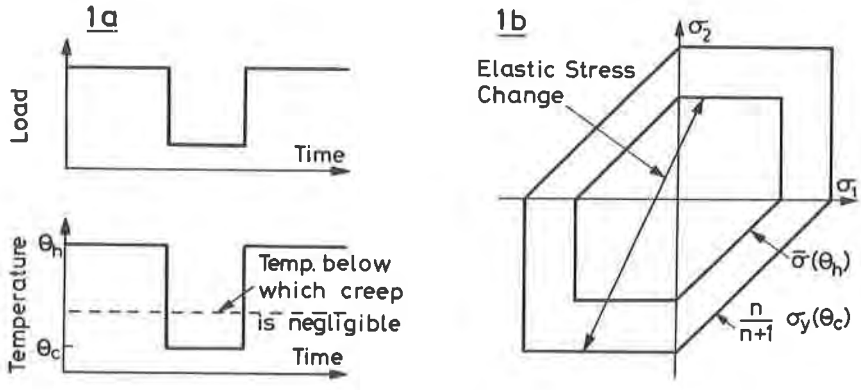

FIG. 1. Schematic of Shakedown Analysis

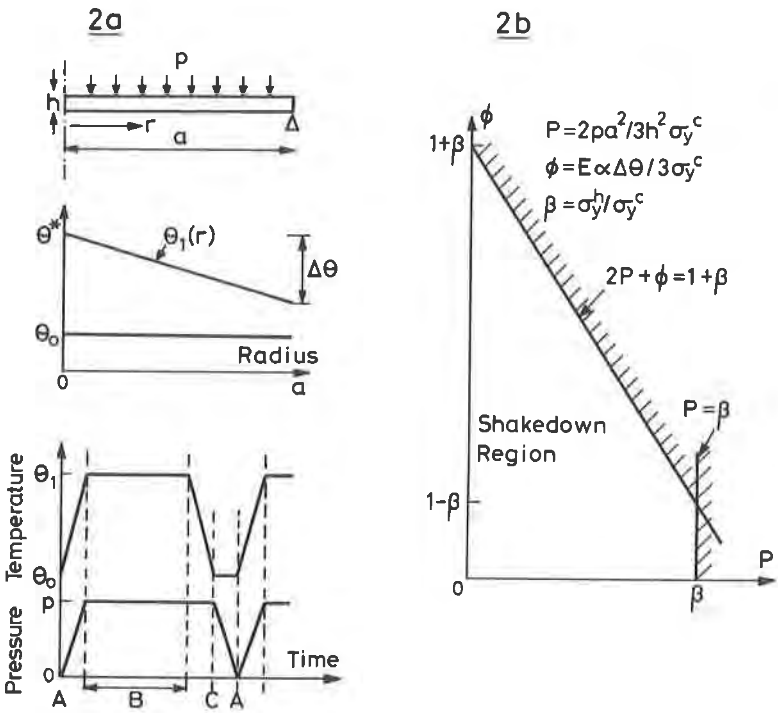

FIG. 2. Cyclic Loading of Circular Plate

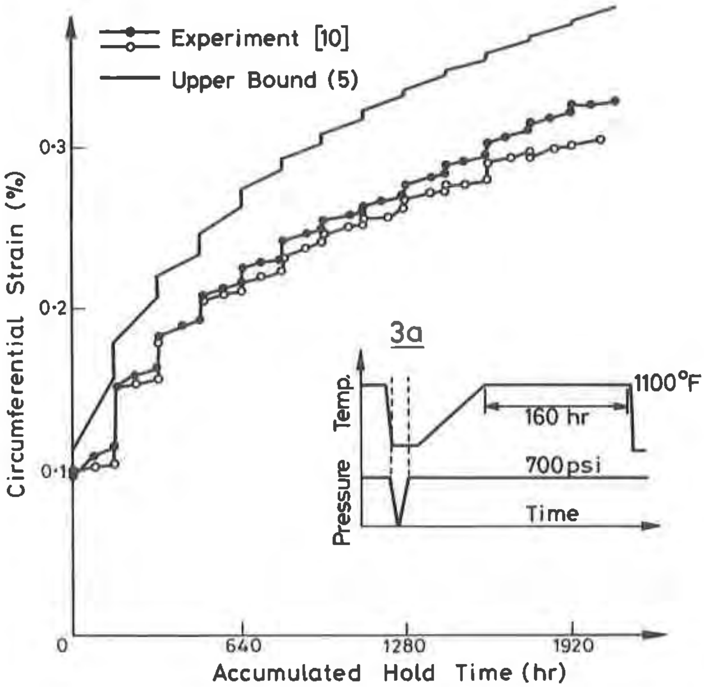

FIG.3 Comparison of Upper Bound and Experiment