ON THE ADEQUACY OF THE STRUCTURAL MODEL OF ELASTO-VISCO-PLASTIC MEDIUM UNDER VARIABLE REPEATED NON-ISOTHERMAL LOADING

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SUMMARY

In the previous paper (see SMiRT-3 Transactions, Paper L 5/7) a version of the mathematical ("structural") model of elasto-visco-plastic medium has been developed. According to it the infinitely small element of body operates as an assemblage of a number of subelements having equal total strains and temperature but different values of the rheological function parameters (similarity coefficients). The convenience of this model for engineering application has been shown.

Some work has been done to verify the adequacy of the model under different operating conditions. The presented paper concerns some results of theoretical and experimental investigations at proportional variable repeated non-isothermal loading.

The generalization of the Masing's principle under such conditions is obtained. The stress-strain curves are determined by both the temperature and strain history and may be described by two one-dimensional functions in accordance with some rules (the part of pre-history being forgotten).

The time-dependent behaviour of the model was investigated under non-symmetric loading condition. The well-known effects of cyclic creep (at stress controled cycle) and cyclic relaxation (strain control) are reflected. The values of cycle parameters (mean stress and amplitude) at which the strain grows unlimitedly may be predicted. The proposed model describes also some peculiarities of cyclic creep and relaxation behaviour connected with the previous deformation history. It is shown that under definite conditions the ordinary direction of the loop translation may be changed to the opposite. Under non-isothermal loading these processes are controled by the temperature values at the moments when the stresses are extreme.

The experimental investigations (the torsion of thin-walled tube specimens) for several structural alloys were carried out. They included the isothermal tests for the model parameters determination and the checking the adequacy of the model at various programs of isothermal and non-isothermal loading. All the experimental data obtained are in sufficiently good correspondence (both qualitative and quantitative) with the behaviour predicted by the model.

1. The constitutive Equations

According to the structural model [1] the behaviour of a body element is described by the following equations:

$$\varepsilon = r + p = \tilde{\varepsilon} = \tilde{r} + \tilde{p}, \quad T = \tilde{T}, \quad E = \tilde{E}, \tag{1}$$

$$\tilde{\sigma} = E\tilde{r}, \tag{2}$$

$$\frac{d\tilde{p}}{dt} = \dot{\tilde{p}} = \Phi(\tilde{\sigma}/\tilde{z}, T), \tag{3}$$

$$\sigma = \langle\tilde{\sigma}\rangle, \quad r = \langle\tilde{r}\rangle, \quad p = \langle\tilde{p}\rangle, \quad \dot{p} = \langle\dot{\tilde{p}}\rangle \tag{4}$$

where ε, r, p are the total, elastic and plastic strains, σ the stress, E the elastic modulus, T the temperature, t the time. The parameters of subelements are marked by tilde. The parameters of a body element are defined as the average of subelement parameters (eq. (4)). The micro-non-homogeneity of the real medium is reflected by the difference of the values of $\tilde{z}$ which is determined by the distribution function $y(\tilde{z})$. The value of $y(\tilde{z}^*)d\tilde{z}$ is the relative quantity of subelements the parameters $\tilde{z}$ of which are in the interval $[\tilde{z}^*; \tilde{z}^* + d\tilde{z}]$. The conditions

$$\int_0^\infty y(\tilde{z})\,d\tilde{z} = \int_0^\infty \tilde{z}\,y(\tilde{z})\,d\tilde{z} = 1 \tag{5}$$

are satisfied. Any parameter average is defined by employing the distribution function $y(\tilde{z})$, e.g.

$$\sigma = \langle\tilde{\sigma}\rangle = \int_0^\infty \tilde{\sigma}(\tilde{z})y(\tilde{z})\,d\tilde{z}. \tag{6}$$

The function Φ coincides with the dependence of the steady creep rate on stress and temperature

$$\dot{\varepsilon}_{st} = \Phi(\sigma, T). \tag{7}$$

As the experiments show the isothermal sections of the surface (7) may be schematically represented by means of a curve in Fig.1. Parameters σ_c and σ_s are some functions of temperature.

If loading corresponds to condition $\dot{\varepsilon} \gg \Phi_s$ (further this process is referred to as a "quick" loading) the subelements are practically perfectly plastic having the yield stresses $\tilde{\sigma}_y = \sigma_s\tilde{z}$. If the strain rate is infinitely small ($\dot{\varepsilon} \approx 0$, "slow" loading) the subelements are also perfectly plastic but their yield stresses are $\tilde{\sigma}_c = \sigma_c\tilde{z}$. Under an increasing load in accordance with eqs. (5), (6)

if $\dot{\varepsilon} \gg \Phi_s$, $t \to \infty$, $\varepsilon \to \infty$, $\tilde{\sigma} \to \sigma_s\tilde{z}$, $\sigma \to \langle\sigma_s\tilde{z}\rangle = \sigma_s$;

if $\dot{\varepsilon} \approx 0$, $t \to \infty$, $\varepsilon \to \infty$, $\tilde{\sigma} \to \sigma_c\tilde{z}$, $\sigma \to \langle\sigma_c\tilde{z}\rangle = \sigma_c$.

These expressions reveal the meaning of the parameters σ_c and σ_s . The temperature dependence of subelements yield stress for quick non-isothermal loading can be written as

$$\tilde{\sigma}_y(T) = E(T)\tilde{r}_y(T) = E(T)\,\theta(T)\,\tilde{r}_y(T_0), \tag{8}$$

where $$\theta(T) = \frac{\tilde{r}_y(T)}{\tilde{r}_y(T_0)} = \frac{\sigma_s(T)\,E(T_0)}{\sigma_s(T_0)\,E(T)}, \quad \theta(T_0) = 1, \tag{9}$$

T_0 is some temperature chosen as basic.

2. Non-isothermal loading analysis

For any history of deformation the stress σ can be found from eq. (6) if the distribution $\tilde{\sigma}(\tilde{z})$ is known. For the non-isothermal loading it is convenient to use the distribution $\tilde{r}(\tilde{z}) = \tilde{\sigma}(\tilde{z})/E(T)$. Then

$$\sigma = E(T)\,r, \quad r = \int_0^\infty \tilde{r}(\tilde{z})\,y(\tilde{z})\,d\tilde{z}. \tag{10}$$

In the space $\tilde{r} - y - \tilde{z}$ the value of elastic strain (10) can be represented as the volume V of a body limited by surfaces $\tilde{r} = 0$, $y = 0$, $\tilde{r} = \tilde{r}(\tilde{z})$, $y = y(\tilde{z})$. For example, under isothermal loading from the initial state ($\tilde{r} = 0$) the distribution $\tilde{r}(\tilde{z})$ is determined by expressions (see Fig.2):

$$\begin{aligned} \tilde{r}(\tilde{z}) &= \tilde{r}_y = \theta(T)\,r_0\,\tilde{z} \qquad \text{for} \quad \tilde{z} \leqslant \frac{\varepsilon}{r_0\,\theta(T)}, \\ \tilde{r}(\tilde{z}) &= \varepsilon \qquad \text{for} \quad \tilde{z} \geqslant \frac{\varepsilon}{r_0\,\theta(T)}, \\ r_0 &= \frac{\sigma_s(T_0)}{E(T_0)}. \end{aligned} \tag{11}$$

For any values ε, T eq. (11) and then eq. (10) allow to calculate the quantity r (if the function $y(\tilde{z})$ is known). On the contrary, having the curve

$$T = T_0, \qquad r = F(\varepsilon) \tag{12}$$

or the values of volume V for each ε (when $\theta = 1$, curves $\tilde{r}(\tilde{z})$ are of type $OA'B$, Fig.3), one can find such volumes for other similar curves $\tilde{r}(\tilde{z})$. For instance, when $T^* \neq T_0$ (OCB, Fig.3) $ED = CD/\theta(T^*)$, $V_{OCB} = \theta(T^*)V_{OEF}$, hence

$$r^* = m\,F(\varepsilon^*/m), \quad \sigma^* = E(T^*)\,r^*, \tag{13}$$

where

$$m = \theta(T^*). \tag{14}$$

Eqs. (13), (14) describe the stress-strain-temperature surface for active ($dp \geqslant 0$) non-isothermal loading. The isothermal sections $r(\varepsilon)$ of it are centeric similar curves. The similarity coefficients are $\theta(T)$. A series of experimental curves $r(\varepsilon)$ at different temperatures can be used for function $\theta(T)$ determination.

After loading reverse in state s^*, T^* part of subelements behave elastically ($\Delta r = \Delta \varepsilon$, GHI , Fig.3) and others (OG) reach their limit stress ($\tilde{r} = -\tilde{r}_y = -r_0\,\theta(T)\,\tilde{z}$). The elastic strain increment curve is OKL, $KM = [\theta(T^*) + \theta(T)]\,NM$, the corresponding value V_{OKL} is $-\frac{KM}{MN} V_{ONP}$ and

$$r - r^* = m\,F\left(\frac{\varepsilon - \varepsilon^*}{m}\right), \tag{15}$$

where

$$m = \theta(T^*) + \theta(T). \tag{16}$$

Eqs. (15), (16) represent the generalization of Masing's principle for non-isothermal loading.

For the description of stress-strain diagrams under arbitrary non-isothermal loading the rules of "forgetting" (or fixing in memory) pre-history must be determined. They can be obtained from the analysis of the $\tilde{r}(\tilde{z})$ curves kinetics and are reduced to the following rules.

Since the moment of the n-th loading reverse the stress-strain diagram is described by eqs. (15), (16) with $r^* = r_n$, $\varepsilon^* = \varepsilon_n$, $T^* = T_n$, where r_n, ε_n, T_n are the values of corresponding parameters at the moment of n-th reverse. They are fixed and kept in the "memory" until one of three states is realised:

a. Another ($n+1$) reverse (a new point of reverse is added to the memory).

b. The "conventional" strain $e = \frac{\varepsilon}{\theta}$ reaches its quantity at the moment of the last but one reverse ($n-1$) kept in the memory. The reverse points n and $n-1$ in the memory are forgotten, the behaviour of media is such as if a part of pre-history $e_{n-1} \to e_n \to e_{n-1}$ did not take place.

c. The value $/e/$ reaches the quantity $/e/_{max}$ kept in the memory. The whole pre-history is forgotten and eqs. (13), (14) are used.

3. Cyclic creep and relaxation

Strictly speaking, the model behaviour peculiarities considered above are correct for a limited number of loading cycles only. With the growth of the cycles number the influence of creep (reflected by the part of the rheologic function Φ in the interval of stress $\sigma_c - \sigma_s$, Fig.1) will be revealed. To begin with, consider the corresponding features of the model behaviour under constant temperature.

If after quick initial loading (curves OAB, Fig.4 and O-1 Fig.5) strain is kept up constant $\varepsilon = \varepsilon_1$ in the part of subelements (for which $\tilde{r} > \tilde{r}_c = \sigma_c / E$) the stress relaxation occurs and $\tilde{r}$ reaches the quantity $\tilde{r}_c$ (asymptotically). The corresponding curve $\tilde{r}(\tilde{z})$ is represented by the line OIB . The same distribution of elastic strain can be also obtained under the loading with strain rate $\dot{\varepsilon} \approx 0$. The volume V_{OIB} is equal to $1/\text{æ}\, V_{OLM}$ where

$$\text{æ} = \frac{\tilde{r}_c}{\tilde{r}_s} = \frac{r_c}{r_s} = \frac{\sigma_c}{\sigma_s}, \tag{17}$$

and the corresponding curve $r(\varepsilon)$ may be expressed as

$$\dot{\varepsilon} \approx 0, \quad r = F_0(\varepsilon) = æ\, F(\varepsilon/æ) \tag{18}$$

(line OB , Fig.5) While keeping up $\varepsilon=\varepsilon_1$, the state point on the plane $r\sim\varepsilon$ displaces from the curve F to F_0 (section $1-1'$. Fig.5). If the hold-time is realized under the constant stress $\sigma_1 = E r_1$ the relaxation of subelements stresses leads to increasing deformation (creep) and the state point displaces to the curve (18) in position $1''$ (Fig.5). In the case of $\sigma_1 > \sigma_c$ the creep is unlimited.

Under quick strain-cycle loading $\varepsilon_1 \rightleftarrows \varepsilon_2$ (hereinafter the indexes 1 and 2 denote the values of material parameters at the loading cycle reverse moments) for the initial cycles the elastic strain distribution curve is displaced from OAB to $OCDE$ (Fig.4) and the loop 1-2 (Fig.5) is described. In a process of cyclic loading the maximum stress relaxation occurs step-by-step. The curve $\tilde{r}(\tilde{z})$ displaces asymptotically to the lines $OFGIB$ and $OCHKE$. This leads to the mean stress relaxation (the loop $1'2'$ in a Fig.5).

Under stress-cycle loading $\sigma_1 \rightleftarrows \sigma_2$ $(\sigma_1 > |\sigma_2|)$ the maximum stress relaxation in the "weak" subelements leads to the cyclic creep (the loop $1''-2''$).

The displacement of histeresis loop down to the curve OB (eq.(18)) occurs if the strain amplitude $\varepsilon_a = \frac{\varepsilon_1-\varepsilon_2}{2}$ is small enough (the volume OFG is neglected). Otherwise the limit loop displacement is determined by the line CG (Fig.6).

$$\bar{F}_0(\varepsilon) = F_0(\varepsilon) + F(\varepsilon_a) - F_0(\varepsilon_a). \tag{19}$$

The line CG is equidistant to DB .

4. The influence of prior history

One can show that under certain conditions the ordinary direction of the cyclic creep and relaxation may be reversed. For example, consider the following loading program: the loading up to some stress level (line OAB, Fig.7), the hold-time at a constant stress up to the strain ε^* (line OCD) and then the strain-cycle loading $\varepsilon_1 \rightleftarrows \varepsilon_2$. In the initial cycles the elastic strain distribution displaces from the line $OEFG$ to the line $OHKLM$. Because of the stress relaxation in the part of subelements the distribution $\tilde{r}(\tilde{z})$ is changed and asymptotically reaches the lines $OTRSFG$ and $OHNPLM$. The shaded areas correspond to the mean stress increment $\Delta\sigma$. The values of ε_1 and ε_2 may be found when $|\sigma_1|<|\sigma_2|$ and cyclic relaxation leads to the mean stress increase. Similarly in the stress-cycle after holdtime the cyclic creep at a positive mean stress may occur in the negative direction.

5. Cyclic creep and relaxation under non-isothermal loading

To generalize the results obtained for the non-isothermal cyclic loading the function $æ = æ(T)$ is used. In accordance with eqs. (15),(16) the main parameters of asymmetric cycle are the following:

$$r_m = \frac{r_1\,\theta(T_2) + r_2\,\theta(T_1)}{\theta(T_1) + \theta(T_2)} , \tag{20}$$

$$r_{ai} = r_i - r_m , \qquad i = 1,2 . \tag{21}$$

These parameters generalize the terms of the mean value r_m and the amplitude for the non-isothermal loading. The sign of the value r_m determines the direction of cyclic creep (or relaxation). The value of limit accumulated strain $\Delta\varepsilon$ (or elastic strain decrease Δr) can be found by applying the scheme in Fig.6. The values of index i in scheme are defined by the following requirement

$$sign(r_a) = sign(r_m). \tag{22}$$

The curves OA , OB are to be described by eq. (13) where $m = \theta(T_i)$ and $m = æ(T_i)\theta(T_i)$ respectively. Under stress-cycle $\{r_1, T_1\} \rightleftarrows \{r_2, T_2\}$ the strain accumulation is unilimited when the following condition is satisfied

$$|r_i| > æ(T_i)r_s(T_i) + \theta(T_i)F\left(\frac{\Delta\varepsilon}{\theta_1+\theta_2}\right) - æ(T_i)\theta(T_i)F\left(\frac{\Delta\varepsilon}{æ(T_i)(\theta(T_1)+\theta(T_2))}\right) \tag{23}$$

The cyclic strain range $\Delta\varepsilon$ is defined by eq.(15) where $m = \theta(T_1) + \theta(T_2)$. This condition corresponds to the schaded area on the plane $\sigma_a \sim \sigma_m$ (Fig.8) where $\sigma_a = \frac{1}{2}(\sigma_1 - \sigma_2)$, $\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_2)$.

It is adopted that $\sigma_1 \geqslant \sigma_2$. The line OB corresponds to the "symmetrical" cyclic stressing ($r_m = 0$). The lines DB and BE are obtained by employing the functions (9), (12), (17).

6. Comparison of experimental results and theoretical predictions

To verify the structural model adequacy the experimental investigations with the Fe-Ni alloy were carried out. At the first stage the functions (9), (12), (17) were determined. A set of isothermal stress-strain diagrams in the range 200°C–700°C was used to obtain the functions F and $\theta(T)$. Determination of the function $æ(T)$ was realized with the help of isothermal relaxation test series. The curves obtained are shown in Figs.9 and 10.

It should be noted that the considered model concerns materials stable under cyclic loading. Therefore before parameters determination tests the cyclic loading was carried out till the hysteresis loop stabilization. To eliminate anisotropic hardening the strain amplitude was gradually decreased after loop stabilisation.

The non-isothermal test is represented by the stress-strain diagram (Fig.11) obtained under strain-cyclic loading. The temperature was changed synchronously with the strain in the range 300–700°C. The line 1 corresponds to the first cycle. During the cyclic loading the displacement of hysteresis loop was observed. The line 2 corresponds to the stabilized state. The dashed lines 1, 2 represent curves obtained by employing the eqs. (17),(18) and the scheme (Fig.6) for evaluating Δr. The effect of the mean value r_m on the cyclic creep direction under non-isothermal loading is represented in Fig.12. The temperature was changed synchronously with strain in the range 350 ÷ 600°C. The diagram (Fig.12a) was obtained for $r_m = 10^{-4} \approx 0$ and accumulation of inelastic strain was not observed. In the cases (b) and (c) the value $r_m = -4.7 \cdot 10^{-4}$ and $r_m = 4.2 \cdot 10^{-4}$ respectively. The loop displacement direction coincides with the sign of r_m. The limit value of $\Delta\varepsilon = -0{,}12\%$ for the case (b) and $\Delta\varepsilon = 0{,}155\%$ for the case (c). The calculated values are -0,12% and 0,12% respectively.

The effect of the prior history on the cyclic creep is illustrated by the test represented in Fig.13. After creep under $\sigma = 169\ MPa$ the stress-cyclic loading was carried out in the interval 169 ⇄ - 130 MPa. During the cyclic loading the loop displacement was observed in the direction opposite to the creep. Inelastic strain accumulated in the course of cyclic creep came to 0,08%, the estimated value was 0,0864%.

Thus, the results of the experimental investigations reported in this paper have shown that the offered structural model is capable to reflect (both qualitatively and quantitatively) the main peculiarities of the behaviour of medium under complex conditions of non-isothermal repeated loading. The method of the model parameters experimental determination and the obtained equations describing the behaviour of medium are sufficiently simple. It permits to apply the proposed approach for engineering calculations of structures subjected to variable or cyclic load and temperature affects.

References

[1] Gokhfeld,D.A. and Sadakov,O.S.,"A mathematical model of medium for analysing the inelastic deforming of structures subjected to repeated actions of load and temperature", Trans. of 3rd Intl.Conf. on SMiRT, London, U.K., 1975, Paper L 5/7.

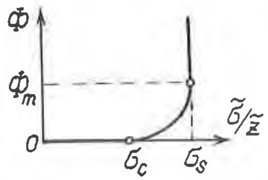

Fig. 1

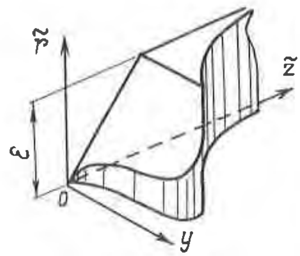

Fig. 2

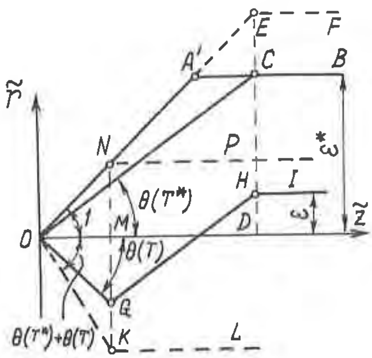

Fig. 3

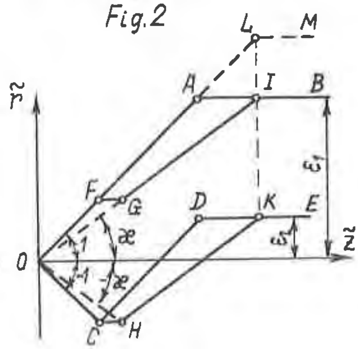

Fig. 4

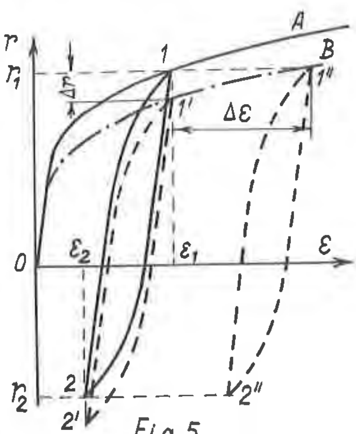

Fig. 5

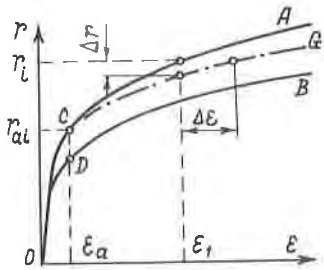

Fig. 6

Fig. 1. Adopted dependence of steady creep rate on stress

Fig. 2. The curve $\tilde{r}(\tilde{z})$ and a body corresponding to the value of elastic strain for initial loading

Fig. 3. The curves $\tilde{r}(\tilde{z})$ under non-isothermal repeated loading. The angles (α) are marked by values of $\frac{1}{r_0} tg\alpha$

Fig. 4. The curves $\tilde{r}(\tilde{z})$ under multiple cyclic loading

Fig. 5. The cyclic creep and relaxation effects

Fig. 6. The scheme for determination of the loop displacement limit value

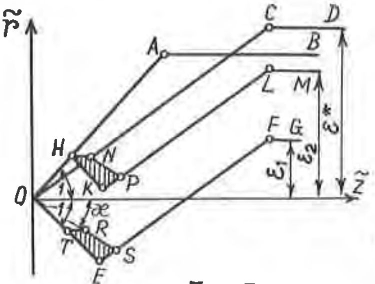

Fig. 7

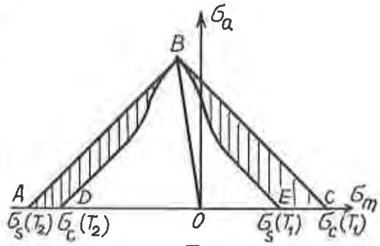

Fig. 8

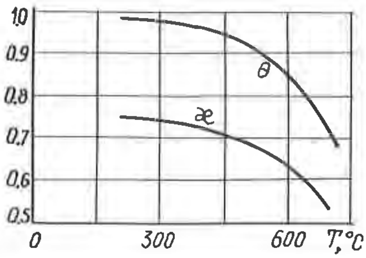

Fig. 9

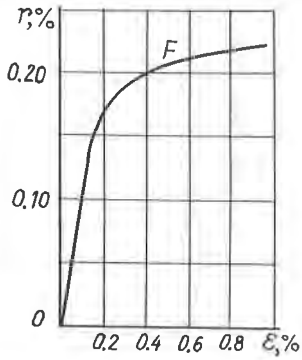

Fig. 10

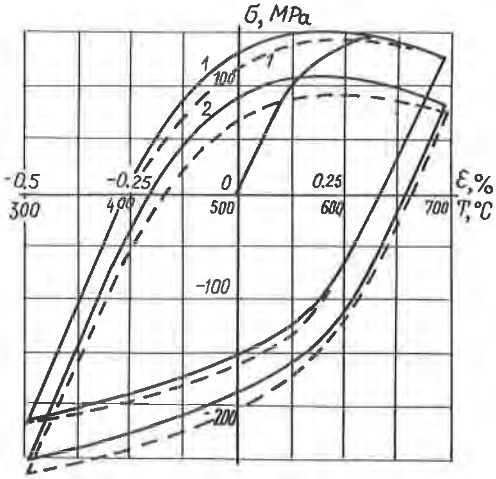

Fig. 11

Fig.7. The curves $\tilde{r}(\tilde{z})$ under complex loading history: OAB the initial loading OCD – the creep up to the strain $\varepsilon=\varepsilon^*$, $OEFG$, $OHKLM$ – the first cycles $\varepsilon_1 \rightleftarrows \varepsilon_2$, $OTRSFG$, $OHNPLM$ – the final cycles $\varepsilon_1 \rightleftarrows \varepsilon_2$

Fig.8. The mean stress – amplitude diagram for determination of a loop displacement limit value under non-isothermal cyclic creep

Fig.9. The functions $\theta(T)$, $æ(T)$ for the $Fe-Ni$ alloy estimated in the tests

Fig.10. The sceleton curve $F(\varepsilon)$ at the basic temperature $T = 100\,°C$

Fig.11. The stress-strain diagram under non-isothermal cyclic loading. The calculated curves are marked by dushed lines

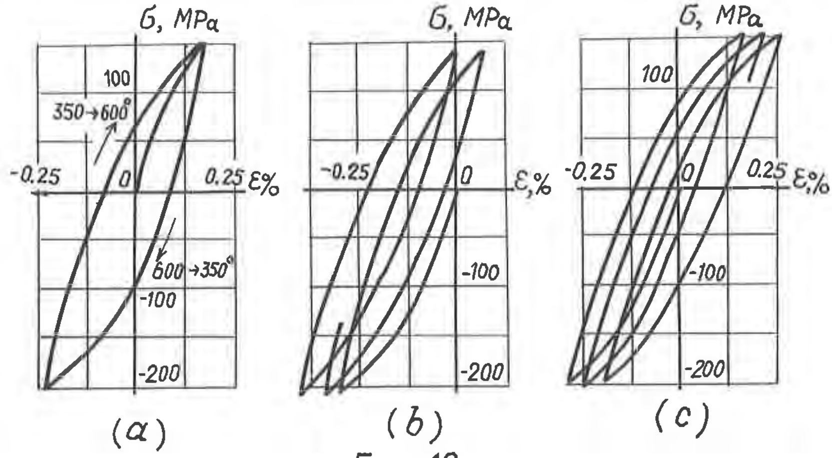

Fig. 12

Fig.12.The non-isothermal stressing under different value of mean elastic strain: (a) $r_m = 0$, (b) $r_m = -4.7 \cdot 10^{-4}$, (c) $r_m = 4.2 \cdot 10^{-4}$

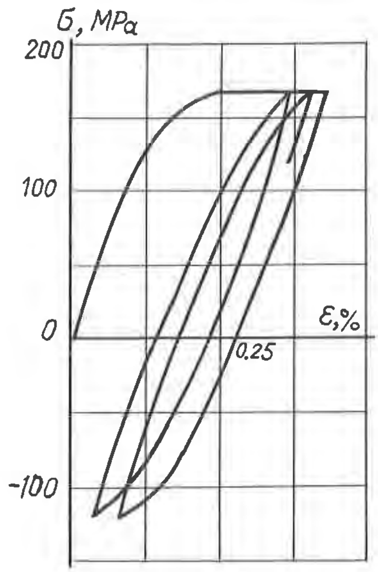

Fig. 13

Fig.13.The stress-cycle loading after the hold-time, $T = 600\,°C$