THE DEVELOPMENT AND USE OF A PIECE-WISE CONTINUOUS FINITE ELEMENT FOR PLATE AND SHELL ANALYSIS

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SUMMARY

The implementation of general-purpose programs for the numerical analysis of plate and shell structures calls for the adoption of finite element stiffness expressions which take account of both lateral distortion and membrane action. It is important that design-oriented programs of the above kind be perfectly general. In particular the element behaviour must be independent of the choice of base axes, and not prone either to singularities or to doubts over convergence with successive mesh refinement. The basic elements should also be mathematically isotropic and the imposition of rigid body displacements should not cause self-straining. Ideally the program should allow the assembly of a wide variety of elements, oriented in any conceivable way and of an freely chosen shape. The present paper documents a procedure for synthesising the latter from a three node primary element which satisfies the above requirements.

Lateral distortion of an element of the kind described calls for curvature changes across its surface. The basic element proposed is conceptually subdivided into congruent sub-regions having the same curvature and twist along adjoining edges, whilst changes of curvature in directions perpendicular to the latter are allowed. Comparison with a cell of grid elements shows perfect accord for homogeneous flexure, which is the laterally deflected state to which each element must tend for a sufficiently fine mesh. The element described has been incorporated into a program based on the sub-routines which UNCLE provides. Results so obtained apparently demonstrate both improved accuracy and convergence by comparison with previous 9 degree of freedom (DOF) plate element formulations. Dissatisfaction with the latter has led to the introduction of higher DOF elements, requiring complex internal integration or interpolation procedures. In order to provide comparisons with the latter, sub-assemblies of the primary element proposed have been envisaged as 'super-elements'. The finite element handling system of UNCLE provides, in fact, input facilities which allow sub-assemblies of elements to be arrayed in terms of 'cells' of basic elements. The conceptual use of super-elements in the above sense has been found to provide comparable accuracy for a given number of DOF, without additional computational or programming effort. Such higher DOF elements automatically satisfy the requirements referred to above in connection with the basic element from which they stem.

Incorporation of the element described into an advanced finite element modelling system has shown that improved accuracy, convergence and versatility are thus attainable. Certain of the novel features of UNCLE provide the user with facilities which are equivalent to a capacity for generating super-elements of unlimited complexity at will, thus extending, in a natural way, the basic piece-wise continuous sub-structure of the primary element proposed.

Finally, reference is made to the modelling of reactor primary circuit components, particularly to establish steady state and transient thermal effects, both under normal-operating and fault conditons.

1. Introduction

The displacement distribution within a finite element is usually characterised by the coefficients of a polynomial expression. Typically, the lateral deflections w(x, y) over the surface of a plate element may be assumed to be:

$$a_1 + (a_2x + a_3y) + (a_4x^2 + a_5xy + a_6y^2) + (a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3) + \ldots$$

The most rudimentary finite element is triangular and, if the lateral and angular deflections of its corners are used as generalised coordinates, there are only nine of the latter to define the above ten coefficients, whereas use of a quadratic expression is inconsistent with the assumption of transverse shears. Gallagher [2] has reviewed attempts to circumvent this problem, which includes either the telescoping or deletion of coefficients, the introduction of additional nodes or the use of further variables, each of which introduces difficulties, uncertainties or limitations. The present paper proposes notional division of each element into sub-regions, as shown in fig 1, so as to represent the laterally deflected shape as a piece-wise continuous quadratic surface. These flexural effects are subsequently combined with membrane action, which is assumed to be homogeneous.

2. Membrane Stiffness in Terms of Generalised Forces and Displacements

A simple triangular plate element of area S and thickness h, subjected to strains $\underset{\sim}{q}$ in its mid-plane experiences associated generalized forces $\underset{\sim}{p}$:

$$\underset{\sim}{q} = (\epsilon_x \; \epsilon_y \; \gamma_{xy})^T , \qquad \underset{\sim}{p} = Sh\,(\sigma_x \; \sigma_y \; \tau_{xy})^T \qquad (1), (2)$$

A further set of generalised displacements is:

$$\underset{\sim}{q}_n = (u_A \; v_A \; u_B \; v_B \; u_G \; v_G)^T \qquad (3)$$

see fig 2. The corresponding generalised forces are:

$$\underset{\sim}{p}_n = (X_A \; Y_A \; X_B \; Y_B \; X_G \; Y_G)^T \qquad (4)$$

so defined that the elastic work W is given by:

$$2W = \underset{\sim}{q}_n^T \, \underset{\sim}{p}_n = \underset{\sim}{q}^T \, \underset{\sim}{p} \qquad (5)$$

The stretch of any side, such as that of the length α opposite to A is given by:

$$\alpha^2 \epsilon_\alpha = \alpha_x^2 \epsilon_x + \alpha_y^2 \epsilon_y + \alpha_x \alpha_y \gamma_{xy} \qquad (6)$$

in which α_x, α_y are the components of α referred to axes Ox, Oy respectively, see fig 2. Three invertible equations are thus obtained to relate $\underset{\sim}{q}$ to the stretch of the connections:

$$\underset{\sim}{q}_c = (\alpha^2\epsilon_\alpha \; \beta^2\epsilon_\beta \; \gamma^2\epsilon_\gamma)^T \qquad (7)$$

$$\underset{\sim}{q} = \underset{\sim}{C} \, \underset{\sim}{q}_c , \qquad \underset{\sim}{q}_c = \underset{\sim}{C}^{-1} \underset{\sim}{q} \qquad (8)$$

The stretch of each edge may be expressed in terms of the nodal point displacements, typically:

$$\alpha^2\epsilon_\alpha = (u_G - u_B)\alpha_x + (v_G - v_B)\alpha_y \tag{9}$$

$$\therefore \quad \underset{\sim}{q}_c = \underset{\sim}{D}\, \underset{\sim}{q}_n \tag{10}$$

The element stiffness matrix $\underset{\sim}{K}$ is based on Hooke's Law:

$$\underset{\sim}{p} = \underset{\sim}{K}\, \underset{\sim}{q} \tag{11}$$

and it thus follows from the work expression that:

$$\underset{\sim}{q}_c^T \underset{\sim}{p}_c = \underset{\sim}{q}_n^T \underset{\sim}{p}_n = \underset{\sim}{q}^T \underset{\sim}{p} = \underset{\sim}{q}^T \underset{\sim}{K}\, \underset{\sim}{q} \tag{12}$$

This enables $\underset{\sim}{K}_c$ in:

$$\underset{\sim}{p}_c = \underset{\sim}{K}_c\, \underset{\sim}{q}_c \tag{13}$$

to be derived from:

$$K_c = \underset{\sim}{C}^T \underset{\sim}{K}\, \underset{\sim}{C} \tag{14}$$

It similarly follows that K_n in:

$$\underset{\sim}{p}_n = \underset{\sim}{K}_n\, \underset{\sim}{q}_n \tag{15}$$

is given by:

$$\underset{\sim}{K}_n = \underset{\sim}{D}^T \underset{\sim}{K}_c\, \underset{\sim}{D} \tag{16}$$

The nodal stiffnesses provide a basis for synthesising corresponding expressions for assemblies of elements, which may form either part of a structure or its whole.

3. Homogeneous Flexure

For thin plates, the membrane stresses can be combined with those which vary proportionately with distance from the mid-plane. Integration through the thickness leads to the definition of associated stress couples M_x, M_y and M_{xy}, which may thus be related to the corresponding curvatures and twist κ_x, κ_y and κ_{xy}. Defining:

$$\underset{\sim}{q} = (\kappa_x\ \kappa_y\ \ 2\kappa_{xy})^T, \qquad \underset{\sim}{p} = S(M_x\ M_y\ M_{xy})^T \tag{17}$$

$$\underset{\sim}{p} = \underset{\sim}{K}\, \underset{\sim}{q} \tag{18}$$

for elastic behaviour. The corresponding generalised displacements for the connections become:

$$\underset{\sim}{q}_c = (\alpha^2\kappa_\alpha\ \ \beta^2\kappa_\beta\ \ \gamma^2\kappa_\gamma)^T \tag{19}$$

so that $\underset{\sim}{q}$ and $\underset{\sim}{q}_c$ are related as before:

$$\underset{\sim}{q}_c = \underset{\sim}{C}^{-1}\, \underset{\sim}{q} \qquad\qquad \underset{\sim}{q} = \underset{\sim}{C}\, \underset{\sim}{q}_c \tag{20}$$

It thus follows that we again have:

$$\underset{\sim}{K}_c = \underset{\sim}{C}^T \underset{\sim}{K} \underset{\sim}{C} \tag{21}$$

applying, in this case, to homogeneous flexure.

4. A Basic Assembly for the Flexure of Plates

The above considerations fail to introduce forces (Z_A Z_B and Z_G) to correspond with the lateral displacements (w_A w_B w_G) since attention has been confined to homogeneous flexure, for which the associated shears are zero. It is thus fundamental to a finite element modelling that changes of curvature be accommodated within the basic assembly. Notional division of the element into sub-regions is therefore proposed, homogeneous flexure being assumed within each of the latter. Continuity of slope and deflection is maintained by implying the same uniform curvature and rate of twist for each abutting edge, with a small step change in curvature in the direction perpendicular to the lines which mark these sub-divisions. The most orderly arrangement of sub-regions is that shown in fig 1. It leads to a central sub-region 0, which is bordered by congruent sub-regions 1, 2 and 3. Since each sub-region has sides of length $\alpha/2$, $\beta/2$ and $\gamma/2$:

$$\underset{\sim}{q}'_c = \tfrac{1}{4}\,(\alpha^2\kappa_\alpha \;\; \beta^2\kappa_\beta \;\; \gamma^2\kappa_\gamma)^T \text{ and } \underset{\sim}{q}' = (\kappa_x \;\; \kappa_y \;\; 2\kappa_{xy})^T \tag{22}$$

are related by:

$$\underset{\sim}{C}'^{-1} = \tfrac{1}{4}\begin{bmatrix} \alpha_x^2 & \alpha_y^2 & \alpha_x\alpha_y \\ \beta_x^2 & \beta_y^2 & \beta_x\beta_y \\ \gamma_x^2 & \gamma_y^2 & \gamma_x\gamma_y \end{bmatrix} \tag{23}$$

in $\underset{\sim}{q}'_c = \underset{\sim}{C}'^{-1}\,\underset{\sim}{q}'$. By defining $\underset{\sim}{q}_c$ as a column of twelve components for the curvatures of the connections and $\underset{\sim}{q}$ similarly, as three components, for each of the four sub-elements in turn:

$$\underset{\sim}{q}_c = \underset{\sim}{C}^{-1}\,\underset{\sim}{q}\;, \qquad \underset{\sim}{q} = \underset{\sim}{C}\,\underset{\sim}{q}_c \tag{24}$$

in which $\underset{\sim}{C}$ and $\underset{\sim}{C}^{-1}$ are diagonal matrices formed from $\underset{\sim}{C}'$ and $\underset{\sim}{C}'^{-1}$ respectively. Our purpose in so doing is to infer $\underset{\sim}{K}_c$ from $\underset{\sim}{K}$ in the same way as before, $\underset{\sim}{K}$ now being a diagonal matrix formed from $\underset{\sim}{K}'$:

$$\underset{\sim}{K} = \begin{bmatrix} \underset{\sim}{K}' & & & \\ & \underset{\sim}{K}' & & \\ & & \underset{\sim}{K}' & \\ & & & \underset{\sim}{K}' \end{bmatrix}, \qquad \underset{\sim}{K}' = \frac{Sh^3\,E}{48(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \tfrac{1}{2}(1-\nu) \end{bmatrix} \tag{25}$$

We thus obtain:

$$\underset{\sim}{K}_c = \underset{\sim}{C}^T \underset{\sim}{K} \underset{\sim}{C} \tag{26}$$

which provides a means of deriving a nodal stiffness matrix $\underset{\sim}{K}_n$, since:

$$\underset{\sim}{q}_c^T\,\underset{\sim}{p}_c = \underset{\sim}{q}_n^T\,\underset{\sim}{p}_n \tag{27}$$

This enables $\underset{\sim}{K}_n$ to be inferred from:

$$\underset{\sim}{K}_n = \underset{\sim}{D}^T \underset{\sim}{K}_c \underset{\sim}{D} \tag{28}$$

if we can find $\underset{\sim}{D}$, in an expression to relate the generalised displacements of the connections to those of the nodes:

$$\underset{\sim}{q}_c = \underset{\sim}{D} \underset{\sim}{q}_n \tag{29}$$

5. Generalised Displacements in Terms of Nodes and Connections

In order to find $\underset{\sim}{D}$ we first define $\underset{\sim}{q}_n$ as:

$$(w_A \ \theta_A \ \phi_A \quad w_B \ \cdots \quad w_G \ \cdots)^T \tag{30}$$

w being the lateral deflection along Oz, θ the corresponding clockwise rotation about Ox and ϕ that about Oy. If the deflected profile of a connection such as DG is expressed as:

$$w = a_1 + a_2 x_\alpha + a_3 x_\alpha^2 \tag{31}$$

in terms of axis Ox_α lying along DG and centred on D, we must write for BD:

$$w = a_1 + a_2 x_\alpha + a_4 x_\alpha^2 \tag{32}$$

to ensure continuity of slope and deflection at D. The slopes and deflections at B and G are thus:

$$\left.\begin{aligned} w_B &= a_1 - \frac{\alpha}{2} a_2 + \frac{\alpha^2}{4} a_4 & \phi_{\alpha B} &= -a_2 + \alpha a_4 \\ w_G &= a_1 + \frac{\alpha}{2} a_2 + \frac{\alpha^2}{4} a_3 & \phi_{\alpha G} &= -a_2 - \alpha a_3 \end{aligned}\right\} \tag{33}$$

The column:

$$\underset{\sim}{q}_1 = (w_B \ \ \phi_{\alpha B} \ \ w_G \ \ \phi_{\alpha G})^T \tag{34}$$

can thus be related to:

$$\underset{\sim}{q}_2 = (a_1 \quad a_2 \quad a_3 \quad a_4)^T \tag{35}$$

$$\therefore \quad \underset{\sim}{q}_1 = \underset{\sim}{A} \underset{\sim}{q}_2 \tag{36}$$

We deduce by inversion that:

$$\underset{\sim}{q}_2 = \underset{\sim}{A}^{-1} \underset{\sim}{q}_1 = \frac{1}{8\alpha^2} \begin{bmatrix} 4\alpha^2 & -\alpha^3 & 4\alpha^2 & \alpha^3 \\ -16\alpha & 4\alpha^2 & 16\alpha & 4\alpha^2 \\ 16 & -4\alpha & -16 & -12\alpha \\ -16 & 12\alpha & 16 & 4\alpha \end{bmatrix} \underset{\sim}{q}_1 \tag{37}$$

The slopes of the edge connections may be related to the reference axes:

$$\phi_\alpha = \frac{1}{\alpha} (-\alpha_y \theta + \alpha_x \phi) \tag{38}$$

and the corresponding curvatures are:

$$\kappa_{\alpha_3} = -2a_3 = -\frac{4}{\alpha^2} w_B - \frac{\alpha_y}{\alpha^2} \theta_B + \frac{\alpha_x}{\alpha^2} \phi_B + \frac{4}{\alpha^2} w_G - \frac{3\alpha_y}{\alpha^2} \theta_G + \frac{3\alpha_x}{\alpha^2} \phi_G \tag{39}$$

$$\kappa_{\alpha_2} = -2a_4 = \frac{4}{\alpha^2} w_B + \frac{3\alpha_y}{\alpha^2} \theta_B - \frac{3\alpha_x}{\alpha^2} \phi_B - \frac{4}{\alpha^2} w_G + \frac{\alpha_y}{\alpha^2} \theta_G - \frac{\alpha_x}{\alpha^2} \phi_G \tag{40}$$

Since the element O lies intermediately between 2 and 3 we write for $\kappa_{\alpha o}$

$$\kappa_{\alpha o} = \tfrac{1}{2} (\kappa_{\alpha_2} + \kappa_{\alpha_3}) \tag{41}$$

and note that continuity requires that $\kappa_{\alpha o} = \kappa_{\alpha_1}$. By proceeding similarly for the β and γ directions a column of curvatures, defined in terms of:

$$\underset{\sim}{q}_c = \tfrac{1}{4} (\alpha^2\kappa_{\alpha_1} \quad \beta^2\kappa_{\beta_1} \quad \gamma^2\kappa_{\gamma_1} \quad \alpha^2\kappa_{\alpha_2} \ldots \alpha^2\kappa_{\alpha_3} \ldots \alpha^2\kappa_{\alpha o} \ldots)^T \tag{42}$$

can be related to:

$$\underset{\sim}{q}_n = (w_A \; \theta_A \; \phi_A \quad w_B \ldots \quad w_G \ldots)^T \tag{43}$$

by means of $\underset{\sim}{D}$ in:

$$\underset{\sim}{q}_c = \underset{\sim}{D} \, \underset{\sim}{q}_n \tag{44}$$

Our object is now achieved; the required nodal stiffness matrix is $\underset{\sim}{D}^T \underset{\sim}{K}_c \underset{\sim}{D}$ ie

$$\underset{\sim}{p}_n = \underset{\sim}{K}_n \underset{\sim}{q}_n = \underset{\sim}{D}^T \underset{\sim}{K}_c \underset{\sim}{D} \, \underset{\sim}{q}_n \tag{45}$$

6. Flexibility and Stiffness of an Equilateral for $\nu = 1/3$

For an equilateral element, disposed as shown in fig 3, the above solution was found to agree with grid-framework methods [3] for uniform flexure across a median, Poisson's ratio being taken as 1/3. Variation of the stress couples across an element gave rise to small discrepancies - see fig 4. No discrepancy was found in the slopes and perfect accord was obtained for homogeneous twisting of the element.

7. Combined Membrane and Flexural Action for an Arbitrarily Oriented Element

The procedure for computing the flexural stiffnesses can be combined with those for membrane action, to relate the component displacements of an element as defined by:

$$\underset{\sim}{q}_n = (u_A \; v_A \; w_A \; \theta_A \; \phi_A \; \psi_A \quad u_B \ldots \quad u_G \ldots)^T \tag{46}$$

to the corresponding generalised forces:

$$\underset{\sim}{p}_n = (X_A \; Y_A \; Z_A \; L_A \; M_A \; N_A \quad X_B \ldots \quad X_G \ldots)^T \tag{47}$$

by an expression of the type:

$$\underset{\sim}{p}_n = \underset{\sim}{K}_n \underset{\sim}{q}_n \tag{48}$$

Transforming to reference axes:

$$\underset{\sim}{p}_o = \underset{\sim}{T}^T \underset{\sim}{p}_n \; , \qquad \underset{\sim}{q}_o = \underset{\sim}{T}^T \underset{\sim}{q}_n \tag{49}$$

and the relationship between the generalised nodal forces $\underset{\sim}{p}_o$ and displacements $\underset{\sim}{q}_o$, referred to these base axes, is:

$$\underset{\sim}{p}_o = (\underset{\sim}{T}^T \underset{\sim}{K}_n \underset{\sim}{T}) \, \underset{\sim}{q}_o \tag{50}$$

8. Test Cases for Plate Bending

The above element has been incorporated by Enderby and Knowles [4] into a program which has been based on the sub-routines which Uncle provides [1]. Results so obtained have been compared with earlier proposals for nine degree of freedom (9 DOF) plate elements due to Bazeley et al [5] and to Clough and Tocher [6]. Fig 5 compares the effect of mesh refinement N on computed deflections w, normalised through the use of $D = Eh^3/12(1-\nu^2)$ in conjunction with lateral load (P) and length of side (a). Coarse-mesh results for pattern B tend to be better than those for A. Subsequent results are therefore presented for the overlay pattern AB; fig 6 compares results for clamped edges and uniformly distributed loads (q). Composite assemblies of elements can be simply handled in the Uncle scheme through the use of cells. Four congruent primary elements may thus be treated by the user as an 18 DOF triangular sub-assembly, having additional nodes at the mid-points of each side. Two such super-elements were combined to form one ply of each square cell for the cases considered. Fig 7 compares results for 18 DOF sub-assemblies with various higher order elements [7]. Super-elements - defined as piece-wise continuous sub-assemblies of the primary element proposed - are seen to be quite competitive with current alternatives.

9. Test Cases involving Membrane Action

Fig 8 shows the computed effect on the initial stiffness of dishing a clamped square plate of sides a to a central deflection w_o, and then subjecting it to a pressure q. The fan of lines obtained depends to some extent on the initial shape; that assumed was based on small deflection theory. The values of w_o/h (= α) correspond to the initial maximum set, as a fraction of the plate thickness; w/h is the corresponding normalised further deflection. The figure also shows derived results for large deflections of an initially flat plate and a comparison with Timoshenko [8], based on a correlation due to Nylander [9]. Good agreement was found, despite the fact that Nylander's construction strictly applies only if the initially deflected plate undergoes affine additional deflection.

10. Applications

Emphasis has been placed on the general character of the above development rather than its specialisation towards particular needs. The latter include structural assessments of sodium-cooled fast reactor primary circuit components where requirements exist for the analysis of primary and thermal stresses in plate and shell structures, both under steady state and transient conditions.

Core components, such as sub-assembly wrappers, give rise to additional considerations, particularly irradiation induced creep and voidage growth effects. Not only does each wrapper lengthen, it also tends to bow progressively as a result of flux gradients. In addition, its cross section distends and distorts at the same time. To the extent that both irradiation and growth effects vary linearly with the number of atomic displacements, the solution of an associated elastic problem provides a basis for assessing these inter-related effects which continue to be major considerations in fast reactor core design.

11. Conclusions

Energy considerations have been used to examine lateral distortion of a triangular plate element, as defined by the transverse displacement and component rotations of each of its corners. The resulting stiffness relationships have been combined with those for membrane action, to derive force and moment expressions which allow the distortion of any plate or shell structure to be derived. The accuracy depends on the degree of mesh refinement and on the extent to which the actual pattern of displacements accords with that assumed. The deformed element shape is envisaged as a piece-wise continuous surface in each of which the state of curvature is uniform. These assumptions become increasingly exact as the stress-couple variation across each element is reduced. Incorporation of the element described into a program based on the Uncle [1] sub-routines provides options for its assembly in cells and arrays. From the user's point of view these are equivalent to a capacity for generating super-elements of unlimited complexity so extending the piece-wise continuous sub-structure of the basic element. The facility described finds applications in assessments of reactor components, particularly where the effects of thermal inertia are mitigated by the extensive use of relatively light plate and shell structures.

Acknowledgement

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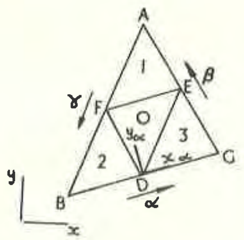

Figure 1 Sub-division of basic element for flexure

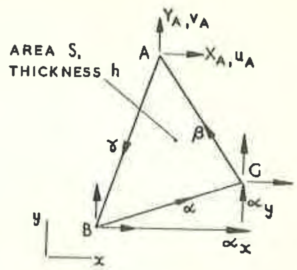

Figure 2 Nodal forces and displacements for membrane action

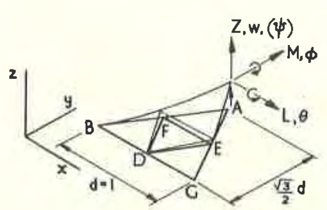

Figure 3 Sub-regions for flexure of an equilateral element

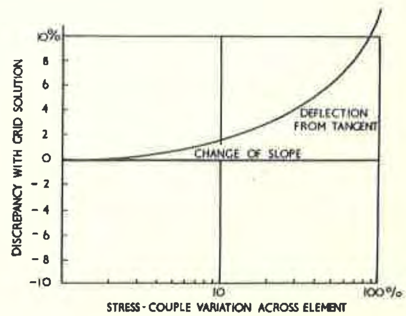

Figure 4 Comparison with grid solution for an equilateral element (ν = 1/3)

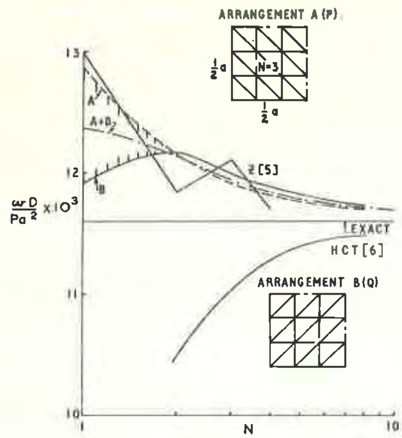

Figure 5 9 DOF sub-assemblies; deflection of a simply supported square plate with point load

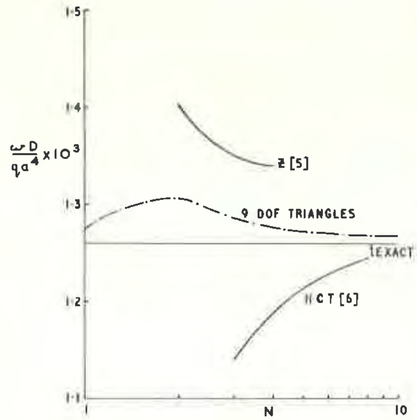

Figure 6 9 DOF sub-assemblies; deflection of uniformly loaded square plate with clamped edges

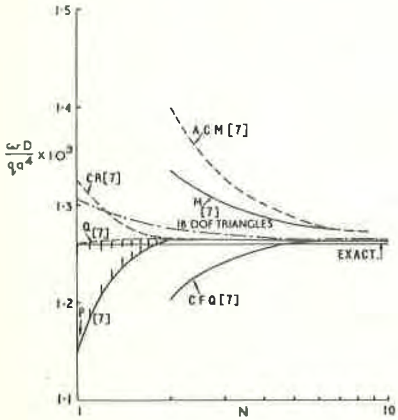

Figure 7 Higher DOF sub-assemblies; deflection of uniformly loaded square plate with clamped edges

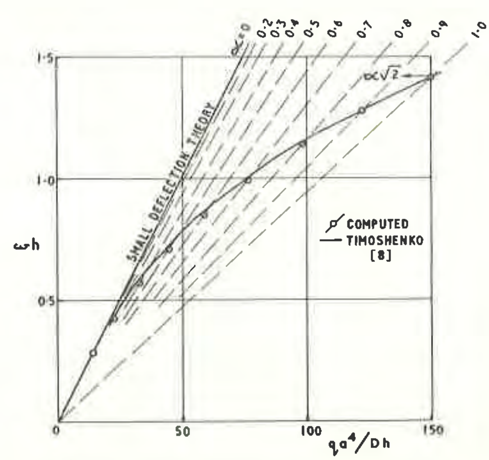

Figure 8 Computed linear deflections for dished plates and derived large deflections (square plate with clamped edges)