

## Applications of Probabilistic Fracture Mechanics to FBR Components

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### 1 INTRODUCTION

Deterministic fracture mechanics has been used for the evaluation of nuclear component integrity as usual. On the other hand, probabilistic fracture mechanics (PFM) becomes more important recently in the design of LWRs. It is expected that safety estimation methods using PFM will be applied to demonstration LMFBR design.

In this study, a probabilistic fracture mechanics code for creep-fatigue crack growth named PCCF was developed, and the effects of loading conditions on failure probability are shown as test analyses of the code.

### 2 FUNCTIONS OF PCCF CODE

#### 2.1 Probabilistic Calculation Methods

The PCCF code uses the Monte Carlo simulation method. The program flow chart is shown in Fig. 1. The values which can be random variables are as follows:

- (1) Constants in the Ramberg-Osgood type elasto-plastic behavior of materials
- (2) Constants in the Norton type creep law
- (3) Initial crack depth :  $a$
- (4) Initial crack aspect ratio :  $a/c$  ( $c$  : half crack length)
- (5) Constants in the fatigue crack growth equation
- (6) Constants in the creep crack growth equation
- (7) Flow stress used in the net section stress criterion.

Any variables above can optionally become random values. Furthermore, stratified sampling can be applied for two of the random variables to improve the calculation efficiency.

The PCCF can treat failure as leakage modes and/or break modes. Leakage is assumed to occur when a crack depth reaches the critical depth determined by users. Break is assumed to occur when the ligament stress in a cracked section exceeds flow stress of materials.

#### 2.2 Creep-Fatigue Crack Growth Analysis

Crack growth by creep-fatigue is analyzed as a sum of creep crack growth and fatigue crack growth. Fatigue crack growth is calculated based on  $\Delta J$ , and creep crack growth is calculated based on  $C^*$ . The loading is separated into fatigue stage and creep stage as shown in Fig. 2.

The  $\Delta J$  value is derived from the J-integral as follows:

$$\Delta J = 4J \quad (1)$$

where the J value is calculated by assuming that half of the stress range is applied.

The PCCF code uses an approximated J-integral evaluation method proposed by Yagawa et al. When the Ramberg-Osgood equation is used to express stress and strain relation of material elasto-plasticity as follows:

$$\frac{\epsilon}{\epsilon_{op}} = \frac{\sigma}{\sigma_{op}} + \alpha_p \left( \frac{\sigma}{\sigma_{op}} \right)^{n_p} \quad \left( \begin{array}{l} \sigma : \text{Stress} \\ \epsilon : \text{Strain} \\ \text{The others are} \\ \text{material constants} \end{array} \right) \quad (2)$$

$$\sigma_{op} = E \epsilon_{op} \quad (E : \text{Young's modulus}) \quad (3)$$

the approximated J-integral is calculated from the following equations:

$$J = J_e + J_p \quad (4)$$

$$J_p = \alpha_p \sigma_{op} \epsilon_{op} g(n_p) (\sigma/\sigma_{op})^{n_p+1} \quad (5)$$

$$J_e = \sigma_{op} \epsilon_{op} g(n_p=1) (\sigma/\sigma_{op})^2 \quad (6)$$

where g is the function of crack size, model size, and  $n_p$ . This g function is given as the table in the PCCF.

Creep J-integral  $C^*$  is also calculated approximately. As for creep, the Norton creep law is used as follows:

$$\frac{\dot{\epsilon}_c}{\dot{\epsilon}_{oc}} = \alpha_c \left( \frac{\sigma}{\sigma_{oc}} \right)^{n_c} \quad (\dot{\epsilon}_c : \text{Creep strain rate}) \quad (7)$$

If fully-plastic solutions of J-integral have been calculated, the approximation of  $C^*$  is obtained from the analogy of J and  $C^*$  as follows:

$$C^* = \alpha_c \sigma_{oc} \dot{\epsilon}_{oc} g(n_c) (\sigma/\sigma_{oc})^{n_c+1} \quad (8)$$

where the g function is the same one as that in equation (5).

These approximations of non-linear fracture mechanics parameters have already been available in the cases of both tensile loads and bending loads. The study on the approximation under combination of tensile and bending loading is being studied by Yagawa.

### 3 ANALYSIS

Test analyses have been performed using the PCCF to quantify the effects of various random variables on failure probability in LMFBR structures. In this paper, some of the results are introduced as examples.

#### 3.1 Input Conditions

An analysis assuming a half-elliptic surface crack in a type 304 stainless steel plate was carried out. Random variables chosen here are initial crack depth  $a_0$  and initial crack ratio  $c_0/a_0$  where  $c_0$  is half crack length at a surface. The input conditions was as follows:

1) Distribution of initial crack depth  $a_0$ :

$$P(a_0) = \frac{\exp(-a_0/\mu)}{\mu\{1-\exp(-h/\mu)\}} \quad (0 \leq a \leq h, \text{ h: thickness}) \quad (9)$$

Average initial crack depth  $\mu = 6.248$  mm

2) Distribution of aspect ratio:

$$P(c_0/a_0) = \frac{\alpha}{\frac{c_0}{a_0} \gamma \sqrt{2\pi}} \exp\left(-\frac{\{\ln(\frac{c_0}{a_0\beta})\}^2}{2\gamma^2}\right) \quad (c_0/a_0 \geq 1) \quad (10)$$

$\alpha=1.419, \beta=1.336, \gamma=0.5382$

3) Thickness of the plate:  $h = 50$  mm

4) Width of the plate:  $2B = 31400$  mm

5) Flow stress:  $\sigma_f = 21.05$  kgf/mm<sup>2</sup>

6) Stress and strain equation: (Eq. (2))

$$\sigma_{OP} = 10.4 \text{ kgf/mm}^2, \epsilon_{OP} = 6.824 \times 10^{-4} \text{ mm/mm}, n_p = 5, \alpha_p = 3.71 \quad (11)$$

7) Fatigue crack growth:

$$da/dN = 2.2 \times 10^{-3} (\Delta J)^{1.66} \quad (12)$$

8) Creep equation: (Eq. (7))

$$\dot{\epsilon}_{OC} = 1 \text{ mm} \cdot \text{mm}^{-1} \cdot \text{hr}^{-1}, \sigma_{OC} = 1 \text{ kgf/mm}^2, \alpha_C = 6.211 \times 10^{-11}, n_C = 6.05 \quad (13)$$

9) Creep crack growth:

$$da/dt = 0.12C^* \quad (14)$$

10) Load history:

Examples of load histories are shown in Fig. 3 and Fig. 4. Ten cases of load conditions were chosen as shown in Table 1. In case 1, creep relaxation was considered as linear stress decrease during hold time to simulate stress in FBR condition. In the other cases, creep relaxation was neglected, and stress levels were parameters. In these analyses, only bending loads were applied without membrane stress.

11) Criterion of leakage:  $a > 0.75$  h

12) Computer:

The computer used in the analyses was IBM 3090. The PCCF is available in CRAY-XMP and will be able to be used in a parallel computer in near future.

### 3.2 Results of the Analyses

The summary of the analyses are shown in Table 1. In the all cases, only leakage occurred. Examples of accumulated leakage probability are shown in Fig. 5 and Fig. 6. Fig. 5 shows the leakage probability of case 1 with creep relaxation.

If the creep relaxation was neglected, the leakage probability increased drastically as shown in Fig. 6. From these results, it is important to consider creep relaxation for probabilistic evaluation of FBRs.

From the results of case 3 through case 10, the relation between bending stress level and leak probability was obtained as shown in Fig. 7. Fig. 7 shows that leak probability is sensitive to stress level. If bending stress exceeds yield stress of material, leak probability increases rapidly.

By using stratified sampling for initial crack depth and ratio, the computational time was reduced. In other problems which is not shown in this paper, probability less than  $10^{-8}$  was able to be analyzed using the PCCF. This means that the Monte Carlo with stratified sampling and approximation of non-linear fracture mechanics parameter is practical for FBR probabilistic evaluations.

#### 4 CONCLUSION

A probabilistic fracture mechanics code PCCF which could analyze half-elliptical crack behavior in a plate under creep-fatigue condition using non linear fracture mechanics parameters was developed. The effects of bending stress level on failure probability was studied using the PCCF as test analyses. As the results, failure mode was leakage not break in all cases analyzed in this study. It is shown that leak probability is sensitive to stress level and increase rapidly around yield stress of materials.

Further study is being carried out to evaluate the integrity of FBR components under more actual conditions by improving the code to be able to analyze fracture parameters under combination of membrane and bending stress.

#### REFERRECES

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Table 1 Load Conditions and Analytical Results

| Case | Load cycles<br>N | Hold time<br>$t_h$ (hr) | Bending stress (kgf/mm <sup>2</sup> ) |              |              | The number<br>of samples | Leakage<br>probability | CPU time<br>(sec) |
|------|------------------|-------------------------|---------------------------------------|--------------|--------------|--------------------------|------------------------|-------------------|
|      |                  |                         | $\sigma_b^1$                          | $\sigma_b^2$ | $\sigma_b^3$ |                          |                        |                   |
| 1    | 120              | 1800                    | -12.22                                | 12.22        | 7.42         | 3672                     | $5.33 \times 10^{-3}$  | 110               |
| 2    |                  |                         | -12.22                                | 12.22        | 12.22        | 960                      | 1.0                    | 8                 |
| 3    |                  |                         | -5.02                                 | 5.02         | 5.02         | 7363                     | $2.14 \times 10^{-3}$  | 74                |
| 4    |                  |                         | -8.32                                 | 8.32         | 8.32         | 5456                     | $2.46 \times 10^{-3}$  | 168               |
| 5    |                  |                         | -9.36                                 | 9.36         | 9.36         | 6396                     | $3.41 \times 10^{-3}$  | 228               |
| 6    |                  |                         | -10.40                                | 10.40        | 10.40        | 3456                     | $1.16 \times 10^{-2}$  | 135               |
| 7    |                  |                         | -10.92                                | 10.92        | 10.92        | 1211                     | $7.05 \times 10^{-2}$  | 42                |
| 8    |                  |                         | -11.44                                | 11.44        | 11.44        | 1303                     | $4.61 \times 10^{-1}$  | 27                |
| 9    |                  |                         | -11.65                                | 11.65        | 11.65        | 710                      | $6.63 \times 10^{-1}$  | 15                |
| 10   |                  |                         | -11.96                                | 11.96        | 11.96        | 960                      | 1.0                    | 10                |

note :  $\sigma_b^1$ ,  $\sigma_b^2$  and  $\sigma_b^3$  are shown in Fig. 4.

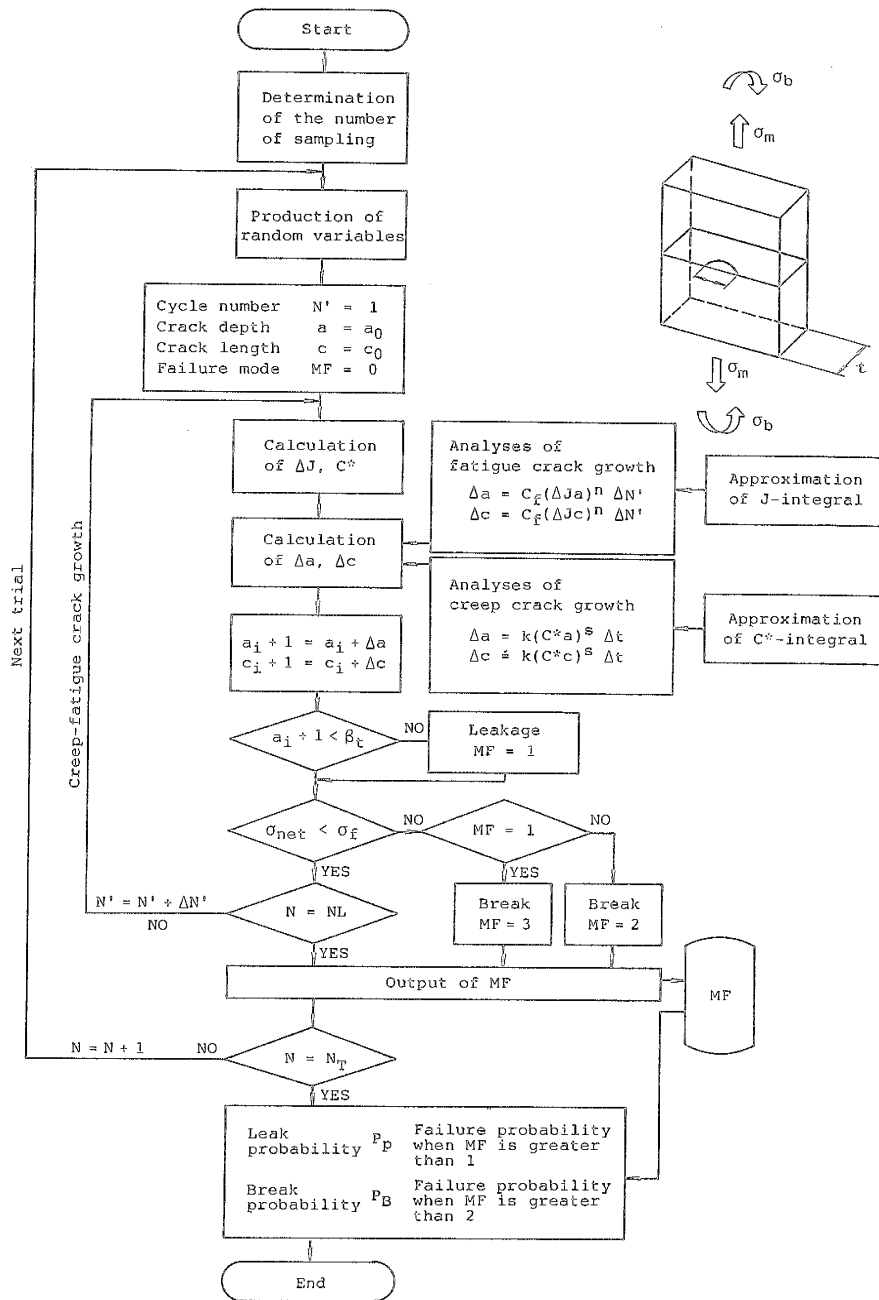


Fig. 1 Program Flow Chart of the PCCF

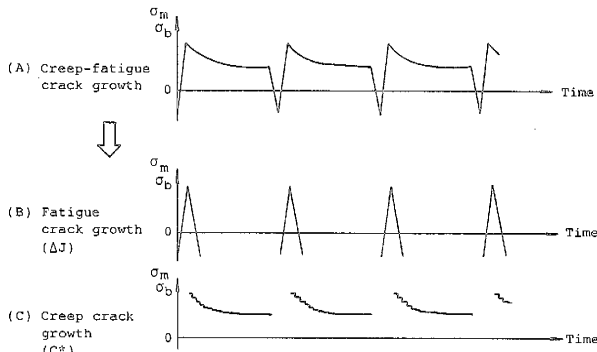


Fig. 2 Separation of Load History into Fatigue and Creep Stages

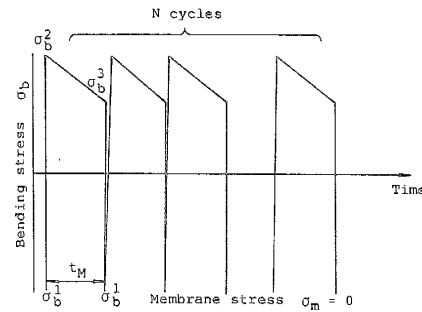


Fig. 3 Example of Load History (Case 1)

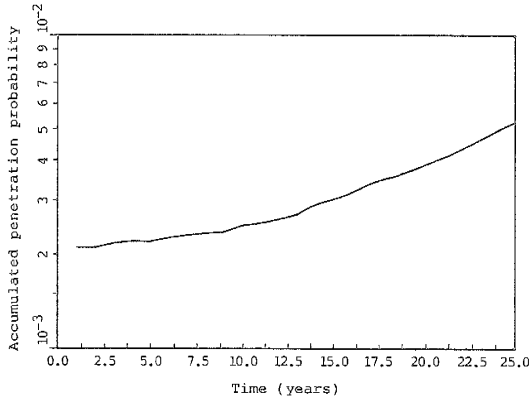


Fig. 5 Accumulated Leakage Probability (Case 1)

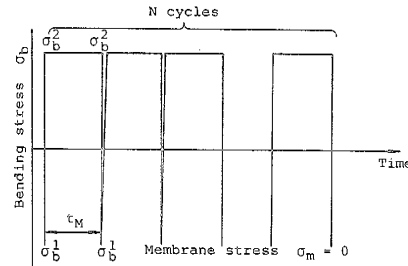


Fig. 4 Example of Load History (Case 2 ~ Case 10)

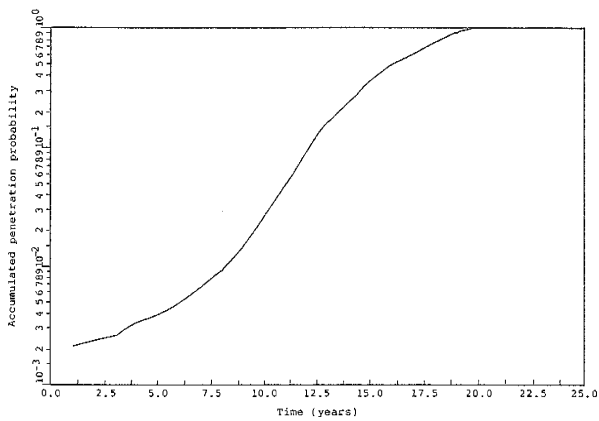


Fig. 6 Accumulated Leakage Probability (Case 2)

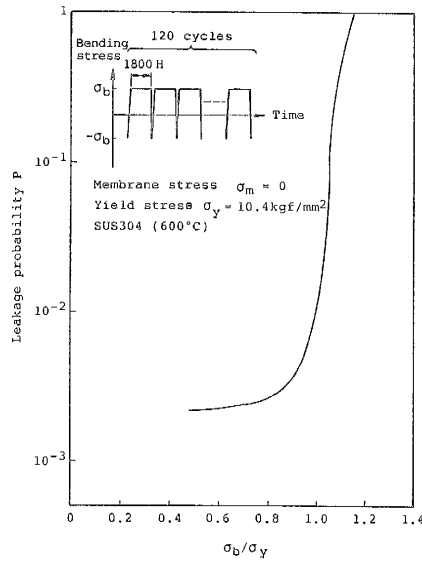


Fig. 7 Relation between Stress Level and Leakage Probability