G 2/7

Sensitivity of Using Blunt and Sharp Crack Models in Elastic-Plastic Fracture Mechanics

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Abstract

J-integral values are calculated for both the blunt (smeared) crack and the sharp (discrete) crack models in elastic-plastic fracture mechanics problems involving metallic materials. A sensitivity study is performed to show the relative strengths and weaknesses of the two cracking models. It is concluded that the blunt crack model is less dependent on the orientation of the mesh. For the mesh which is in line with the crack direction, however, the sharp crack model is less sensitive to the mesh size. Both models yield reasonable results for a properly discretized finite-element mesh. A subcycling technique is used in this study in the explicit integration scheme so that large time steps can be used for the coarse elements away from the crack tip. The savings of computation time by this technique are reported.

1. Introduction

Cracking of structural members has been an age-long problem. This is no exception in the nuclear power plant. Cracks have been observed in pressure vessels, pipings, containment structures and virtually all other structural members. From a safety point of view, it is very important to know how cracks behave under various loading conditions so that the safety of the plant can be properly assessed.

Traditionally, cracks in metal components have been studied as sharp (discrete) cracks in finite element models where the crack surfaces are treated as the boundary of the finite element mesh. Cracks in concrete structures, on the other hand, have been treated as blunt cracks where the cracks are assumed to be uniformly distributed, or smeared, in a finite element model. The latter is done partly because fracture mechanics of concrete is not yet well understood and quantified, and partly because a concrete structure is usually very large and may contain many small cracks. Therefore, it is often judged to be unnecessary and uneconomical to model the details of every single crack in the concrete structure.

Aside from their different applications, the blunt and sharp crack models are really dealing with the same problem from a continuum mechanics point of view. Concrete is a very complex and heterogeneous material which consists of cement and aggregate, so that a crack moving through a concrete structure would encounter regions of different properties. However, once the properties are smeared, the finite element model in either approach reflects a homogeneous medium with the average equivalent material properties. Therefore, both approaches are really modeling cracks in a continuum. It is not surprising that both blunt and sharp crack models have been successfully applied to concrete structural problems. It can also be expected that they should be applicable to metal structural problems at least in limited cases.

Indeed, this has been shown in a study recently undertaken by the authors [1]. The J-integral values computed for the blunt and the sharp crack models were compared with those obtained in an international round robin on elastic-plastic fracture mechanics using a three point bend problem of a steel specimen [2]. In that study, it is found that the J-integral is path independent but not mesh independent for both the blunt and the sharp crack models. Namely, the J-values are approximately the same for different integration loops in the same mesh, but they are different when different meshes are used.

In this paper, mesh sensitivity study of both the blunt and the sharp crack models is described. An attempt is made to establish the area where the blunt crack model can offer a good alternative to the sharp crack model. In addition, the sub-cycling technique [3] is used in the explicit integration scheme so that large time steps can be used for the coarse elements away from the crack tip. The savings of computing time by using the blunt crack and the sub-cycling techniques are also reported.

2. J-Integral Approach

If a crack of length a is assumed to advance in the x-direction, the rate of energy change can be related to the well-known J-integral whether the crack is modeled as a sharp crack or a blunt crack [4,5]

$$J = \int \left(W\, dy - T \cdot \frac{\partial u}{\partial x}\, d\ell\right) , \tag{1}$$

where x and y are Cartesian coordinates with y perpendicular to the crack surface, W is the strain energy, T is the surface traction, u is the displacement, $d\ell$ is a line segment in an arbitrary integration loop surrounding the crack tip. In the J-integral evaluation, it is noted that the surface traction vanishes on the blunt crack surface just as on the sharp crack surface. Therefore, the numerical integration for the blunt crack model is similar to that for the sharp crack model.

The J-integral is path dependent if extensive plasticity exists. In that case, a modified concept called J* integral was introduced by Blackburn [6].

$$J^* = \int_{\Gamma_3} \left\{\frac{1}{2}\sigma_{ij}\frac{\partial u_i}{\partial x_j}dx_2 - T_i\frac{\partial u_i}{\partial x_1}d\ell\right\} + \lim_{\rho\to 0}\iint\left\{\frac{1}{2}\sigma_{ij}\frac{\partial^2 u_i}{\partial x_1\partial x_j} - \frac{1}{2}\frac{\partial\sigma_{ij}}{\partial x_1}\frac{\partial u_i}{\partial x_j}\right\}dS , \tag{2}$$

where S is the area enclosed by Γ_1 and Γ_3, Γ_1 is a circle of radius ρ around the crack tip, and Γ_3 is another contour beyond Γ_1 also surrounding the crack tip. The Cartesian coordinates are denoted by x_1 and x_2. Tensor notation is used in Eq. (2).

The J* integral is equal to the J integral for an elastic case. Hence, the amount of difference between the J and J* integral values is also an indication of the degree of nonlinear deformation of the structures.

3. Explicit-Explicit Partition

When more than one integration time step is used in explicit integration, the computer program uses a procedure called explicit-explicit partitioning or subcycling [3]. This method is advantageous whenever different parts of the mesh have substantially different maximum frequencies, particularly when there is only a small portion of the total mesh with the highest frequency, because this avoids many evaluations of the nodal forces. This situation is often encountered in modeling cracks.

In using the subcycling procedure, the mesh may be integrated with an arbitrary number of different time steps. The time step is assigned by element group. The following restrictions apply:

i. All time steps must be integer multiples of the smallest time step, Δt_{min}.

ii. If any node is shared by elements with two or more different time steps, these time steps must be integer multiples of each other.

The essence of the procedure is as follows. Within a cycle, whenever the master clock t_{MAS} is incremented by Δt_{min}, all elements are checked. Any element which is in a group that is not ahead of the master time (i.e., if the solution time of the element group $t_G < t_{MAS}$) is updated. This update involves the calculation of new velocity strains, stresses and internal forces, and the element internal forces are assembled into the global internal force matrix; also, the group time t_G is updated by Δt_G. At the same time, the element group time is compared to the time step assigned to any nodes connected to the element (all nodes are assigned a zero time increment at the beginning of the cycle). Therefore, nodes connected to two element groups will be updated with the largest time step.

After all of the elements have been updated to the time of the master clock, the nodes are checked. Any node whose clock is behind the master clock ($t_N < t_{MAS}$) is updated using the time step stored for that node, Δt_N.

The time step is chosen as follows: the time step for all nodes of all elements must satisfy

$$\Delta t_I = \frac{2}{\omega_{max,e}} \left((1+\mu_e^2)^{1/2} - \mu_e \right) \tag{3}$$

where $\omega_{max,e}$ is the maximum frequency of element e, and μ_e the fraction of critical damping for this frequency. The largest time step is used in updating the node for any element in the mesh.

4. Numerical Results

A three point bend problem reported in a calculational round robin in elastic-plastic fracture mechanics [2] is chosen for studying the sensitivity of the blunt crack and the sharp crack models. The specimen is 2.54 cm wide, 2.54 cm thick and 10.16 cm long with a 1.27 cm long center crack. The material properties used in the study are given by

$$\varepsilon = \frac{\sigma}{E} + \varepsilon_p , \tag{4}$$

where

$$\varepsilon_p = 0 \qquad \text{(for } \sigma < 275 \text{ MPa)}$$

$$= \left(\frac{\sigma}{B_o}\right)^n \qquad \text{(for } \sigma > 275 \text{ MPa)}$$

$E = 2.148 \times 10^5$ MPa, B_o = 827.3 MPa, and n = 10.

Several meshes are set up for numerical calculation. The main thrust in setting up these meshes is not to arrive at the optimum finite element discretization. Rather, it is to obtain a reference mesh which yields reasonable results and then perform a relative comparison on the blunt and the sharp crack models.

4.1 Reference Mesh

The reference finite element meshes with the blunt crack and the sharp crack models are shown in Figs. 1 and 2, respectively. The integration path for the J and J* integrals are shown as bold lines in the figures. In both cases, the lower three rows of elements are treated as the first group in the subcycling procedure while the rest of the elements belong to the second group. The calculated loading point displacement, crack mouth opening displacement and the J and J* values are plotted in Figs. 3, 4 and 5, respectively. Also shown in the figures are the values reported by the organizations participating in the calculational round robin. It is noted that the current J and J*-integrals calculations yield very reasonable results as compared with those obtained in the round robin. These values will be used as the reference points in the following study.

With the subcycling procedure, 581 seconds CPU time is needed for one computer run. This is compared with 651 seconds CPU time if no subcycling is used. Considering the fact that the mesh is relatively coarse, one can expect more saving if the mesh is further refined.

4.2 Effect of Mesh Orientation

In this study, the finite element discretization is done such that the orientation of the mesh is not parallel or perpendicular to the direction of the crack as shown in Figs. 6 and 7. Therefore, the crack has to be represented by a zig-zag band in the blunt crack model. The results of the J integral calculation are plotted in Fig. 5. Contrary to what one might suspect, the zig-zag blunt crack model results in less change in the J-values as compared to the sharp crack model.

4.3 Effect of Mesh Size

Two different ways of mesh refinement are implemented in order to examine the effects of mesh size on the J-value calculation. The first is to simply change the size of the lower three rows of elements (group 1 in the subcycling procedure) in Figs. 1 and 2. The second is to refine the mesh near the crack tip as shown in Fig. 8. The effects of these changes on the calculated J-values are shown in Fig. 9. From this figure, two trends are evident: (1) the sharp crack model is less sensitive to the mesh size, and (2) local mesh refinement is an effective way of improving the sharp crack model whereas the blunt crack model depends more on the refinement of all the elements representing the crack.

When the subcycling procedure is used with the mesh shown in Fig. 8, 108 seconds CPU time is needed for one computer run as compared with 201 seconds CPU time without subcycling procedure.

5. Conclusions

A sensitivity study is conducted to investigate the influence of mesh size and orientation on the performance of the blunt crack and the sharp crack models used in elastic-plastic fracture mechanics problem. It is concluded that both models yield reasonable results with proper finite element discretization. The sharp crack model is found to be less sensitive to the change of grid size whereas the blunt crack model is less sensitive to the change of grid orientation. The sharp crack model can be better improved by refinement of elements surrounding the crack tip. For situations where the cracking configuration is simple and well defined, the sharp crack model is clearly better. On the other hand, finite element discretization is easier with the blunt crack model. In cases where the cracking configuration is complex, the blunt crack model is easier to implement and will yield reasonable results without getting into the details of the crack.

The subcycling technique has been used in this study. It is found that subcycling yields noticeable saving of the computer time even for a simple finite element mesh with a two to one ratio in two time groups as shown in Fig. 1. Saving of 50% CPU time is obtained when the subcycling technique is used with the mesh shown in Fig. 8.

6. Acknowledgments

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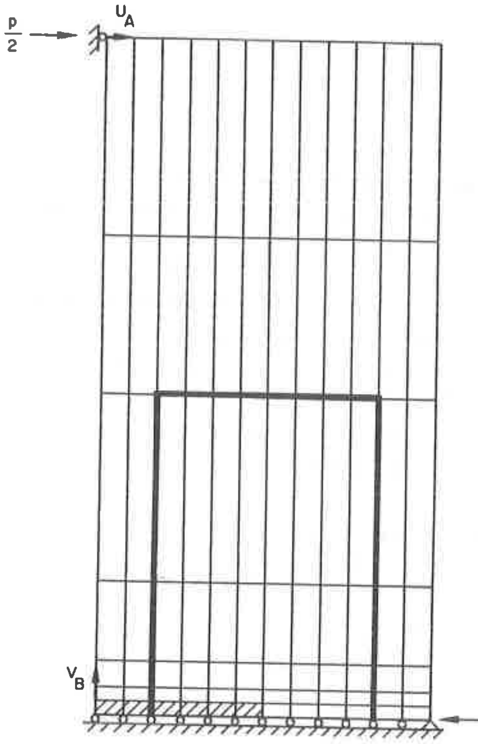

Fig. 1. Finite Element Mesh with the Blunt Crack Model (Shaded area represents the blunt crack)

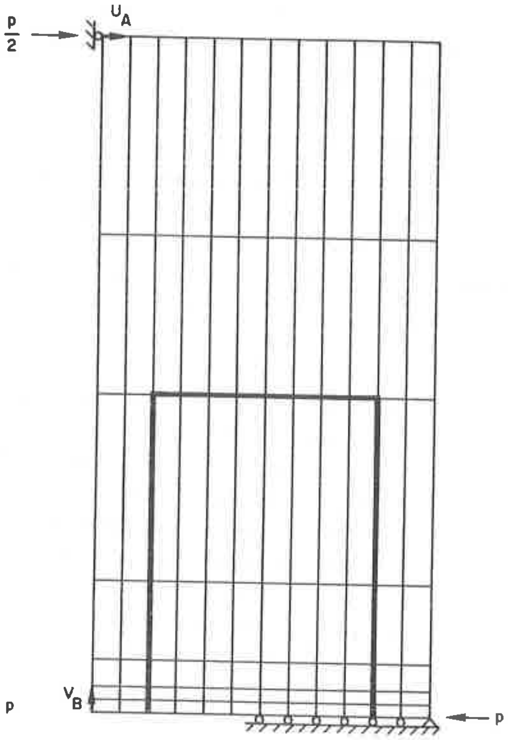

Fig. 2. Finite Element Mesh with the Sharp Crack Model

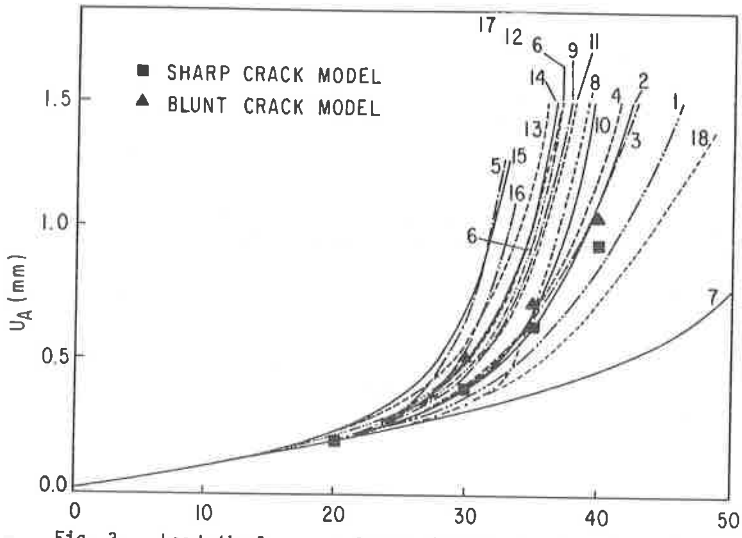

Fig. 3. Load-displacement Curves (Curves are from Ref. 2)

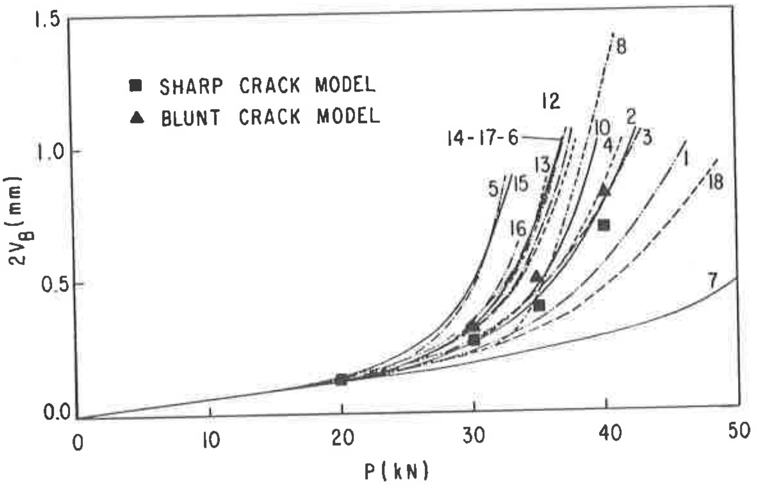

Fig. 4. Crack Mouth Opening Versus Load (Curves are from Ref. 2)

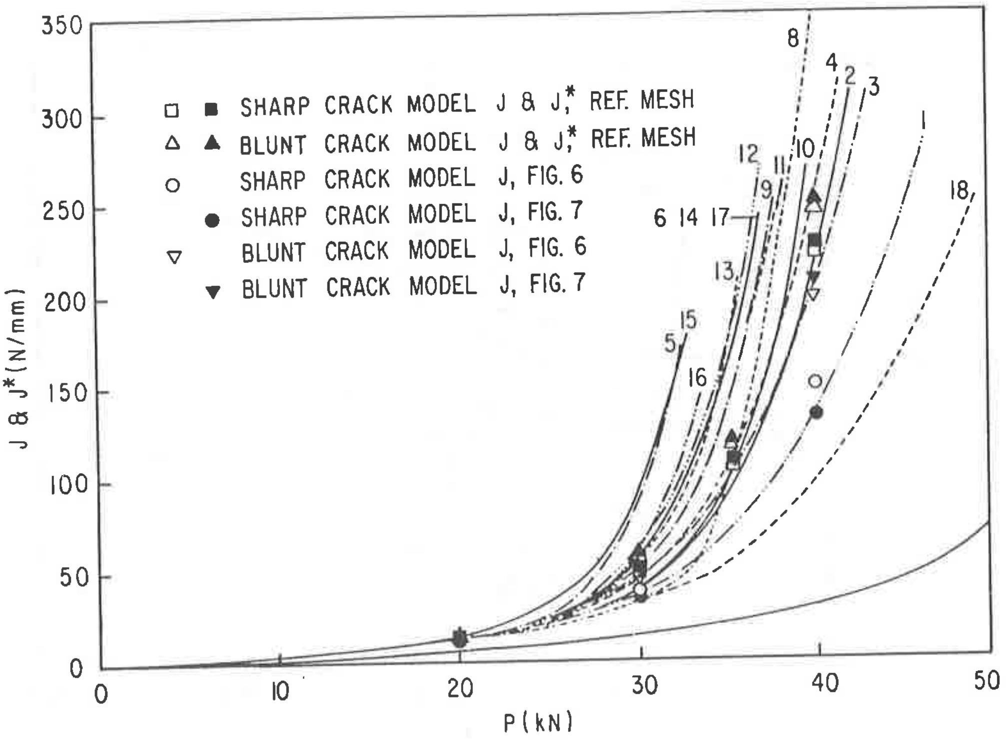

Fig. 5. J and J* Integrals Versus Load (Curves are from Ref. 2)

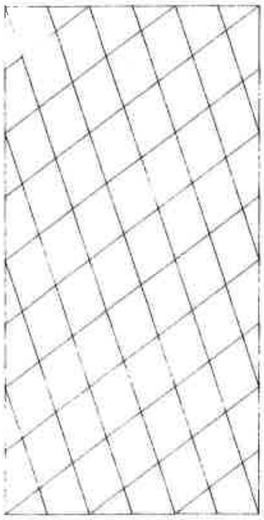

Fig. 6.
Skewed Finite Element Mesh
(Shaded area represents the blunt crack)

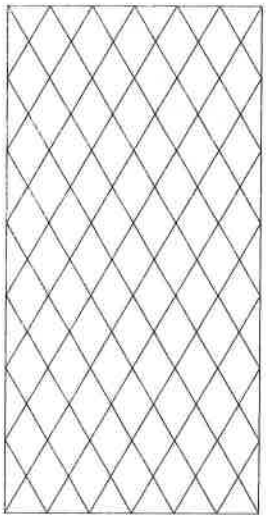

Fig. 7.
Skewed Finite Element Mesh
(Shaded area represents the blunt crack)

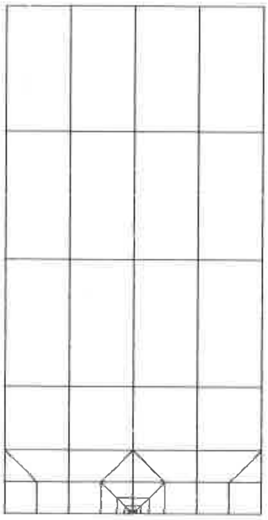

Fig. 8.
Finite Element Mesh with Refinement at the Crack Tip
(Shaded area represents the blunt crack)

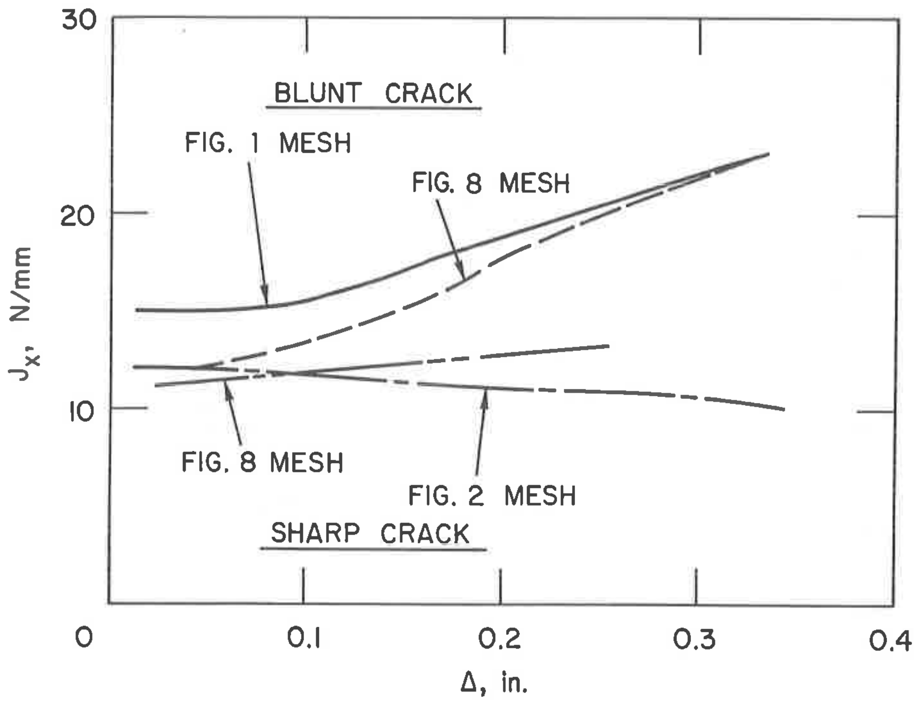

Fig. 9. J Integral Versus Crack Size