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Improved generation and application of artificial time histories and PSD functions

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ABSTRACT

Using new approximate Transient Random Vibration (TRV) theory models, a multi-purpose computer program *PowerSpec* was constructed for improved generation and application of artificial accelerograms (AA's) and power spectral density (PSD) functions optimally matching design response spectra (DRS) and satisfying USNRC regulatory requirements.

The paper describes the new approximate models, and the improvements that were built or can be added in *PowerSpec* for the generation of AA's and IRS. Non physical aspects in regulatory DRS such as the USNRC 1.60 are also identified..

INTRODUCTION

The Random Vibration (RV) theory is a powerfull method not only to generate directly In-Structure Response Spectra (IRS), but also to generate Artificial Accelerograms (AA's) matching given Design Response Spectra (DRS).
The process starts for both applications with a generation of a PSD matching single or multiple damping DRS.
Two existing RV theory paradoxes being the so-called ZPA or high frequency paradox, and the damping paradoxes were resolved using a new approximate TRV model, which shows from the information built in the DRS at different damping, that the process is amplitude and frequency evolutionary.
This allows improved applications of the RV theory for generation of IRS, and the generation of AA's behaving in all aspects like real ones.
Besides the improved generation of PSD functions optimally matching 2 damping DRS, several improved features have or can be added to the program *PowerSpec* to generate AA's satisfying the USNRC SRP 3.7.1 DRS and PSD enveloping requirements and phase relationships between support AA's.

THEORETICAL ASPECTS

General DRS - PSD Relationship

The pseudo absolute acceleration DRS can in its most general way be written in function of a non-stationary relative displacement response PSD S_{un} (ω, t) as:

$$S_{pA}^2(\omega_n, \xi_n) = \omega_n^4 . \max_t \left[\eta_n^2(t) . \int_0^\infty S_{un}(\omega, t)\, d\omega \right] \quad (1)$$

For light modal dampings and a peaked $|H_n|^2$ Eq. (1) can be approximated by:

$$S_{pAn}^2 = \omega_n^4 . \eta_n'^2 . \int_0^\infty |H_n|^2 \; G(\omega)\, d\omega \quad (2)$$

The problem is then to find the separable PSD part G(ω) from S(ω, t) from Eq. (2).

Resolution of the PSD Paradox

The classical solution of the problem to find G(ω) from the DRS S_{pAn}^T involves successive iterations starting with the broadband solution of Eq. (2) as a first guess, and iterations as per Eq. (4).

$$G(\omega)(o) = \frac{\pi}{4} \frac{\omega_n}{\xi_n} \left(\frac{S_{pAn}^T}{\eta'_n} \right)^2 \quad (3)$$

$$G(\omega)(I+1) = G(\omega)(I) . \left(\frac{S_{pA}^T(\omega)}{S_{pA}(\omega)} \right)^2 \quad (4)$$

This leads to the ZPA paradox, i.e. when the final PSD G(ω) is used in Eq. (2) the S_{pAn} remains higher than the S_{pAn}^T in the high frequency regions, particularly the ZPA.

Writing an approximation of Eq. (2) in a matrix format with the same $\omega_i = \omega_n$ points i.e.:

$$\left\{ \frac{S_{pAi}^2}{\eta_i'^2} \right\} = [\quad M \quad]\{G_i\}$$

and performing the inversion operation:

$$\{G_i\} = [\quad M \quad]^{-1} \left\{ \frac{S_{pAi}^2}{\eta_i'^2} \right\}$$

P. Mertens has succeeded to find an approximate inversion integral equation between G(ω) and $S_{pA}(\omega)$ in an analytical format (see Ref.[1]).

Figure 1 shows the result of using this inversion integral solution G(ω)(o) versus the classical broadband solution. As clearly seen the ZPA paradox has dissapeared.

Resolution of the Damping Paradox

Using the most simple non-stationary expression being a rectangular unit function of duration t(ω)depending on the frequency, and the simple Rosenblueth and Bustamente model of the peak factor being:

$$\eta_n^2 = \eta_o^2 \cdot \left(1 + \frac{2}{\xi_n \omega_n t}\right)^{-1} \tag{5}$$

We obtain by putting t in Eq. (5) dependent on ω_n:

$$\eta'^2_n = \eta_o^2 \left(1 + \frac{2}{\xi \omega_n t(\omega_n)}\right)^{-1} \tag{6}$$

Using the broadband solution for S_{pAn} at 2 different damping, ξ_n (n = i, j) we obtain $t(\omega)$ from:

$$t(\omega) = \frac{2}{\omega}\left(\frac{S_{pA}^2(\omega,\xi_i) - S_{pA}^2(\omega,\xi_j)}{\xi_j S_{pA}^2(\omega,\xi_i) - \xi_i \cdot S_{pA}^2(\omega,\xi_j)}\right) \tag{7}$$

Figure 2 shows the result of the application of Eq. (7) to calculate the separable PSD $G(\omega)$ from the DRS of USNRC R.G. 1.60 at 2 different dampings, compared to a classical solution.

As noted the 2 PSD's now coincide at 2 different dampings, which is not the case using the classical RV theory approach and which solves the damping paradox.

Non-Physical Aspects of Idealized DRS

With the models developed above, we should for physical reality of a DRS obtain that:

- <u>The PSD $G(\omega)$ should always be positive.</u> It was found that the USNRC R.G. 1.60 spectrum at 2% damping satisfies the criterium, but at higher damping, e.g. the 5% spectrum it does not at the high frequencies.
- <u>The $t(\omega)$ PSD determined from Eq. (7) should:</u>
  - have a reasonable strong motion time and total duration
  - be usually a declining function in function of frequency
  - be independent of the damping pairs ξ_i, ξ_j.

  It was found for the USNRC R.G. 1.60 spectrum that the first (about 20 sec. and 70 sec.) and second conditions are satisfied (however the decline is too rapid and the total duration too high (70 sec.); indicating that a less time dependent peak factor model than the Rosenblueth one, should be used). The third condition was not satisfied for 3 dampings 2%, 5%, 7%.

Non-physicalities in the idealized DRS were often found, especially concerning the $t(\omega)$ PSD. It has no impact when working in the frequency domain with only 2 dampings, but serious impact in the real time domain, necessitating a different pragmatic approach.

Extension to MDOF Systems

The frequency evolutionary model described above can simply be extended to MDOF systems by using an equivalent modal damping, and a constant peak factor.

$$\xi'_n = \xi_n + \frac{2}{\omega_n\, t(\omega)} \text{ in the SDOF complex TFF } H_n^* = \left[\left(\omega_n^2 - \omega^2\right) + j\,2\xi'_n\,\omega_n\omega\right]^{-1} \tag{8}$$

Figure 4 shows the improvement realized for the calculation of an IRS.

Physical Relationships for AA Generation

Amplitude evolutionary aspects

It was found in Ref. [2], that after studying several real accelerograms that the amplitude time history could be very well approximated by:

$$A(t) = t^{\gamma} e^{-\alpha t} \tag{9}$$

where t^{γ} and α are constant parameters characteristic for a certain earthquake.

Frequency evolutionary aspects

It was also found in Ref. [2] that the zero and peak crossings $N_o(t)$, $N_p(t)$ of several earthquakes could very well be fitted by:

$$N = N_1 . (1 - e^{-xt}) \tag{10}$$

were x_o, x_p are zero and peak crossing decay rates, characteristic for a certain earthquake.

Consequently the most appropriate physical model for the non-separable (in t and ω) PSD of the accelerogram is given by:

$$A^2 (\omega, t) = t^{\gamma} . e^{-\alpha(\omega).t} \tag{11}$$

Relationships were developed between t(ω) found from the DRS, and α(ω). It also turns out mathematically that the best fit of the amplitude of Eq. (9) with a unit step function leads to $\gamma = 2$.

Also approximate relationships between α(ω) and the strong motion time, total duration, x_o, x_p were developed which allows to use these parameters as an input rather than coming from Eq. (7) which in most cases turns out to be irrealistic because of the non-physicality of the idealized input DRS.

Method for AA Generation

Non-frequency evolutionary

The method used in *PowerSpec* generates the non-frequency evolutionary AA a(t) from:

$$a(t) = \sqrt{2}\, A(t) . \sum_i G^{1/2}(\omega_i) \Delta\omega_i^{1/2} . \sin(\omega_i t + \theta_i) \tag{12}$$

in which

A(t)	=	$t^{\gamma} e^{-\alpha t}$
θ_i	=	a random phase angle (O, 2π)
G(ω)	=	the one sided PSD

The summation term is generated using the Inverse Digital Fourier Transform (IDFT) (see Ref. [3]).

Frequency evolutionary (Ref. [4])

In this case the non-separable part of the PSD is given by:

$$A^2 (\omega, t) = t^{\gamma} e^{-\alpha(\omega)t}$$

Assuming $\alpha(\omega)$ a constant = α_j in the frequency bands (ω_j, ω_{j+1}), Eq. (12) can be written as:

$$a(t) = \sqrt{2}\,.\sum_j (t^\gamma\ e^{-\alpha_j t}) \sum_{i(j)} G_j^{1/2}(\omega_i)\,\Delta\omega_{i(j)}^{1/2}\,.\sin\left(\omega_{i(j)}.t + \theta_{i(j)}\right) \qquad (19)$$

If ω is subdivided in J frequency bands, the calculation needs J times the generation by IDFT of the terms in the summation.

<u>*Deterministic Phase Angle*</u>

The use of multiple IDFT in the J frequency bands can be avoided by multiplication of the $G_i(\omega)$ with a suitable TFF squared, and a deterministic phase angle φ_i (ω) to be added to the random phase angle θ_i.

The use of a deterministic phase angle as an input is also of great use to generate multiple support accelerograms which maintain their phase relationship amongst each other, or to simulate the beating found in real floor accelerograms.

The deterministic phase angle has not yet been implemented in *PowerSpec* version 2.0.

THE PROGRAM *POWERSPEC*

The flowchart of the current version 2.0 of the multipurpose computer program *PowerSpec* is shown in Figure 3.

It currently contains 6 modules:

- <u>Module 1</u>: (FTE): generates from 2 DRS at 2 dampings the $f.t_e(f)$ ($\omega.t(\omega)$) function.
- <u>Module 2</u>: (PSD): generates from the DRS and the FTE the separable PSD $G(\omega)$.
- <u>Module 3</u>: (PSD to AA): generates from the PSD the AA.
- <u>Module 5</u>: (PSD to RS): generates from the $\omega.t(\omega)$ and the separable PSD $G(\omega)$ the RS.
- <u>Module 7</u>: (TH to RS): generates the RS from a time history.
- <u>Module 8</u>: (TH to PSD): generates from a time history the separated PSD.

As seen it can work in the frequency domain (TRV analysis), and in the time domain (AA, RS, PSD generation).

In the frequency domain: IRS generation

A (currently separate) TFF module is necessary to calculate the IRS. This operation can easly be performed even with a spreadsheet but needs very fine frequency spacing. The next step is to calculate from the response PSD (= input PSD x $|\text{TFF}|^2$) the IRS using module 5.
Figure 4 shows the improvement that is realised for the calculation of the IRS of a typical system using the new TRV theory versus the classical RV theory and when compared to multiple AA generation using *PowerSpec.*

In the time domain: AA generation

As well multiple (up to 99) in one run or single (1) AA generation, RS and PSD calculations are possible with *PowerSpec* 2.0.

This aspect is very important for the calculation of the target PSD (being the multiple AA, average PSD) and the minimal PSD requirement (80% of target) to satisfy the USNRC SRP 3.7.1 PSD enveloping requirement for single AA generation.

Iterations for matching the single or average multiple AA's RS with the single damping target DRS is performed automatically in module 3.

Figure 5 shows a typical accelerogram generated, whereas Figure 6 shows the result of the improved possibilities of *PowerSpec* to match the DRS with the average RS from 10 AA's, as well as the spread min./max. of individual AA's.

Iterations can also be performed to optimally generate AA's which:

- envelope the single damping RS and minimum PSD requirement of a single AA
- envelope the multiple damping RS.

Such multiple condition iterations are impossible or very difficult to perform using AA generating methods in the time domain starting from a real accelerogram.

Noteworthy is also that the statistical independency of several AA's is inherently guaranteed to its maximum. The program contains the standard peak raising/folding technique to limit the ZPA (Ref. [3]), as well as a double method to correct the AA to produce realistic velocity and displacement TH of the accelerogram TH (Ref. [1]).

SUMMARY AND CONCLUSIONS

Using new improved TRV models, and practical realistic relationships, the program *PowerSpec* 2.0 was developed, for direct generation of IRS and AA's, the latter being able to satisfy also the minimal PSD requirements as now required by the USNRC SRP 3.7.1.
The improvements using the new theory were demonstrated. For AA generation the program offers many possibilities not existing with programs such as those based on the iterative adaptation of a accelerogram in the time domain.

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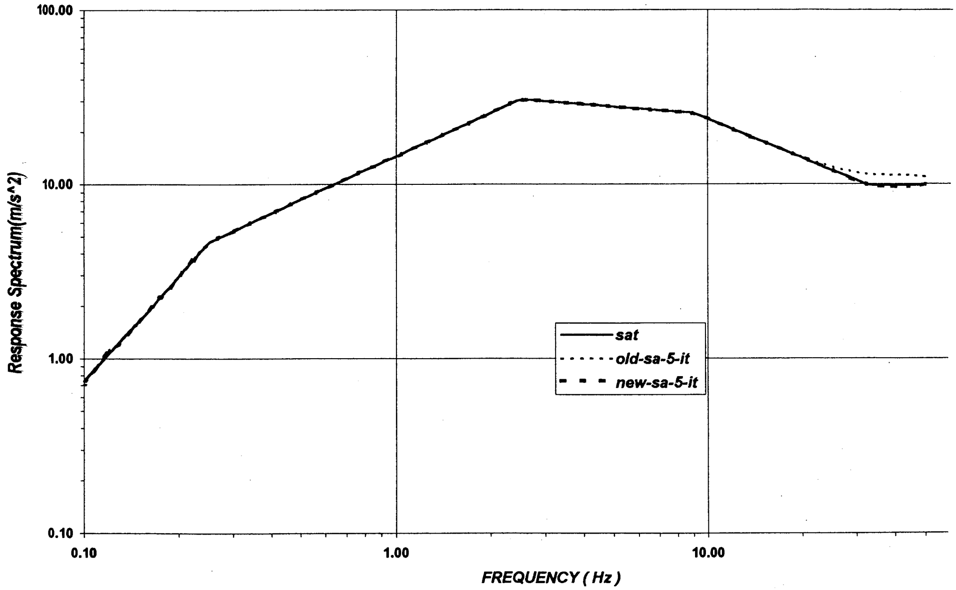

Fig. 1: Illustration of the resolution of the ZPA paradox

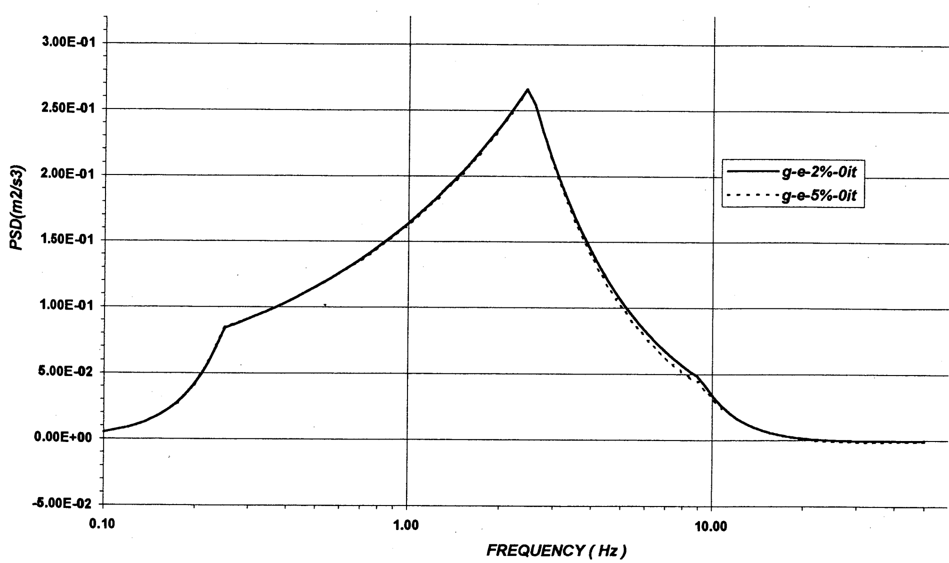

Fig. 2: Illustration of the Resolution of the damping paradox

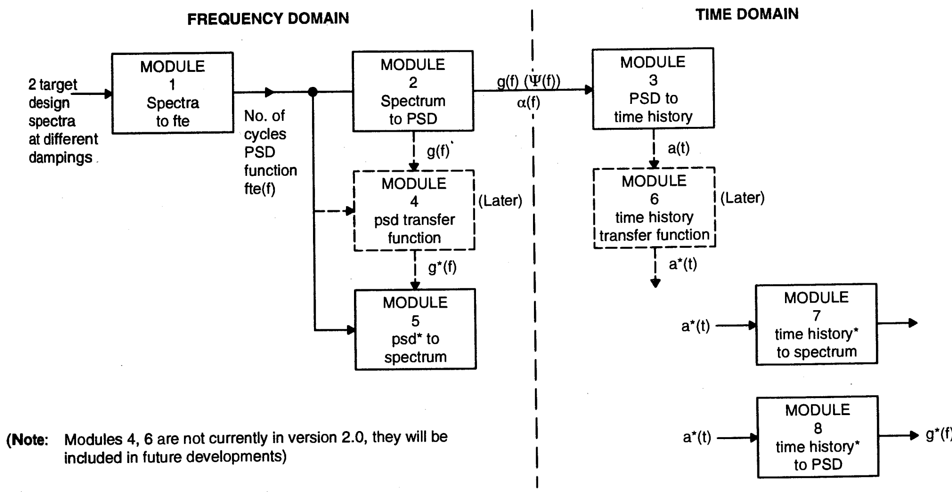

Fig. 3: Flowchart of the program *PowerSpec* 2.0

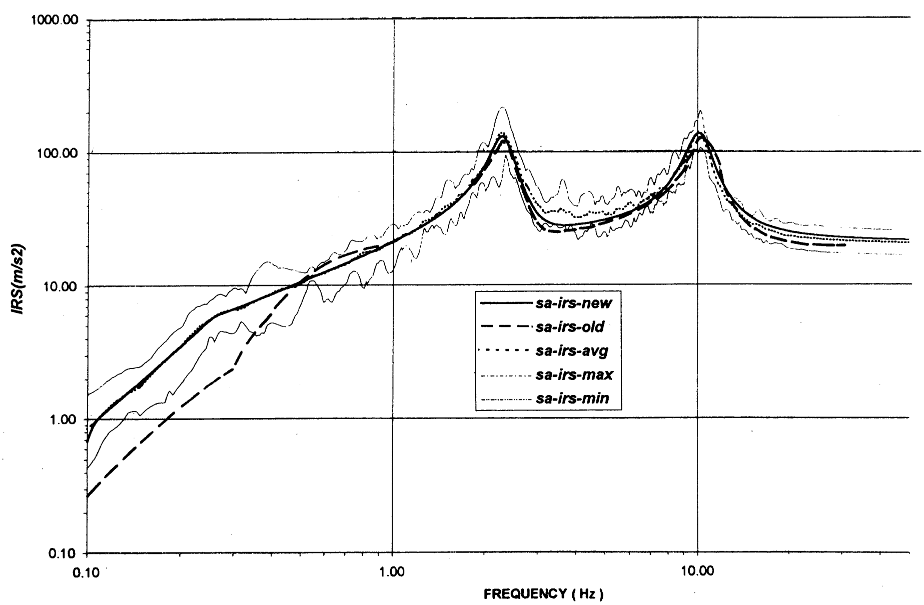

Fig.4: Illustration of improved IRS calculation

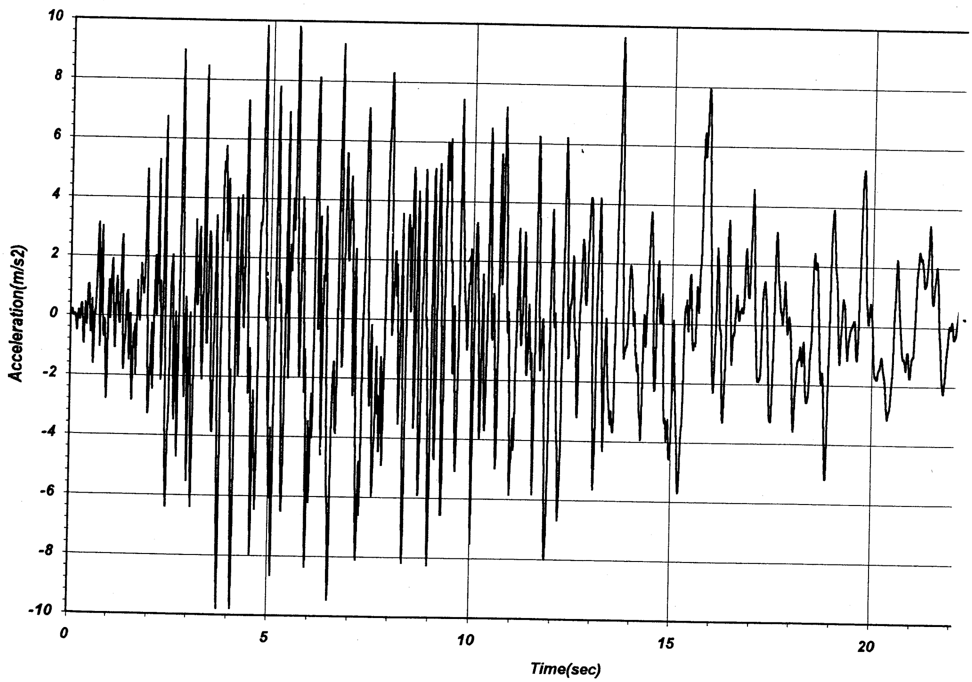

Fig.5: Illustration of improved AA generation

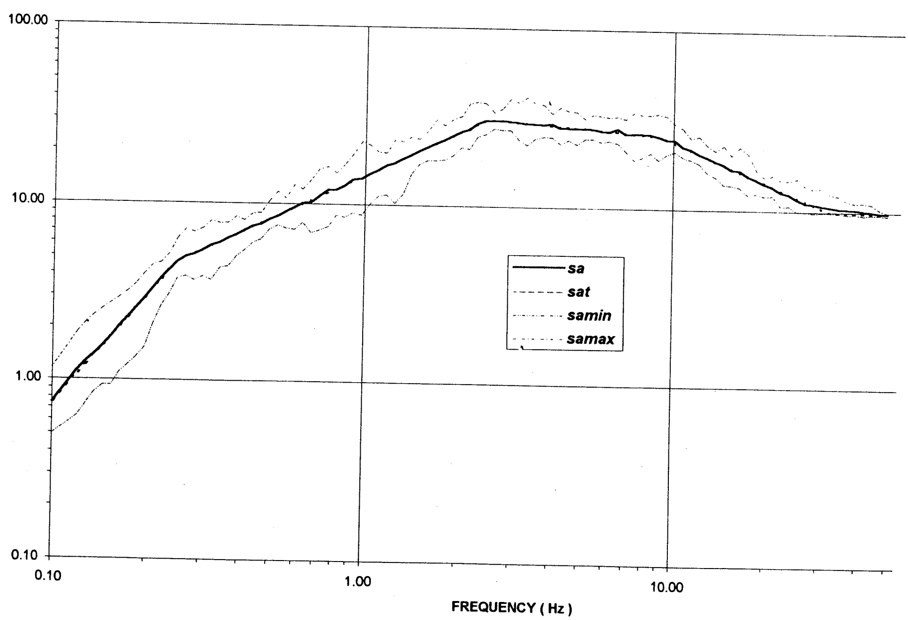

Fig.6: Illustration of improved multiple (10) AA response spectrum matching capabilities