The Stress and Stress Intensity Factors Computation by BEM and FEM Combination for Nozzle Junction Under Pressure and Thermal Loads

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INTRODUCTION

Linear elastic fracture analysis based upon the stress intensity factor evalution has been successfully applied to safety assessments of cracked structures. The nozzle junction are usually subjected to high pressure and thermal loads simultaneously. In validity of linear elastic fracture analysis, K can be decomposed into K^F (caused by mechanic loads) and K^T (caused by thermal loads). Under thermal transient loading, explicit analysis (say by the FEM or BEM) of K tracing an entire history respectively for a range of crack depth may be much more time consuming. The techniques of weight function,Rice,J.R. et al.,(1972), provide efficient means for transforming the problem into the stress computation of the uncracked structure and generation of influence function (for the given structure and size of crack) . For the high accuracy in computation of high stress gradient or the stress intensity factors for the crack front of the nozzle structures, there still remains some numerically difficulties. The boundary element methods and finite element methods have been widely adopted for this purpose. Due to the complexity in the configuration of the structure and the practical working condition, it is not always efficient to solve the whole structure or trace an entire history of loads by the sole means of either the finite element or boundary element methods. A combination of boundary element and finite element methods will preserve the advantages of both schemes but reduce the disadvantages by using either method alone.

In this paper, a combination of BE-FEM has been used for the analysis of the cracked nozzle structure by techniques of weight function. The influence functions are obtained by coupled BE-FEM and the uncracked structure stress are computed by finite element methods. Since the boundary element method has been adopted for the domain with crack, a set of singular elements, including crack elements of 'line traction singularity',elements of 'point singularity' and the transition elements are used in neighbourhood of the crack. Under thermal loads,the analysis of temperature and stress fields of the nozzle structures is performed by the unified formulation for two kinds of finite elements. Some results of stress intensity factors computations, in case of high pressure, steady temperature loading and thermal shock, are given.

THE BOUNDARY ELEMENT METHODS OF 3D CRACK ELASTIC ANALYSIS

For the case of three dimension elastic problems without body forces, the boundary integral equations can be written as :

$$c_{ij}(p)u_j(p) = \int_S U^*_{ij}(p,q)t_j(q)dS(q) - \int_S T^*_{ij}(p,q)u_j(q)dS(q) \qquad i,j=1,2,3 \qquad (1)$$

For the boundary elements,the coordinate, displacement and traction components can be interpolated in the following manner:

$$x_i = N^e x_i^e, \quad u_i = \Phi^e u_i^e, \quad t_i = \Psi^e t_i^e \tag{2}$$

After the discretization around the boundary S, the boundary integral equations can be rewritten as:

$$c_{ij}(p)u_j(p) + \sum_{b=1}^{NE}\sum_{e}\int_{S_b} T_{ij}^S(p,q)\Phi^e(\xi_1,\xi_2)J(\xi_1,\xi_2)d\xi_1 d\xi_2 u_j^e$$

$$= \sum_{b=1}^{NE}\sum_{e}\int_{S_b} U_{ij}^S(p,q)\Psi^e(\xi_1,\xi_2)d\xi_1 d\xi_2 t_j^e \tag{3}$$

By applying the specified boundary conditions, equation (3) can be rearranged in matrix form:

$$\underline{A}\underline{x} = \underline{f} \tag{4}$$

Where $\underline{x}$ is the boundary unknown and $\underline{A}$,$\underline{f}$ can be found in many BEM literatures,say,Du,Q.H.,et al (1982).

In this paper,the structures are always discretized by the second order isoparametric boundary elements. The traction singular elements with 'line singularity',Luchi,M.l. et al.,(1983),are used around the crack boundary while the triangular boundary elements with 'point' singularity are used for the free surface at the crack tip. The transition elements have been used between singular elements and the isoparmetric boundary elements. The stress intensity factors can be obtained from the computed stress field near the crack. The techniques have been checked by some specific examples in order to verify that it should be less in cost and more effective than previous studies,Cen,Z.Z. et al.,(1988). Considering the compatibility of the stress and displacements on the interface of the boundary elements and the finite elements, the coupling of the two methods is carried out,Cen,Z.Z. et al.,(1987).

THE ANALYSIS OF THE HEAT TRANSFER BY FINITE ELEMENT METHODS

The governing equations and boundary conditions of temperature for heat transfer are:

$$div(K grad T) + \rho Q = \rho c T_{,t} \qquad in V \tag{5}$$

$$T = \overline{T} \qquad on S_1 \tag{6}$$

$$(K grad T)\cdot n = \overline{q} \qquad on S_2 \tag{7}$$

$$(K grad T)\cdot n = h(T_a - T) \qquad on S_3 \tag{8}$$

Using a general scheme of FEM such as a Galerkin formulation operation on the heat transfer equation and boundary conditions, the matrix equation of FEM be written as follows:

$$\underline{C}\dot{\underline{T}} + \underline{K}\underline{T} = \underline{F} \tag{9}$$

Where $\underline{C}$ is the matrix of heat capacity, $\underline{K}$ is the matrix of heat conduction , $\underline{F}$ is the column of thermal loads.

The code TAP for the unified finite element analysis of temperature and stress fields has been established by another colleague in Tsinghua Group, Zhang,Y.,(1988). In transient heat transfer analysis,the time integration algorithm has been used and the time steps has been provided by the minimum eigenvalues. The transient thermal stress are computed by quasi-stable process at each time step (the coupled thermal-mechanical effect has been ignored).

TECHNIQUES OF WEIGHT FUNCTION

For the purpose of illustration, a nozzle junction is considered in Fig. 1. Tractions $\sigma_{ij}=x^i y^j$ were applied on the crack surface. The corresponding stress intensity factors $K_{ij}(\theta)$ as the influence functions were computed around the crack front by BE-FEM. Once the practical working condition is given, we can obtain the hoop stress distribution of the uncracked structure on the surface of the crack by FEM/BEM.

Let the hoop stress on the crack surface:

$$\sigma_{33}(x,y)=\sum c_{ij}x^i y^j \tag{10}$$

The stress intensity factors K(θ) under practical working condition can be written as:

$$K(\theta)=\sum c_{ij}K_{ij}(\theta) \tag{11}$$

SOME COMPUTATIONAL RESULTS

This example is taken for the case of the nozzle junction with the corner crack. the coupling analysis of finite elements and boundary elements was performed with discretization shown in FIG. 1, where t=1cm, t_1=0.5cm, h=10cm, d=4cm, D=16cm, E=2.1×10^6 kg/cm^2, ν=0.3,the heat extension coefficient α =0.0000125cm/cm.°C, heat conduct coefficient k=0.0836Cal/cm.s.°C. The results include:

1) The nozzle structure subjected to internal pressure
p=100kg/cm^2, a/r =0.4 and 0.2, the stress intensity factors computed directly along the crack front was given in FIG. 2. FIG. 3 gives comparison of normalized K obtained by this paper and results from other authors (the difference of the configuration of structures is ignored)

2) The influence functions of the cracked nozzle structure
Considering the crack surface of the structure subjected to $\sigma_{ij}=x^i y^j$, (i,j =0,1,2), Table 1 gives the influence functions $K_{ij}(\theta)$, where a/r=0.2.

When the structure is subjected to internal pressure ,the difference of the stress intensity factors obtained by techniques of weight function and that from direct computation is less than 3 percent for all point. And thus it has been proved the validity of techniques for using the weight function.

3) The stress intensity factors under internal pressure and steady thermal load
The pressure is 100 kg/cm^2, the temperature on internal surface is 100°C, the temperature on the external surface is 300°C, Fig. 4 gives the distribution of K along the crack front.

4) The stress intensity factors under thermal shock.

The working condition orients toward the simulation of some accidential cases, where the temperature on internal surface is descended from 500°C to 100°C during 0.001 sec., and the external surface is insulated. The result of analysis of the example has shown that the system would be steady after 42 sec. or thirteen steps of computation. The stress intensity factors of the entire process are given in Fig. 5.

CONCLUSION

With some sort of improved techniques for the BEM involved the stress intensity factors of a complicated component such as nozzle junction has been computed. The combination of FEM and BEM has been suggested as an efficient method for solveing the stresses and stress intensity factors of the nozzle structure not only under high pressure but also under thermal shock.

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Table.1 Influnce functions of the nozzle junction (kg/cm$^{3/2}$)

$2\theta/\pi$	K_{00}	K_{10}	K_{01}	K_{20}	K_{11}	K_{02}
0.000	.729960	.209930	.040750	.069540	.008420	.005810
0.125	.783710	.217140	.073460	.070000	.018200	.010630
0.250	.742880	.196070	.099900	.060310	.026080	.016790
0.375	.742150	.179150	.128960	.050480	.031870	.026810
0.500	.734800	.155390	.154890	.038430	.034020	.038260
0.625	.736270	.128630	.177650	.026790	.031770	.050070
0.750	.733680	.099070	.193690	.016640	.025890	.059660
0.875	.722080	.072620	.213680	.010510	.017960	.069000
1.000	.740390	.042260	.210400	.006120	.008520	.069570

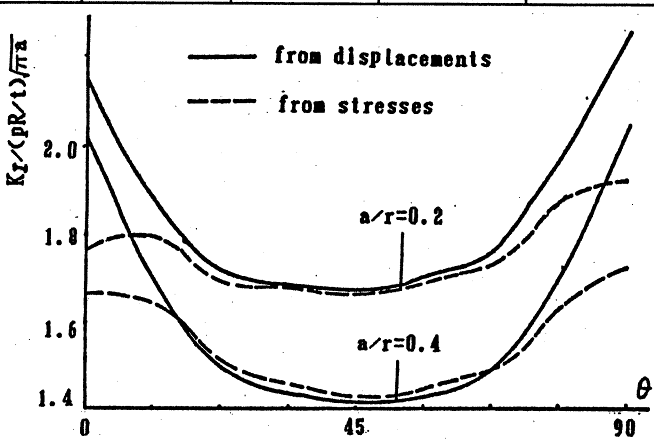


Fig. 2 K_I along the crack front

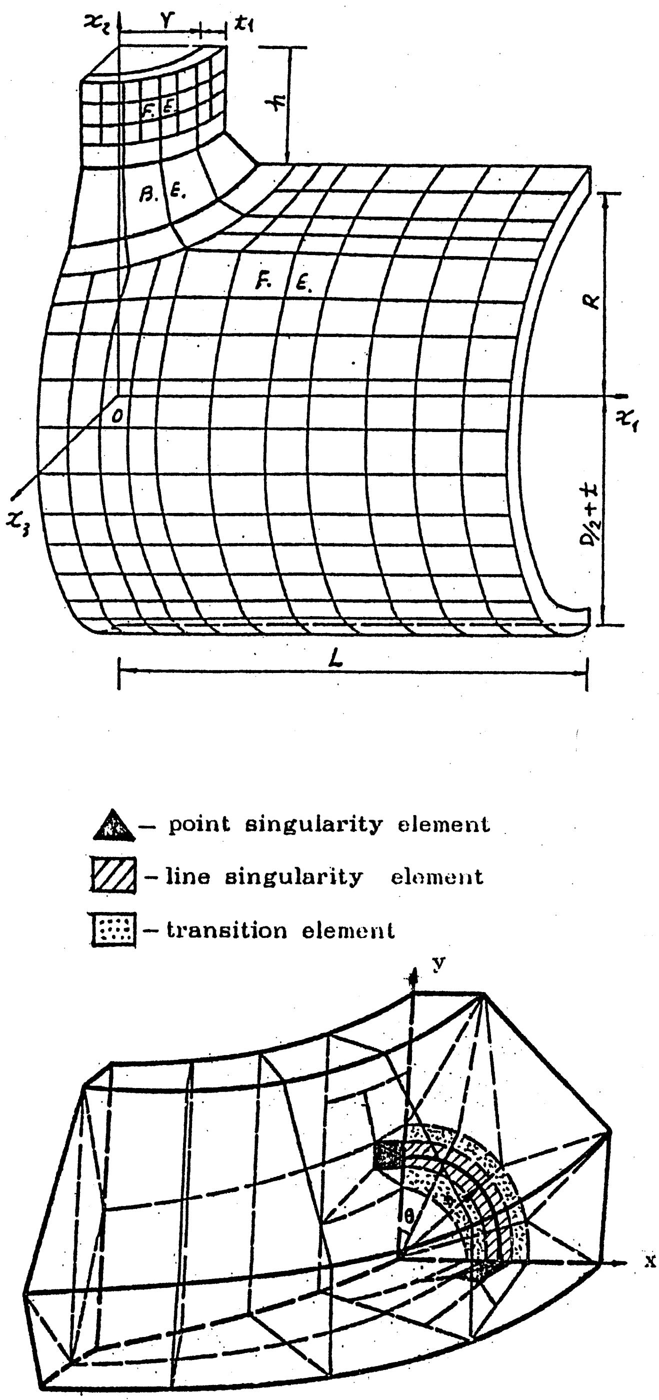


Fig. 1 The nozzle with the corner crack

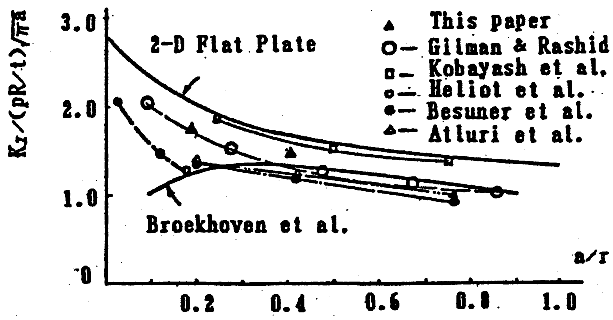


Fig. 3 Comparison of normalized K_I

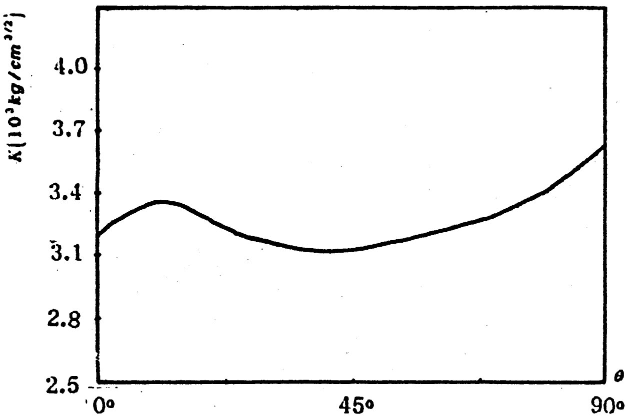
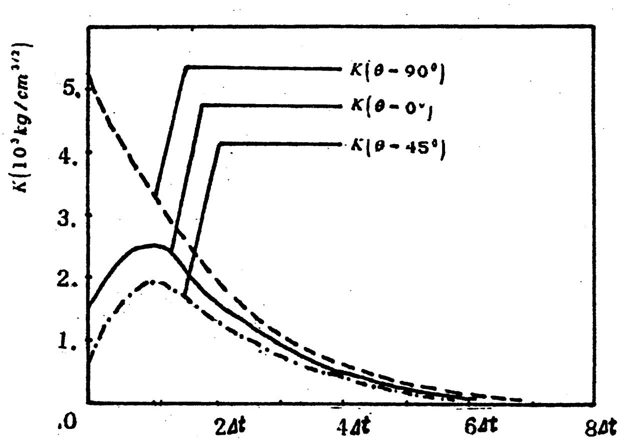


Fig. 4 The distribution of K along the crack front



($\Delta t = 3.44 s$)

Fig. 5 K of the entire process

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