Various Aspects of Response of the Cantilever Beam with Hysteresis Damping

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ABSTRACT

The steady-state response of piping systems with nonlinear boundary conditions is of great importance in nuclear engineering applications. Collision characteristics is one of the most important nonlinear characteristics. In this paper, the nonlinear piping system with triangular and quadrilateral characteristics are introduced. And, as one span model of piping system, a beam clamped at one end and colliding with the stop at the other end is dealt with. An analytical method of approximate solution for the response of the beam is presented. In triangular hysteresis loop characteristics is the most important model and quadrilateral hysteresis loop characteristics is more practical model for collision characteristics. The resonance curves of the beam with various support are shown and are compared with those of the results for various nonlinear model.

INTRODUCTION

The forced response of the continuous systems with nonlinear boundary conditions is of great importance in nuclear engineering applications[1]. The piping systems have nonlinear boundary conditions[2]. Collision characteristics is one of the most important nonlinear characteristics observed at supports and joints. It is very difficult to find the analytical solution of the equation of motion for the given nonlinear boundary conditions because in the response analysis, bilinear force-penetration characteristics in which stiffness increases during collision is usually used[3] or the coefficient of restitution is assumed to be constant[4]. However, in those models, energy loss or duration of collision are not considered.

In this paper, an analytical method for approximate solution of steady-state response of the piping system with symmetrical collision characteristics is presented. As one span model of piping system, a beam clamped at one end and colliding with the stop at the other end is dealt with. The stop is assumed to have hysteresis loop characteristics in order to consider the energy loss in a collision and duration of collision. For hysteresis loop characteristics, triangular and quadrilateral characteristics are introduced. In triangular hysteresis loop characteristics, the coefficient of restitution is not dependent on velocity response at the beam end. In quadrilateral hysteresis loop characteristics, the coefficient of restitution depends on velocity response at beam end[5]. Quadrilateral hysteresis loop characteristics is more practical model for collision characteristics. Force of restitution is expanded into the Fourier series. Only fundamental terms of the Fourier series are considered.

The resonance curves of the beam with various support having hysteresis loop characteristics are obtained by the method of approximate solution and are compared with those of the results for various nonlinear boundary conditions.

ANALYTICAL METHOD FOR THE SYSTEM WITH QUADRILATERAL HYSTERESIS LOOP CHARACTERISTICS

A simplified dynamical model of the piping system with symmetrical collision characteristics is shown in Fig.1. Namely, this model consists of a beam with one end simply supported, with two-sided amplitude constrained by the hysteresis loop characteristics (Fig.2) at the other end. Let ρ be the mass density, A the cross-sectional area and EI the modulus of flexural rigidity. The equation of motion for transverse free vibration of a beam can be written as follows:

$$\frac{\partial^{2} y}{\partial t^2} + \frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4} = 0 \tag{1}$$

The relations between y and z as shown in Fig.1 are given by

$$y = z + y_0 \cos \omega t \tag{2}$$

Hence we have

$$\frac{\partial^{2} z}{\partial t^2} + \frac{EI}{\rho A}\frac{\partial^4 z}{\partial x^4} = y_0 \omega^2 \cos \omega t \tag{3}$$

The solution of Eq.(1) is assumed as:

$$y = \sum_{n=1}^{\infty} X_n(x) \cos n\omega t \tag{4}$$

A formal solution of Eq.(3) can be expressed as follows:

$$z = -y_0 \cos\omega t + \sum_{n=1}^{\infty} \left(A_n \cosh\lambda_n x + B_n \sinh\lambda_n x + C_n \cos\lambda_n x + D_n \sin\lambda_n x\right)\cos n\omega t \tag{5}$$

where A_n, B_n, C_n and D_n are constants to be determined in each particular case from the boundary conditions of the beam, where

$$\lambda_n{}^4 \ell^4 = Z_1{}^4 n^2 \Omega_1{}^2,\ \Omega_1 = \omega/\omega_1 \tag{6}$$

and

$$\omega_1 = \frac{Z_1{}^2}{\ell^2}\sqrt{\frac{EI}{\rho A}},\ Z_1 = 1.8751 \tag{7}$$

The boundary conditions for this case are as follows:

$$1)\ x = 0,\ z = 0 \tag{8}$$

$$2)\ x = 0,\ \frac{dz}{dx} = 0 \tag{9}$$

$$3)\ x = \ell,\ \frac{d^2 z}{dx^2} = 0 \tag{10}$$

$$4)\ x = \ell,\ EI\frac{d^3 z}{dx^3} = f(z_\ell, \dot{z}_\ell) \tag{11}$$

where z_ℓ is the transverse displacement at the beam end (x= ℓ) and the force of restitution $f(z_\ell, \dot{z}_\ell)$ is defined by the piecewise-linear characteristics as shown in Fig.2:

$$f(z_\ell,\dot{z}_\ell)\begin{cases} = K_1(z_\ell - e_0);\ e_0 \le z_\ell \le (e_0+\delta_0),\ \dot{z}_\ell > 0 \quad \text{(I)} \\ = K_1\delta_0 + K_2(z_\ell - e_0 - \delta_0);\ e_0+\delta_0 \le z_\ell \le z_{\ell\max},\ \dot{z}_\ell \ge 0 \quad \text{(II)} \\ = K_3(z_\ell - z_{\ell 3});\ z_{\ell 3} \le z_\ell \le z_{\ell\max},\ \dot{z}_\ell \le 0 \quad \text{(III)} \\ = 0;\ -e_0 \le z_\ell \le z_{\ell 3},\ \dot{z}_\ell < 0 \quad \text{(IV)} \\ = K_1(z_\ell + e_0);\ -\delta_0 \le z_\ell \le 0,\ \dot{z}_\ell < 0 \quad \text{(V)} \\ = -K_1\delta_0 + K_2(z_\ell + e_0 + \delta_0);\ -z_{\ell\max} \le z_\ell \le -(e_0+\delta_0),\ \dot{z}_\ell \le 0 \quad \text{(VI)} \\ = K_3(z_\ell + z_{\ell 3});\ -z_{\ell\max} \le z_\ell \le -z_{\ell 3},\ \dot{z}_\ell \ge 0 \quad \text{(VII)} \\ = 0;\ -z_{\ell 3} \le z_\ell \le e_0,\ \dot{z}_\ell > 0 \quad \text{(VIII)} \end{cases} \tag{12}$$

where $z_{\ell\max}$ denotes the maximum dispalcement at the end (x= ℓ). K_1, K_2 and K_3 are spring constants of nonlinear support as shown in Fig.2 and δ_0 is the maximum deformation by the spring constant K_1. And then, $z_{\ell 3}$ is written as follows:

$$z_{\ell 3} = e_0 + \left(1 - \frac{K_1}{K_3}\right)\delta_0 + \left(1 - \frac{K_2}{K_3}\right)\left(z_{\ell\max} - e_0 - \delta_0\right) \tag{13}$$

In quadrilateral hysteresis loop characteristics, the coefficient of restitution depends on velocity response at the beam end. The coefficient of restitution ε is the ratio of the retreating velocity v_2 to the approaching velocity v_1 and is given as the following equation.

$$\left.\begin{aligned} \varepsilon &= v_2/v_1 = 1.0,\ v_1 \le \bar{v} \\ \varepsilon &= v_2/v_1 = \sqrt{K_2/K_3 + (1 - K_2/K_1)(K_1/K_3)(\bar{v}/v_1)^2} \end{aligned}\right\} \tag{14}$$

which $\bar{v}$ denotes the critical velocity distinguishing a perfectly elastic collision from an imperfectly elastic one, which is

$$\bar{v} = \delta_0\sqrt{K_1/m} \tag{15}$$

Relations between the coefficient restitution and velocity are shown in Fig.3.

From Eqs.(5) and the boundary condition Eq.(8), A_n and C_n are expressed as

$$C_1 = y_0 - A_1 \quad (n = 1) \tag{16}$$

$$C_n = -A_n \quad (n = 2,3,.....) \tag{17}$$

Then Eq.(5) is written as

$$z = y_0(\cos\lambda_1 x - 1)\cos\omega t + \sum_{n=1}^{\infty}\{A_n(\cosh\lambda_n x - \cosh\lambda_n x) + B_n(\sinh\lambda_n x - \sin\lambda_n x)\}\cos n\omega t \tag{18}$$

Using the boundary condition Eq.(9), the following equations are obtained.

$$(\cosh\lambda_1\ell + \cos\lambda_1\ell)A_1 + (\sinh\lambda_1\ell + \sin\lambda_1\ell)B_1 = y_0\cos\lambda_1\ell \quad (n = 1) \tag{19}$$

$$(\cosh\lambda_n\ell + \cos\lambda_n\ell)A_n + (\sinh\lambda_n\ell + \sin\lambda_n\ell)B_n = 0 \quad (n = 2,3,.....) \tag{20}$$

In this paper, the steady-state vibration is dealt with. Once the vibration of the beam becomes steady and periodic, the nonlinear force of restitution $f(z_\ell, \dot{z}_\ell)$ becomes also periodic and can be represented as a periodic function $g(\theta)$ of θ with the period 2π. And θ is defined by the following equation:

$$\theta = \omega t - \alpha \tag{21}$$

where α is the phase lag angle.

This periodic function $g(\theta)$ must satisfy the conditions of the given characteristics of the nonlinear force of restitution equation(12), which is in this case, to be written as the following equations:

$$f(z_\ell,\dot{z}_\ell)\begin{cases} \equiv g(\theta) = K_1(z_\ell - e_0);-(\theta_1+\theta_2) \le \theta \le -\theta_2 \quad (I) \\ \equiv g(\theta) = K_1\delta_0 + K_2(z_\ell - e_0 - \delta_0);-\theta_2 \le \theta \le 0 \quad (II) \\ \equiv g(\theta) = K_3(z_\ell - z_{\ell 3});0 \le \theta \le \theta_3 \quad (III) \\ \equiv g(\theta) = 0;\theta_3 \le \theta \le \pi - (\theta_1+\theta_2) \quad (IV) \\ \equiv g(\theta) = K_1(z_\ell + e_0);\pi - (\theta_1+\theta_2) \le \theta \le \pi - \theta_2 \quad (V) \\ \equiv g(\theta) = -K_1\delta_0 + K_2(z_\ell + e_0 + \delta_0);\pi - \theta_2 \le \theta \le \pi \quad (VI) \\ \equiv g(\theta) = K_3(z_\ell + z_{\ell 3});\pi \le \theta \le \pi + \theta_3 \quad (VII) \\ \equiv g(\theta) = 0;\pi + \theta_3 \le \theta \le 2\pi - (\theta_1+\theta_2) \quad (VIII) \end{cases} \tag{22}$$

where θ_1, θ_2 and θ_3 denote the range of the phase angle θ as shown in Fig.4. In the forgoing, one period 2π of the resulting vibration is divided into eight intervals. During the first, the second and the third intervals and the fifth, the sixth and the seventh intervals of length θ_1, θ_2 and θ_3, respectively, the beam end moves with nonlinear force of restitution of spring. And during the fourth and the eighth intervals of $\pi - (\theta_1 + \theta_2 + \theta_3)$, the beam moves without force of restitution.

A Fourier series expansion for periodic function $g(\theta)$ is assumed as:

$$g(\theta) = \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \tag{23}$$

In this paper, let the function be approximated by

$$g(\theta) = a_1\cos n\omega t + b_1 \sin n\omega t \tag{24}$$

As shown in reference(), when $g(\theta)$ is approximated by only the fundamental terms of the Fourier expansion, approximate solution agree with exact solution for relatively low ratio of nonlinear parameters, K_1/k, K_2/k and K_3/k, used in this paper[6]. And k is equivalent stiffness of the beam defined by Eq.(34).

From the boundary condition equation (11), the following equation is obtained.

$$(\sinh\lambda_1\ell - \sin\lambda_1\ell)A_1 + (\cosh\lambda_1\ell + \cos\lambda_1\ell)B_1 = \frac{a_1\cos\theta + b_1\sin\theta}{EI\lambda_1\cos(\theta+\alpha)} - y_0\sin\lambda_1\ell \tag{25}$$

From Eqs.(10) and (25), B_1 and D_1 are obtained as:

$$A_1 = \frac{y_0}{\Delta_1}(1 + \cosh\lambda_1\ell\cos\lambda_1\ell + \sinh\lambda_1\ell\sin\lambda_1\ell) - \frac{a_1\cos\theta + b_1\sin\theta}{EI\lambda_1{}^3\Delta_1\cos(\theta+\alpha)}(\sinh\lambda_1\ell + \sin\lambda_1\ell) \tag{26}$$

where

$$\Delta_1 = 2(1+\cosh\lambda_1\ell\cos\lambda_1\ell) \tag{27}$$

$$B_1 = -\frac{y_0}{\Delta_1}(\cosh\lambda_1\ell\sin\lambda_1\ell+\sinh\lambda_1\ell\cos\lambda_1\ell)+\frac{a_1\cos\theta+b_1\sin\theta}{EI\lambda_1{}^3\Delta_1\cos(\theta+\alpha)}(\cosh\lambda_1\ell+\cos\lambda_1\ell) \tag{28}$$

Equation(18) can be written as:

$$z = y_0(\cos\lambda_1 x-1)\cos\omega t+\{A_1(\cosh\lambda_1 x-\cosh\lambda_1 x)+B_1(\sinh\lambda_1 x-\sin\lambda_1 x)\}\cos\omega t \tag{29}$$

Substituting Eqs.(26) and (28), z is given as:

$$z = y_0 N_x\cos(\theta+\alpha)+\frac{1}{EI\lambda_1{}^3\Delta_1}\{(\cosh\lambda_1\ell+\cos\lambda_1\ell)(\sinh\lambda_1 x-\sin\lambda_1 x) \\ -(\sinh\lambda_1\ell+\sin\lambda_1\ell)(\cosh\lambda_1 x-\cos\lambda_1 x)\}(a_1\cos\theta+b_1\sin\theta) \tag{30}$$

where

$$N_x = \frac{\begin{array}{l}\{2(\cos\lambda_1 x-1)(1+\cosh\lambda_1\ell\cos\lambda_1\ell)\\ +(1+\cosh\lambda_1\ell\cos\lambda_1\ell+\sinh\lambda_1\ell\sin\lambda_1\ell)(\cosh\lambda_1 x-\cos\lambda_1 x)\\ -(\cosh\lambda_1\ell\sin\lambda_1\ell+\sinh\lambda_1\ell\cos\lambda_1\ell)(\sinh\lambda_1 x-\sin\lambda_1 x)\}\end{array}}{2(1+\cosh\lambda_1\ell\cos\lambda_1\ell)} \tag{31}$$

Displacement of beam end z_ℓ is

$$z_\ell = y_0 N_\ell\cos(\theta+\alpha)+M_1\frac{a_1}{k}\cos\theta+M_1\frac{b_1}{k}\sin\theta \tag{32}$$

where

$$\left.\begin{array}{l} N_\ell = \dfrac{(1-\cos\lambda_1\ell)(\cosh\lambda_1\ell-1)}{1+\cosh\lambda_1\ell\cos\lambda_1\ell} \\ M_1 = \dfrac{3\lambda_1\ell(\cos\lambda_1\ell\sinh\lambda_1\ell-\cosh\lambda_1\ell\sin\lambda_1\ell)}{Z_1{}^4\Omega_1{}^2(1+\cosh\lambda_1\cos\lambda_1\ell)}\end{array}\right\} \tag{33}$$

k is equivalent stiffness of the beam and given as:

$$k = \frac{3EI}{\ell^3} \tag{34}$$

Meanwhile the switching-over conditions from one to another of the eight intervals (I), (II), (III), (IV), (V), (VI), (VII) and (VIII) are expressed as:

$$\theta = -(\theta_1+\theta_2),\ z_\ell = e_0 \quad (\text{VIII}\rightarrow\text{I}) \tag{35}$$

$$\theta = -\theta_2,\ z_\ell = e_0+\delta_0 \quad (\text{I}\rightarrow\text{II}) \tag{36}$$

$$\theta = 0,\ \dot z_\ell = 0,\ z_\ell = z_{\ell\max} \quad (\text{II}\rightarrow\text{III}) \tag{37}$$

$$\theta = \theta_3,\ z = z_{\ell 3} \quad (\text{III}\rightarrow\text{IV}) \tag{38}$$

$$\theta = \pi-(\theta_1+\theta_2),\ z_\ell = -e_0 \quad (\text{IV}\rightarrow\text{V}) \tag{39}$$

$$\theta = \pi-\theta_2,\ z_\ell = -(e_0+\delta_0) \quad (\text{V}\rightarrow\text{VI}) \tag{40}$$

$$\theta = \pi,\ \dot z_\ell = 0,\ z_\ell = -z_{\ell\max} \quad (\text{VI}\rightarrow\text{VII}) \tag{41}$$

$$\theta = \pi+\theta_3,\ z = -z_{\ell 3} \quad (\text{VII}\rightarrow\text{VIII}) \tag{42}$$

Using Eqs.(32) and (37), z_ℓ is given as:

$$z_\ell = \Gamma \cos\theta \tag{43}$$

where

$$\Gamma = y_0 N_\ell \cos\alpha + \frac{a_1}{k} M_1 \tag{44}$$

And,

$$\sin\alpha = \frac{1}{y_0 N_\ell}\frac{b_1}{k} M_1 \tag{45}$$

$$\cos\alpha = \frac{1}{y_0 N_\ell}\left(\Gamma - \frac{a_1}{k} M_1\right) \tag{46}$$

And the nondimensional Fourier coefficients are defined as follows:

$$x_1 = \frac{a_1}{k\Gamma},\ y_1 = \frac{b_1}{k\Gamma} \tag{47}$$

From Eqs.(45), (46) and (47), the phase lag angle α is expressed as:

$$\alpha = \tan^{-1}\left(\frac{M_1 y_1}{1 - M_1 x_1}\right) \tag{48}$$

From Eqs.(35), (43), (45), (46) and (47), the amplitude of response at the beam end Γ and that of sinusoidal excitation y_0 is determined as:

$$\frac{\Gamma}{e_0} = \frac{y_0}{e_0}\frac{N_\ell}{\sqrt{(1 - M_1 x_1)^2 + (M_1 y_1)^2}} \tag{49}$$

$$\frac{y_0}{e_0} = \frac{\sqrt{(1 - M_1 x_1)^2 + (M_1 y_1)^2}}{N_\ell \cos(\theta_1 + \theta_2)} \tag{50}$$

Mode shape Z_x is obtained by using (6), (30), (34), (47) and (49) and substituting $\theta = 0$ as:

$$Z_x = \Gamma\left\{\sqrt{(1 - M_1 x_1)^2 + (M_1 y_1)^2}\,\frac{N_x}{N_\ell}\cos\alpha + \frac{3\lambda_1 \ell}{Z_1^4 \Omega_1^2}\frac{I_x}{\Delta_1} x_1\right\} \tag{51}$$

where

$$I_x = (\cosh\lambda_1\ell + \cos\lambda_1\ell)(\sinh\lambda_1 x - \sin\lambda_1 x) - (\sinh\lambda_1\ell + \sin\lambda_1\ell)(\cosh\lambda_1 x - \cos\lambda_1 x) \tag{52}$$

From Eq.(6)

$$\lambda_1^4 x^4 = Z_1^4 \Omega_1^2 x^4 / \ell^4 \tag{53}$$

Using Eqs.(46) and (49), Eq.(51) is written as:

$$Z_x = \Gamma\left\{\frac{N_x}{N_\ell}(1 - M_1 x_1) + \frac{3\lambda_1 \ell}{Z_1^4 \Omega_1^2}\frac{I_x}{\Delta_1} x_1\right\} \tag{54}$$

From Eqs.(13), (35), (36), (38) and (43),

$$\cos\theta_2 = \cos(\theta_1 + \theta_2) + \frac{\delta_0}{e_0}\cos(\theta_1 + \theta_2) \tag{55}$$

$$\cos\theta_3 = 1 - \frac{K_1}{K_3}\frac{\delta_0}{e_0}\cos(\theta_1 + \theta_2) - \frac{K_2}{K_3}(1 - \cos\theta_2) \tag{56}$$

Since $g(\theta)$ given by Eq.(22) is symmetrical, nondimensional $g(\theta)$ is expressed as follows using Eqs.(35), (36), (37), (38), (43) and (47).

$$\left.\begin{aligned}
&\frac{g(\theta)}{k\Gamma}=x_1\cos\theta+y_1\sin\theta=\frac{K_1}{k}\{\cos\theta-\cos(\theta_1+\theta_2)\} \quad ;-(\theta_1+\theta_2)\le\theta\le-\theta_2\\
&\frac{g(\theta)}{k\Gamma}=x_1\cos\theta+y_1\sin\theta=\frac{K_1}{k}\{\cos\theta_2-\cos(\theta_1+\theta_2)\}+\frac{K_2}{k}(\cos\theta-\cos\theta_2) \quad ;-\theta_2\le\theta\le 0\\
&\frac{g(\theta)}{k\Gamma}=x_1\cos\theta+y_1\sin\theta=\frac{K_3}{k}(\cos\theta-\cos\theta_3) \quad ;0\le\theta\le\theta_3\\
&\frac{g(\theta)}{k\Gamma}=x_1\cos\theta+y_1\sin\theta=0 \quad ;\theta_3\le\theta\le\pi-(\theta_1+\theta_2)
\end{aligned}\right\} \tag{57}$$

Applying a technique similar to that for determining Fourier coefficients, namely, multiplying both sides of Eq.(57) by cosθ and sinθ integrating through the whole period of 2π, the nondimensional coefficients x_1 and y_1 are obtained as follows:

$$x_1=\frac{2}{\pi}\left[\frac{K_1}{k}\left\{\frac{\theta_1+\sin\theta_2\cos\theta_2-\sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)}{2}\right\}+\frac{K_2}{k}\left(\frac{\theta_2-\sin\theta_2\cos\theta_2}{2}\right)+\frac{K_3}{k}\left(\frac{\theta_3-\sin\theta_3\cos\theta_3}{2}\right)\right] \tag{58}$$

$$y_1=\frac{2}{\pi}\left[\frac{K_1}{k}\left\{\frac{\cos^2\theta_2-\cos^2(\theta_1+\theta_2)}{2}-\cos\theta_2+\cos(\theta_1+\theta_2)\right\}+\frac{K_2}{k}\left(\frac{1}{2}\sin^2\theta_2-1+\cos\theta_2\right)-\frac{K_3}{k}\left(\frac{1}{2}\sin^2\theta_3-1+\cos\theta_3\right)\right] \tag{59}$$

ANALYTICAL METHOD FOR THE SYSTEM WITH TRIANGULAR HYSTERESIS LOOP CHARACTERISTICS

For the system with triangular hysteresis loop characteristics as shown in Fig.5, intervals (I) and (V) do not exist, that is, $\theta_1=0$, $K_1=0$. In this case, the coefficient of restitution is not dependent of velocity response at the beam end and give as:

$$\varepsilon=\sqrt{K_2/K_3} \tag{60}$$

From Eq.(56),

$$\cos\theta_3=1-\frac{K_2}{K_3}(1-\cos\theta_2) \tag{61}$$

From Eqs.(58) and (59),

$$x_1=\frac{2}{\pi}\left\{\frac{K_2}{k}\left(\frac{\theta_2-\sin\theta_2\cos\theta_2}{2}\right)+\frac{K_3}{k}\left(\frac{\theta_3-\sin\theta_3\cos\theta_3}{2}\right)\right\} \tag{62}$$

$$y_1=\frac{2}{\pi}\left\{\frac{K_2}{k}\left(\frac{1}{2}\sin^2\theta_2-1+\cos\theta_2\right)-\frac{K_3}{k}\left(\frac{1}{2}\sin^2\theta_3-1+\cos\theta_3\right)\right\} \tag{63}$$

NUMERICAL EXAMPLES

Figures 6,7 and 8 show the resonance curve of the system with quadrilateral hysteresis loop characteristics with the amplitude Γ/e_0 of the beam and versus frequency ratio Ω_1 for excitation ratio $y_0/e_0=1.0$. Fig.6 shows the resonance curve for very low ratio of nonlnear parameters $K_1/k=0.04, K_2/k=0.02, K_3/k=0.1$. The resonance curve is almost same as that of the system without collision. Figure 7 and Figure 8 show the resonance curves for low ratio of nonliear parameters $K_1/k=4, K_2/k=2, K_3/k=10$ and high ratio of nonlinear parameters $K_1/k=4, K_2/k=2, K_3/k=10$, respectively. In Fig.7, first peak is not clear. In fig.8, firs peak and second peak is not clear. Figures 9 and 10 show the resonance curve of the system with triangulrar hysteresis loop characteristics for excitation ratio $y_0/e_0=1.0$. Figure 7 and Figure 8 are the resonance curves for low ratio of nonliear parameters $K_1/k=3, K_2/k=10$ and high ratio of nonlinear parameters $K_1/k=30, K_2/k=100$, respectively. Comparing Figs.7 and 8 with Figs.9 and 10, the resonsnce curves of the system with quadrilateral hysteresis loop charactersitics are almost same as those with triangular hysteresis loop characteristics. Fig.11 shows mode shapes of the system with quadrilateral hysteresis loop charactersitics.

CONCLUSIONS

An analytical method for approximate solution of steady-state response of a clamped beam, one span of the the piping system, with symmetrical collision characteristics at the other end is presented. The stop is assumed to have hysteresis loop characteristics in order to consider the energy loss in a collision and duration of collision. For hysteresis loop characteristics, triangular and quadrilateral characteristics are introduced. The resonance curves of the beam with various support having hysteresis loop characteristics are obtained by the method of approximate solution and are compared with those of the results for various nonlinear boundary conditions.

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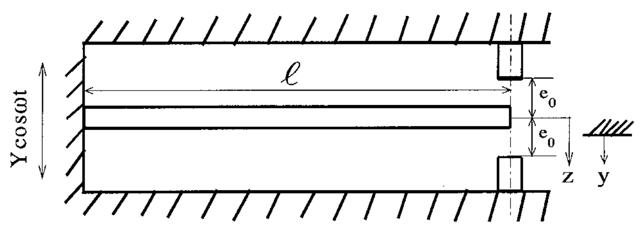

Fig.1 Analytical model of cantilever with collision

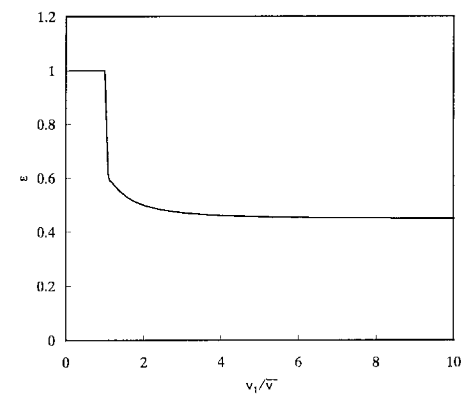

Fig.3 Relation between coefficient of restitution and velocity response at beam end.
$(K_1/k = 4, K_2/k = 2, K_3/k = 10)$

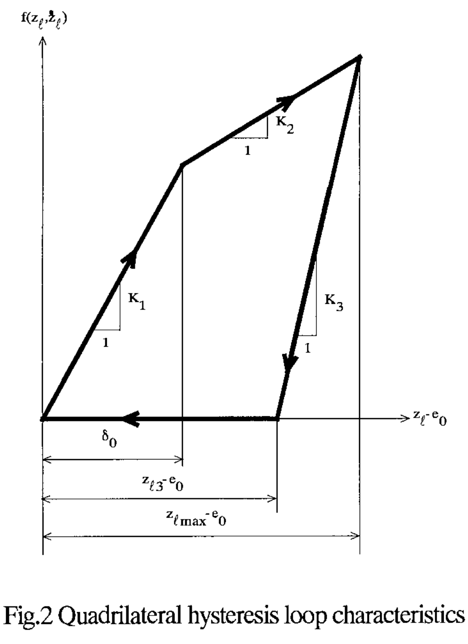

Fig.2 Quadrilateral hysteresis loop characteristics

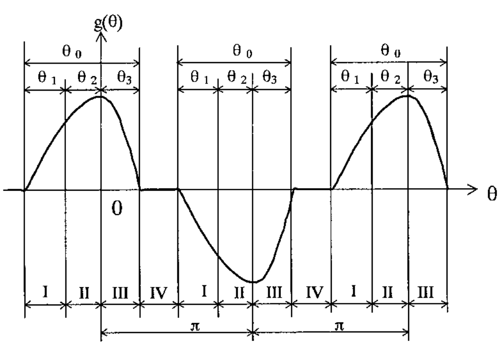

Fig.4 Waveform of force of restitution at beam end

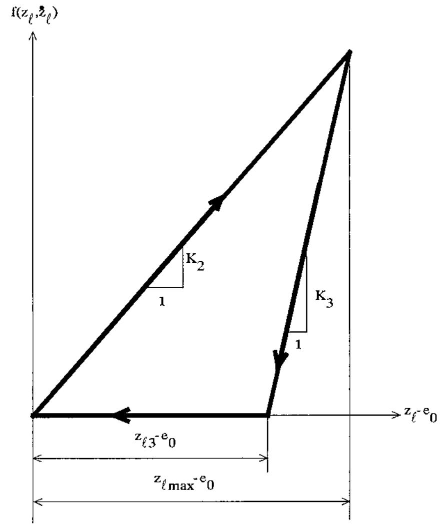

Fig.5 Triangular hysteresis loop characteristics

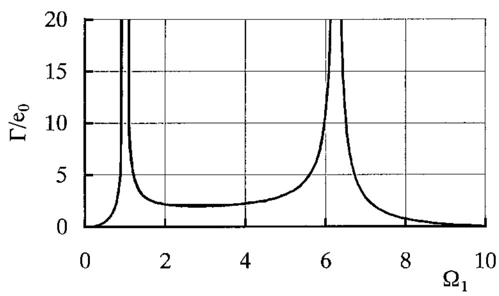

Fig.6 Resonance curve of cantilever with quadrilateral hysteresis loop characteristics
$\left(K_1/k = 0.04, K_2/k = 0.02, K_3/k = 0.1, y_0/e_0 = 0.1\right)$

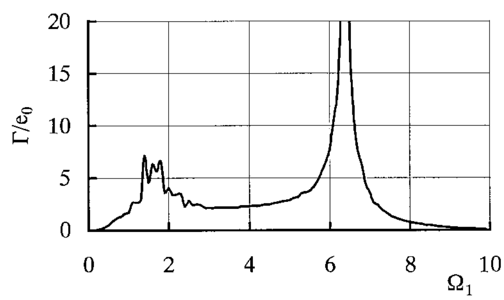

Fig.7 Resonance curve of cantilever with quadrilateral hysteresis loop characteristics
$\left(K_1/k = 4, K_2/k = 2, K_3/k = 10, y_0/e_0 = 1.0\right)$

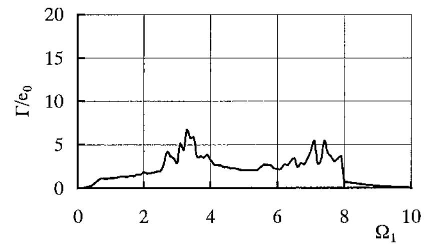

Fig.8 Resonance curve of cantilever with quadrilaeral hysteresis loop characteristics
$\left(K_1/k = 40, K_2/k = 20, K_3/k = 100, y_0/e_0 = 0.1\right)$

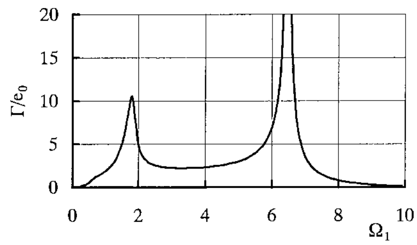

Fig.9 Resonance curve of cantilever with triangular hysteresis loop characteristics
$\left(K_1/k = 3, K_2/k = 10, y_0/e_0 = 0.1\right)$

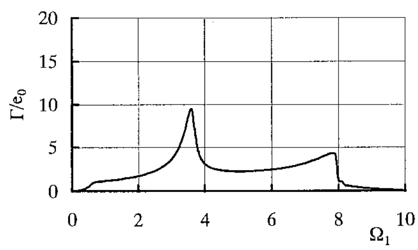

Fig.10 Resonance curve of cantilever with triangular hysteresis loop characteristics
$\left(K_1/k = 30, K_2/k = 100, y_0/e_0 = 0.1\right)$

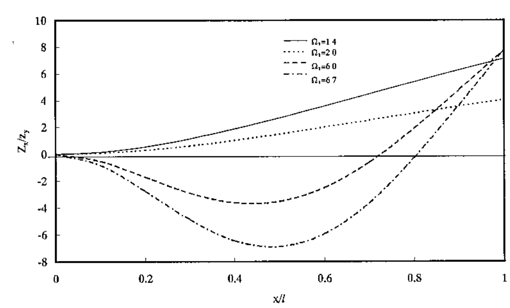

Fig.11 Mode shapes of cantilever with quadrilateral hysteresis loop characteristics
$\left(K_1/k = 4, K_2/k = 2, K_3/k = 10, y_0/e_0 = 1.0\right)$